Optimal design of flexible mechanisms using the Equivalent Static Load method and a Lie group formalism.

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Abstract

Historically, the optimization of mechanisms started with the selection of several configurations whereupon structural optimization was performed based on representative loading conditions coming from the designer experience for each posture [8]. This approach is doubtful since a few configurations can hardly represent the overall motion and the optimal design entirely depends on the designer’s choices.

With the evolution of multibody system (MBS) analysis, Bruns and Tortorelli [4] proposed an approach combining rigid MBS analysis and optimization techniques to design optimal components. The optimization procedure was performed with load cases evaluated during the MBS analysis. They illustrated their method on the design of a slider-crank mechanism loaded with the maximum tensile force calculated during the simulation.

Considering the time-dependent loading conditions coming from the MBS analysis, the optimization problem is rather complex and a lot of research has been conducted to remove this time dependency. For instance, Oral and Kemal Ider [7] investigated the representation of the constraints either by the most critical constraint or summarized with a Kresselmeier-Steinhauser function. An important breakthrough has been made by Kang, Park and Arora [6] who proposed a method to define Equivalent Static Load (ESL) to optimize flexible mechanisms. For each time step and for each component to design, they define an ESL which produces the same displacement field as the one generated by the dynamic load at the considered time step. However, even if the concept is totally general, it has been developed for MBS based on a floating frame formalism. This formalism separates the elastic coordinates from the coordinates describing the global motion of the bodies which enables to define the ESL for each optimized component by simply isolating some parts of the equations of motion. Indeed, this formalism gives the component deformation with respect to its material (body-attached) frame whereupon the ESL definition is unambiguous.

In the last years, finite element analysis and MBS simulation have been merged into a unified nonlinear finite element approach which can account for the full flexibility of the different components and enables the analysis of mechanism deformation undergoing large and fast joint motions [5]. Applying the ESL method to MBS modeled by the nonlinear finite element method is not straightforward since the decoupling characteristics given by the floating frame formalism are not present. To recreate artificially these characteristics, we propose to introduce a post-processing step to define the ESL without perturbing the analysis itself [9].

In this research, we investigate the definition of ESL for flexible MBS formulated on a Lie group. The Lie group theory offers several advantages for the finite element analysis of systems with large rotations variables [1, 2]. Firstly, the equations of motion are derived and solved directly on the nonlinear manifold, without an explicit parameterization of the rotation variables, which leads to important simplifications in the formulations and algorithms. Secondly, displacements and rotations are represented as increments with respect to the previous configuration, and those increments can be expressed in the material frame. Therefore, geometric nonlinearities are automatically filtered from the relationship between incremental displacements and elastic forces, which strongly reduces the fluctuations of the iteration matrix during the simulation [3].

In this work, we first present the evaluation of ESL for a MBS formulated on a Lie group. As mentioned previously, the definition of material frames is embedded in the Lie group formalism wherein component strain is computed. Thus, ESL can be properly defined. Then, a comparative study of three optimization methods is presented. We compare the optimization method considering the time dependent constraint, the method considering the ESL using a “classical” nonlinear finite element formalism and
the one considering the ESL using a Lie group finite element formalism. The study is performed on the mass minimization of a 2-dof robot subjected to a tracking trajectory constraint inspired from Ref. [6]. Figure 1 illustrates the kinematic model of the robot.

![Kinematic model of the 2-dof robot.](image)

**Figure 1: Kinematic model of the 2-dof robot.**

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**References**


