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Suppression of Aeroelastic Instability with a Nonlinear Energy Sink: Theory

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Limit cycle oscillations (LCOs) commonly require restrictions on the operation of high-performance aircraft and have the potential to cause structural damage or failure. This paper summarizes recent theoretical findings on the application of passive and targeted nonlinear energy transfer ("nonlinear energy pumping") for the reduction or elimination of LCOs in self-excited systems. This NES has been used successfully to suppress LCOs of an elastically mounted, rigid airfoil in flow. The theoretical results are in agreement with experimental studies of a practical airfoil with an attached NES. These experimental results, presented in full in a separate paper, verify the capacity of the NES to reduce or even eliminate these undesired oscillations and to extend the operating speed range of the wing.

I. Introduction

CLASSICAL linear theory predicts that a flexible aircraft structure will exhibit divergent response at flow speeds above a critical "flutter speed," implying that catastrophic failure will occur when this flight speed is exceeded. Fortunately, real structures are often sufficiently nonlinear, displaying hardening stiffness for example, that their response at supercritical speeds takes the form of a steady limit cycle oscillation rather than diverging. Even when they do not cause damage, such oscillations are extremely undesirable because they limit the operating envelope of high-performance aircraft.[1,2,3,4] A large body of literature exists on the analysis of aircraft LCO and various means proposed for its suppression; some works most relevant to the present study are reviewed in Hill et al.[5]

A necessary condition for limit cycle oscillation in general is the availability of energy to sustain the self-excited motion. In the particular case of aircraft LCO, the energy available from high-speed flow is huge. It is therefore much better to prevent the onset of an LCO than to attempt to eliminate it after large steady-state oscillatory motion has developed. We summarize below the results of a rigorous analysis showing how nonlinear energy pumping can be used to extract energy from an aeroelastic system during the earliest stage of LCO response, thus avoiding any need to fight a developed limit cycle. An accompanying paper[5] presents experimental results obtained with a structurally nonlinear wind-tunnel model, demonstrating a significant increase in the critical speed, as well as improved stability at high subcritical speeds, following the addition of a nonlinear energy sink.

II. Nonlinear Energy Pumping and LCO Triggering in Self-Excited Systems

Nonlinear energy pumping refers to the controlled (or targeted) irreversible transfer of vibrational energy from a primary system to a passive nonlinear energy sink (NES). The essentially nonlinear (nonlinearizable)

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stiffness and damping in the NES makes it possible to localize the energy through resonance capture and to dissipate the transferred energy.[6,7,8] Recently, a two-degree-of-freedom (2–DOF) nonlinear system consisting of a grounded linear oscillator coupled to an NES was studied to obtain the very complicated bifurcation structure of its nonlinear normal modes.[9] Furthermore, in Kerschen et al.[10] it was shown that there exist at least three mechanisms of energy pumping; namely, *one-to-one and subharmonic resonance captures, and energy pumping initiated by nonlinear beating phenomena*. It was also shown that the transient dynamics and energy transfer can be systematically interpreted and understood by studying the topology and bifurcations of the underlying Hamiltonian system.

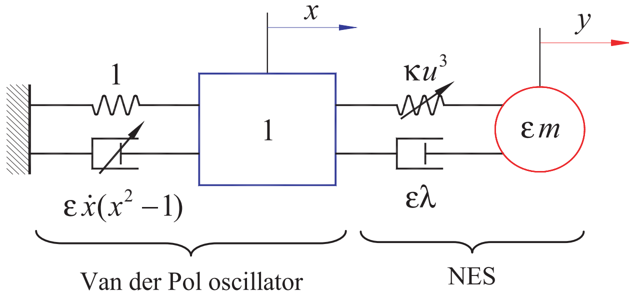


Figure 1. VDP oscillator coupled with an ungrounded NES.

Lee et al.[11] showed the applicability of nonlinear energy pumping to suppress the limit cycle oscillation (LCO) in a van der Pol (VDP) oscillator (Fig. 1), which is analogous to a nonlinear aeroelastic problem. The equations of motion of this system can be written

$$\begin{aligned} &\ddot{x} - \epsilon\dot{x}\left(1-x^2\right) + x + \epsilon\lambda\left(\dot{x}-\dot{y}\right) + \kappa\left(x-y\right)^3 = 0 \\ &\epsilon m\ddot{y} + \epsilon\lambda\left(\dot{y}-\dot{x}\right) + \kappa\left(y-x\right)^3 = 0 \end{aligned} \tag{1}$$

where x and y are, respectively, the displacements of the VDP and NES masses; ϵm, the mass ratio between the NES and VDP masses; $\epsilon\lambda$, the damping coefficient; and κ, the nonlinear stiffness coefficient. The uncoupled VDP oscillator has the LCO $x(t) = 2\cos\omega t + \mathcal{O}(\epsilon)$ where $\omega = 1 + \mathcal{O}(\epsilon^2)$.

