Attribution des cours à option, modèle avec préférences numériques

Elective Course Allocation with numerical preferences

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Plan

- Elective course allocation problem
- Mathematical formulation
- First 2-year results
- Emerging strategies, problem classification and methods
- Further work
Elective course allocation problem

Students must take a number of courses to complete their curriculum.

The classical Assignment Problem:
• each student has preferences on bundle of courses
• each course accepts a maximum number of students (seats)
• no use of monetary transfer (artificial currency is allowed)

Additional characteristics:
• students have preferences on individual courses
• courses schedules are predetermined
• students availability is known
Elective course allocation problem

Set S of N students $s_i$
Set C of M courses $c_j$ with integral capacities $q = (q_1, ..., q_M)$.

Each student $s_i$ has a set of permissible schedules $\Psi_i$ and a utility function $u_i$
Non-permissible schedules have utility of zero
No peer effects on the utility function

Each student $i$ has a preference function: $C_j \succ_i C_k$

A solution is an allocation $x$ of courses to students: $x_{ij} = 1$ if course $j$ is allocated to student $i$

A solution is characterized with stability, efficiency and fairness
Elective course allocation problem

Constraints
Taking a number of elective courses is mandatory to fulfill the curriculum.

Courses:
• each course has a weight proportional to its workload (2 or 3 ECTS)
• courses schedule is known (1rst, 2nd semester or both)
• each course can be offered multiple times (occurences) according to different schedules
• each course has a minimum and a maximum capacity (seats)

Each student
• has 2 years to achieve a given number of ECTS (according to their curriculum)
• decides how many ECTS they make each year
• specifies a maximum workload for any course (accept a 3 ECTS course or not)
• has to set a preference value for each course offered
• provides his own availability (1rst, 2nd semester or both)
• can not take 2 courses at the same schedule

• a course cannot be granted to a student if his preference value for this course is null
• some assignments are predefined, some are forbidden
Elective course allocation problem

Numerical preferences $p_i$ are used
- each student $i$ has a budget of 100
- at least 5 courses must be given 1
- at least 5 courses must be given more than 10

Possible objectives:

welfare: \[ \max W(x) = \sum_i u_i x_i \]

fairness: \[ \max \min_i u_i x_i \]

equity: \[ \min \max_{i,j} (u_j x_j - u_i x_i) \]
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Mathematical formulation

\[
\sum_{o} x_{s,o} T_{o,c} \leq 1 \quad \forall s,c
\]
student take a course only once (1)

\[
x_{s,o} \leq 1 - p_{o,1} a_{s,1} \quad \forall s,o
\]
availability semester 1 (2)

\[
x_{s,o} \leq 1 - p_{o,2} a_{s,2} \quad \forall s,o
\]
availability semester 2 (3)

\[
\sum_{s} x_{s,o} T_{o,c} \leq \sum_{c} q_{c}^\text{max} T_{o,c} y_{o} \quad \forall o
\]
course maximum capacity (4)

\[
\sum_{s} x_{s,o} T_{o,c} \leq \sum_{c} q_{c}^\text{max} T_{o,c} y_{o} \quad \forall o
\]
course minimum capacity (5)

\[
\sum_{c,o} x_{s,o} T_{o,c} W_c \geq R_s \quad \forall s
\]
minimum number of ECTS required (6)

\[
\sum_{c,o} x_{s,o} T_{o,c} W_c \leq R_s + V_s \quad \forall s
\]
maximum number of ECTS accepted (7)

\[
\sum_{o} x_{s,o} \leq M_s \quad \forall s
\]
maximum number of courses accepted (8)

\[
\sum_{o} x_{s,o} T_{o,c} \geq I_{s,c} \quad \forall s,c
\]
imposed courses (9)

\[
x_{s,o} \leq F_{s,o} \quad \forall s,o
\]
forbidden courses (10)

\[
x_{s,o} \leq \sum_{c} T_{o,c} P_{s,c} \quad \forall s,o
\]
not desired courses (11)

\[
x_{s,o_1} + x_{s,o_2} \leq 2 - D_{o_1,o_2}
\]
simultaneous courses (12)
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First 2-year results

Data sets:

List of courses and schedule

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# First 2-year results

Data sets:

- number of students: \(M\)
- number of occurrences of courses: \(N\)

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<td>(N)</td>
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<td>42</td>
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• **Emerging strategies, problem classification and methods**
• Further work
• Literature
Emerging strategies, problem classification and methods

Related problems

Marriage problem

Women 1 : 1 Men
each set has preferences on the other set

College admission problem

Students N : 1 College
students give preferences on colleges
colleges have preferences on students

are related to Course allocation

Students N : N Courses
students give preferences on courses
courses do not have preferences on students
Emerging strategies, problem classification and methods

**Marriage problem**

an assignment \( x \) is unstable if

there exist a man \( m \) and a woman \( w \) who prefer each other to their assignment at \( x \)

*Theorem (Alvin Roth, 1985):*

*there exist no stable matching procedure for the marriage problem that makes it a dominant strategy for all agents to state their true preferences*

**College admission problem**

Students \( N : 1 \) College

an assignment is S-Optimal stable if there is no assignment \( x' \) such that \( x' \) is preferred for all student in \( S \)

*Theorem holds (generalization, A. Roth 1985)*

**Course allocation problem**

Students \( N : N \) Courses

students give preferences on courses / courses do not have preferences on students

*Theorem holds (generalization)*
Emerging strategies, problem classification and methods

Students use strategies and partially reorder their preferences taking into account course popularity levels.

Allocation mechanisms produce solutions with a lower welfare when a fraction of students are manipulating their preferences instead of a truthful play (Budish, 2011).

Literature shows that the only mechanisms that are Pareto efficient and strategy proof are dictatorships (Budish et al. 2012, Kominers et al. 2010, etc.) but those mechanisms are not equitable.

Many schools use bid mechanisms or the Harvard Draft procedure although these methods are not strategy-proof.

A bid mechanism using agents allows an uniform use of strategy and achieve a better equity.

Budish proposed recently a procedure based on approximate competitive equilibrium with equal incomes (budgets of students are not equal) that proves to be resistant to strategy.
Emerging strategies, problem classification and methods

Example at Harvard Business School (Budish, Cantillon, 2012)

Three steps procedure:
Step 1: students are asked their preferences to size the course offer
Step 2 students are asked to bid on courses the procedure outputs an assignment given as an information to students
-> courses are given a gross value
Step 3: students are asked to bid and the output is definitive
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• Literature
Further work

What is the Holy Grail?

In a perfect world all students are happy with their bid/preference decisions and the resulting allocation.

Reach a Nash equilibrium?
- perfect information of all students about other student strategies
- perfect information about the minimum preference required to obtain a course
- the final solution no student is willing to change his strategy
- use agents to avoid students strategy mistakes

How?
- In depth analysis of the relation between mechanisms based on preference ordering and mechanisms based on numerical preferences
- Develop a fast procedure that provides a optimal stable allocation
- Propose an web-based bid/preference system with real-time output
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Literature


