Attribution des cours à option, modèle avec préférences numériques

Elective Course Allocation with numerical preferences

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Plan

- Elective course allocation problem
- Mathematical formulation
- First 2-year results
- Emerging strategies, problem classification and methods
- Further work

Elective course allocation problem

Students must take a number of courses to complete their curriculum.

The classical Assignment Problem:

- each student has preferences on bundle of courses
- each course accepts a maximum number of students (seats)
- no use of monetary transfer (artificial currency is allowed)

Additionnal characteristics :

- students have preferences on individual courses
- courses schedules are predetermined
- students availability is known

Elective course allocation problem

Set S of N students s_i
Set C of M courses c_j with integral capacities q = (q_1 , ..., q_M).

Each student s_i has a set of permissible schedules Ψ_i and a utility function u_i

Non-permissible schedules have utility of zero

No peer effects on the utility function

Each student i has a preference function: $c_j \succ_i c_k$

A solution is an allocation x of courses to students : x_{ij} = 1 if course j is allocated to student i

A solution is characterized with stability, efficiency and fairness

Elective course allocation problem

Constraints

Taking a number of elective courses is mandatory to fulfill the curriculum.

Courses:

- each course has a weight proportional to its workload (2 or 3 ECTS)
- courses schedule is known (1rst, 2nd semester or both)
- each course can be offered multiple times (occurences) according to different schedules
- each course has a minimum and a maximum capacity (seats)

Each student

- has 2 years to achieve a given number of ECTS (according to their curriculum)
- decides how many ECTS they make each year
- specifies a maximum workload for any course (accept a 3 ECTS course or not)
- has to set a preference value for each course offered
- provides his own availability (1rst, 2nd semester or both)
- can not take 2 courses at the same schedule

- a course cannot be granted to a student if his preference value for this course is null
- some assignments are predefined, some are forbidden

Elective course allocation problem

Numerical preferences p_i are used

- each student *i* has a budget of 100
- at least 5 courses must be given 1
- at least 5 courses must be given more than 10

Possible objectives :

welfare :

$$\max W(x) = \sum_i u_i x_i$$

faireness:

$$\max \min_i u_i x_i$$

equity:

$$\min \max_{i,j} (u_j x_j - u_i x_i)$$

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Mathematical formulation

$\sum_o x_{s,o} T_{o,c} \le 1 \quad \forall s,c$	student take a course only once	(1)
$x_{s,o} \le 1 - p_{o,1} a_{s,1} \quad \forall s,o$	availability semester 1	(2)
$x_{s,o} \le 1 - p_{o,2} a_{s,2} \quad \forall s,o$	availability semester 2	(3)
$\sum_s x_{s,o} T_{o,c} \le \sum_c q_c^{\max} T_{o,c} y_o \quad \forall o$	course maximum capacity	(4)
$\sum_s x_{s,o} T_{o,c} \le \sum_c q_c^{\max} T_{o,c} y_o \quad \forall o$	course minimum capacity	(5)
$\sum_c \sum_o x_{s,o} T_{o,c} W_c \ge R_s \quad \forall s$	minimum number of ECTS required	(6)
$\sum_c \sum_o x_{s,o} T_{o,c} W_c \le R_s + V_s \quad \forall s$	maximum number of ECTS accepted	(7)
$\sum_o x_{s,o} \le M_s \quad \forall s$	maximum number of courses accepted	(8)
$\sum_o x_{s,o} T_{o,c} \ge I_{s,c} \quad \forall s,c$	imposed courses	(9)
$x_{s,o} \le F_{s,o} \quad \forall s,o$	forbidden courses	(10)
$x_{s,o} \le \sum_c T_{o,c} P_{s,c} \quad \forall s,o$	not desired courses	(11)
$x_{s,o_i} + x_{s,o_j} \le 2 - D_{o_i,o_j}$	simultaneous courses	(12)

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First 2-year results

Data sets :