Using the complexification technique[12] and averaging over the linearized natural frequency of the VDP oscillator (i.e., performing single-frequency averaging over e^{jt}), it was analytically shown that the LCO suppression (or elimination) mechanism is composed of a series of resonance captures and escapes (Fig. 2). In Fig. 2, ϕ denotes the phase difference between the VDP and NES oscillators. The energies were computed by the following relations:

$$\begin{aligned} &E(0) = [\dot{x}(0)^2 + \varepsilon m\dot{y}(0)^2]/2 + x(0)^2/2 + \kappa[x(0)-y(0)]^4/4 \equiv E_0 \\ &E_d^{VDP}(t) = \varepsilon\int_0^t \dot{x}(\tau)^2\left(x(\tau)^2-1\right)d\tau,\ E_d^{NES}(t) = \varepsilon\lambda\int_0^t\left(\dot{x}(\tau)-\dot{y}(\tau)\right)^2 d\tau \\ &E^{VDP}(t) \triangleq E_0^{VDP} - E_d^{VDP}(t), E^{NES}(t) \triangleq E_0^{NES} - E_d^{NES}(t) \\ &E^{total}(t) = E(0) - E_d^{VDP}(t) - E_d^{NES}(t) \end{aligned} \tag{2}$$

Initially the dynamics is captured close to the 3:1 resonance manifold of the underlying Hamiltonian system, and the resulting transient resonance capture (TRC) is sustained for a certain period of time as the total energy decreases. As the energy reaches the value where the S31-branch meets the S11-branch, the dynamics escapes from 3:1 TRC and engages in 1:1 TRC (Fig. 2 (d)). As energy further decreases and the LCO is completely eliminated, the dynamics escapes from 1:1 resonance capture at the last stage of the motion. Examining the corresponding energy exchanges in Fig. 2 (c), we can clearly identify the reasons for LCO elimination. That is, the energy dissipation by the NES counterbalances the energy supply provided by the nonlinear (negative) damping in the VDP oscillator. The NES passively adjusts the rate of energy dissipation so as to precisely counterbalance the energy input fed by the VDP oscillator. Moreover, the rates of energy dissipation and generation asymptotically reach steady-state values. Therefore, for LCO elimination it is necessary that energy pumping from the VDP oscillator to the NES should occur fast enough and be strong enough to overcome the energy input by the nonlinear damping.

Finally, LCO suppression and its robustness were addressed by means of steady-state bifurcation analysis. By proving the existence of Hopf bifurcation, complete elimination of LCOs is possible for some parameter ranges; and by examining the stability of super-/subcritical LCOs, robustness can be examined.

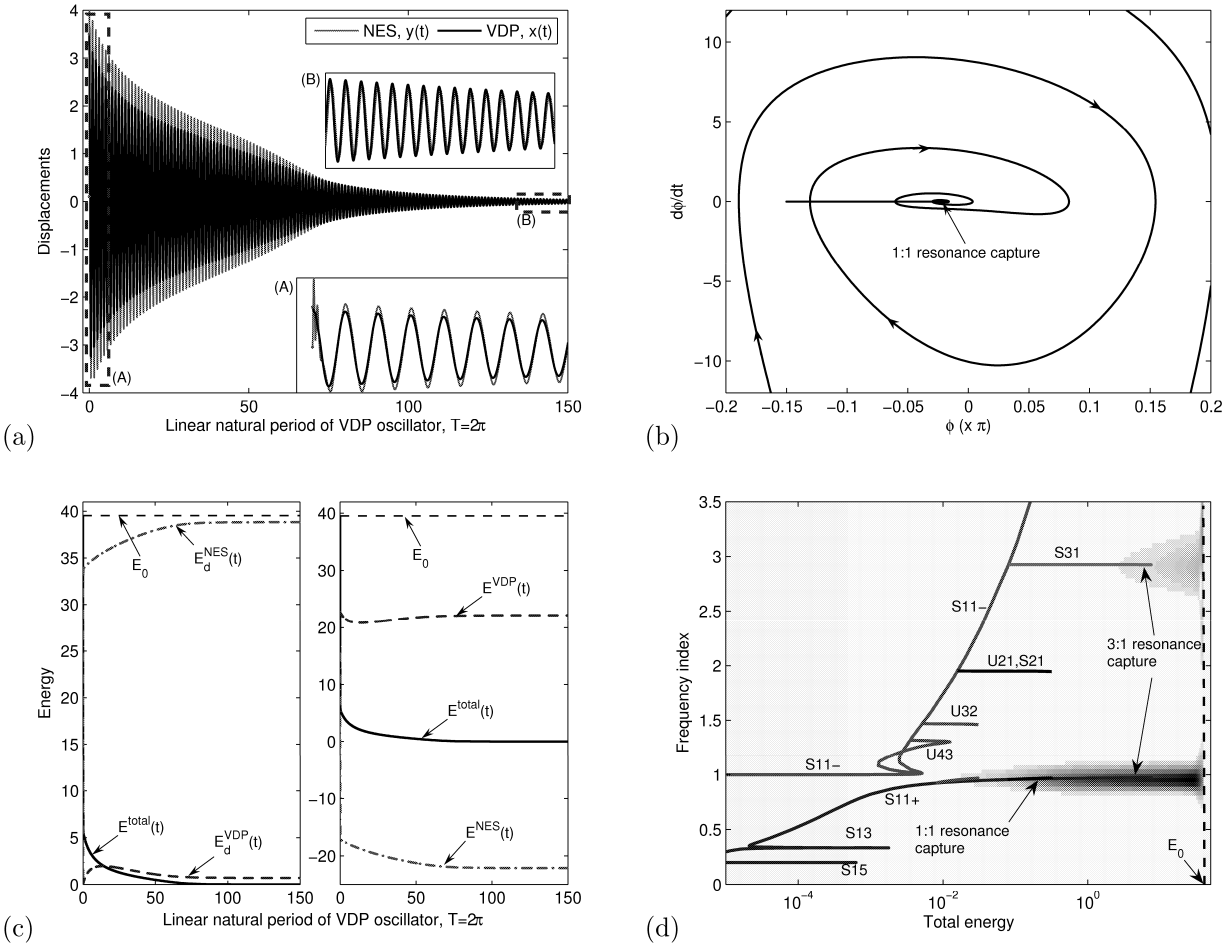


Figure 2. Transient dynamics when the LCO of the VDP oscillator is eliminated ($\epsilon m = 5\%, \kappa = 1.0, \epsilon\lambda = 0.1$), for initial displacements $(x(0), y(0)) = (3.5, 0.1)$ and zero initial velocities: (a) Time series; (b) 1:1 phase interaction; (c) instantaneous energy exchanges; and (d) dynamics on the frequency–energy plot.

The importance of this study lies not only in showing that LCO of the VDP oscillator can be suppressed by means of passive nonlinear energy pumping, but also in the whole methodology that can be directly utilized in studying suppression of aeroelastic instability.

Lee et al.[13] studied triggering mechanism of LCOs due to aeroelastic instability of a rigid wing in flow (Fig. 3 without the NES part), for which the nondimensional equations of motion can be written

$$\begin{aligned} &y'' + x_\alpha \alpha'' + \Omega^2 y + \xi_y y^3 + \mu C_{L,\alpha}\Theta\left(y' + \Theta\alpha\right) = 0 \\ &r_\alpha^2 \alpha'' + x_\alpha y'' + r_\alpha^2 \alpha + \xi_\alpha \alpha^3 - \gamma\mu C_{L,\alpha}\Theta\left(y' + \Theta\alpha\right) = 0 \end{aligned} \tag{3}$$

where quasi-steady aerodynamic theory, subsonic flight conditions and small motions have been assumed in the derivation. The following system parameters were considered:

$$x_\alpha = 0.2,\ r_\alpha = 0.5,\ \gamma = 0.4,\ \Omega = 0.5,\ \mu = (10\pi)^{-1},\ C_{L,\alpha} = 2\pi,\ \xi_y = \xi_\alpha = 1 \tag{4}$$

which gives a flutter speed of $\Theta_F = 0.87$.

Since the LCO responses possess multiple frequency components, a three-frequency averaging technique was employed for analytical study of triggering mechanism (Fig. 4). It was shown that a cascade of resonance captures constitutes the triggering mechanism due to aeroelastic instability of rigid wings in flow. That is, the LCO triggering mechanism consists of a combination of different dynamic phenomena, taking place in three main stages or regimes: attraction to transient resonance captures (TRCs), escapes from these captures and, finally, entrapments into permanent resonance captures (PRCs). The general conclusion was that an initial excitation of the heave mode by the flow acts as the triggering mechanism for the excitation of the pitch mode through nonlinear interactions resulting from the resonance captures and escapes. The eventual excitation of the pitch mode signifies the appearance of LCOs of the in-flow wing.