List of courses and schedule

16-Oct		20-Oct	23-Oct		30-Oct		03-Nov		06-Nov		10-Nov		13-Nov	
pm	soir	am	pm	soir	pm	soir	am	pm	pm	soir	am	pm	pm	s
PREE 1			PREE 1		PREE 1		TECT 1	TECT 1	PREE 1		TECT 1	TECT 1		J
CREM 1	METG 1		CREM 1	METX 1	CREM 1	METG 1			CREM 1	METX 1				M
SCIA 1			SCIA 1		SCIA 1				SCIA 1				SCIA 1	
													TOPC 1	
ARGX 1			ARGX 1						ARGX 1				ARGX 1	
VALP 1			VALP 1		VALP 1				VALP 1					
TECP 1			TECP 2		TECP 1				TECP 2				TECP 1	
ARTM 1			ARTM 1		ARTM 1				ARTM 1					
COMN 1			COMN 2		COMN 1				COMN 2				COMN 1	
RESS 1			RESS 1		RESS 1				RESS 1					
TECV 1			TECV 1						TECV 1				TECV 1	
COOD 1		SUWE 1	SUWE 1		COOD 1				SUWE 1				COOD 1	
			ENER 1						ENER 1					
			FECO 1		FECO 1				FECO 1				FECO 1	
					COCL 1						COCL 1		COCL 1	
									PECC 1				PECC 1	

First 2-year results

Data sets :

number of students : M
number of occurences of courses : N

	2011-BAC2	2011-Masters	2012-BAC	2012-Masters
M	246	532	432	335
N	25	42	58	48
seats available	582	796	1129	942
ECTS available	1164	1951	2258	2169
R (required ECTS)	0	0	1378	1019
V (volonteered ECTS)	804	2660	674	543
Mean score	47	46	62	60
Max score	80	80	84	82

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Emerging strategies, problem classification and methods

Related problems

Marriage problem

Women 1 : 1 Men

each set has preferences on the other set

College admission problem

Students N : 1 College

students give preferences on colleges

colleges have preferences on students

are related to Course allocation

Students N : N Courses

students give preferences on courses

courses do not have preferences on students

Emerging strategies, problem classification and methods

Marriage problem

an assignment *x* is unstable if

there exist a man *m* and a woman *w* who prefer each other to their assignment at *x*

Theorem (Alvin Roth, 1985):

there exist no stable matching procedure for the marriage problem that makes it a dominant strategy for all agents to state their true preferences

College admission problem

Students N : 1 College

an assignment is S-Optimal stable if there is no assignment x' such that x' is preferred for all student in S

Theorem holds (generalization, A. Roth 1985)

Course allocation problem

Students N : N Courses

students give preferences on courses / courses do not have preferences on students

Theorem holds (generalization)

Emerging strategies, problem classification and methods

Students use strategies and partially reorder their preferences taking into account course popularity levels.

Allocation mechanisms produce solutions with a lower welfare when a fraction of students are manipulating their preferences instead of a truthful play (Budish, 2011)

Literature shows that the only mechanisms that are Pareto efficent and strategy proof are dictatorships (Budish et al. 2012, Kominers et al. 2010, etc.) but those mechanisms are not equitable.

Many schools use bid mechanisms or the Harvard Draft procedure although these methods are not strategy-proof.

A bid mechanism using agents allows an uniform use of strategy and achieve a better equity

Budish proposed recently a procedure based on approximate competitive equilibrium with equal incomes (budgets of students are not equal) that proves to be resistant to strategy.

Emerging strategies, problem classification and methods

Example at Harvard Business School (Budish, Cantillon, 2012)

Three steps procedure:

Step 1 : students are asked their preferences to size the course offer

Step 2 students are asked to bid on courses the procedure outputs an assignment given as an information to students

-> courses are given a gross value

Step 3: students are asked to bid and the output is definitive

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Further work

What is the Holy Grail ?

In a perfect world all students are happy with their bid/preference decisions and the resulting allocation.

Reach a Nash equilibrium ?

- perfect information of all students about other student strategies
- perfect information about the minimum preference required to obtain a course
- the final solution no student is willing to change his strategy
- use agents to avoid students strategy mistakes

How ?

- In depth analysis of the relation between mechanisms based on preference ordering and mechanisms based on numerical preferences
- Develop a fast procedure that provides a optimal stable allocation
- Propose an web-based bid/preference system with real-time output

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Literature

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