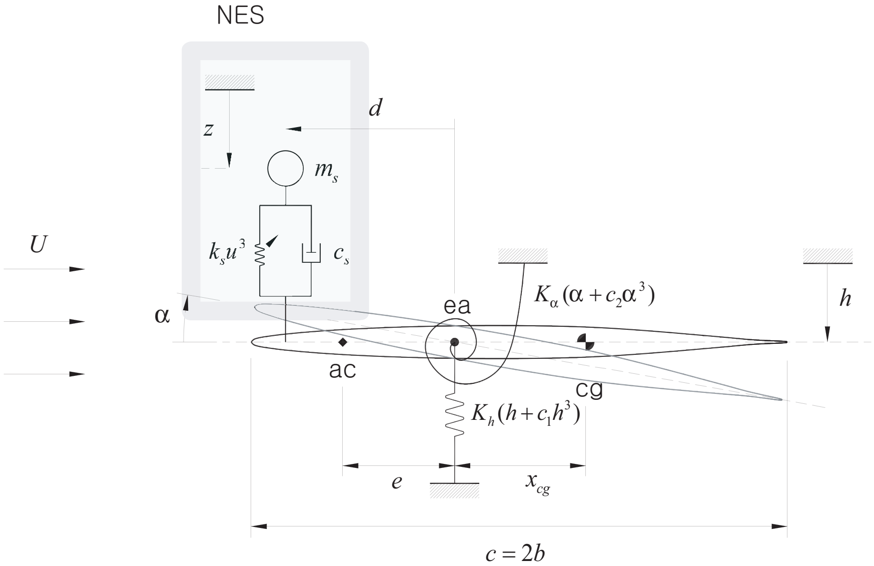


Figure 3. Two-degree-of-freedom rigid wing with offset attachment of an NES.

Based on the study of LCO suppressions in a VDP oscillator and of the LCO triggering mechanism due to aeroelastic instability, we finally propose an NES attachment as in Fig. 3,

$$\begin{aligned}
&y'' + x_\alpha \alpha'' + \Omega^2 y + \xi_y y^3 + \mu C_{L,\alpha}\Theta\left(y' + \Theta\alpha\right) + \epsilon\lambda\left(y' - \delta\alpha' - v'\right) + C\left(y - \delta\alpha - v\right)^3 = 0 \\
&r_\alpha^2 \alpha'' + x_\alpha y'' + r_\alpha^2 \alpha + \xi_\alpha \alpha^3 - \gamma\mu C_{L,\alpha}\Theta\left(y' + \Theta\alpha\right) + \delta\epsilon\lambda\left(\delta\alpha' + v' - y'\right) + \delta C\left(\delta\alpha + v - y\right)^3 = 0 \\
&\epsilon v'' + \epsilon\lambda\left(v' + \delta\alpha' - y'\right) + C\left(v + \delta\alpha - y\right)^3 = 0
\end{aligned} \tag{5}$$

where the fundamental is that the NES can interact with the heave mode before the heave mode triggers the pitch mode, extracting energy fed by the flow to the heave. Due to its offset from the elastic axis, the NES can also interact with the pitch mode.

Considering the same parameters as before and performing a numerical parametric study, we find that there exist three basic LCO suppression mechanisms: (i) repeated burst-out and suppression (Fig. 5); (ii) intermediate suppression (Fig. 6); and (iii) complete suppression (Fig. 7). The first mechanism will determine critical boundaries for proper design of NES parameters if we recall the results from the VDP oscillator on the frequency-energy plot. Furthermore, utilizing a two-frequency averaging method, we can show analytically that each suppression mechanism derives from similar behavior as in the case of LCO suppression of the VDP oscillator (Figs. 5–7). That is, each is composed of a series of resonance captures and escapes from the superharmonic to subharmonic order.

A synoptic presentation of the three basic LCO suppression mechanisms identified in the numerical simulations is provided in the following, focusing only on their main dynamical features.

- **The first suppression mechanism** (Fig. 5) This mechanism is characterized by *a recurrent series of suppressed burst–outs of the heave and pitch modes of the wing, followed by eventual complete suppression of the aeroelastic instabilities.* In the initial phase of transient burst–outs, a series of developing instabilities of predominantly the heave mode is effectively suppressed by proper transient 'activation' of the NES, which tunes itself to the fast frequency of the developing aeroelastic instability; as a result, the NES engages in 1:1 TRC with the heave mode, passively absorbing broadband energy from the wing, thus eliminating the burst–out. In the later phase of the dynamics, the energy fed by the flow does note appear to directly excite the heave and pitch modes of the wing, but, instead, to get transferred directly to the NES until the wing is entirely at rest and complete LCO suppression is achieved. At the initial stage of the recurrent burst–outs, at time instants when the pitching LCO is nearly eliminated, most of the energy induced by the flow to the wing is absorbed directly by the NES

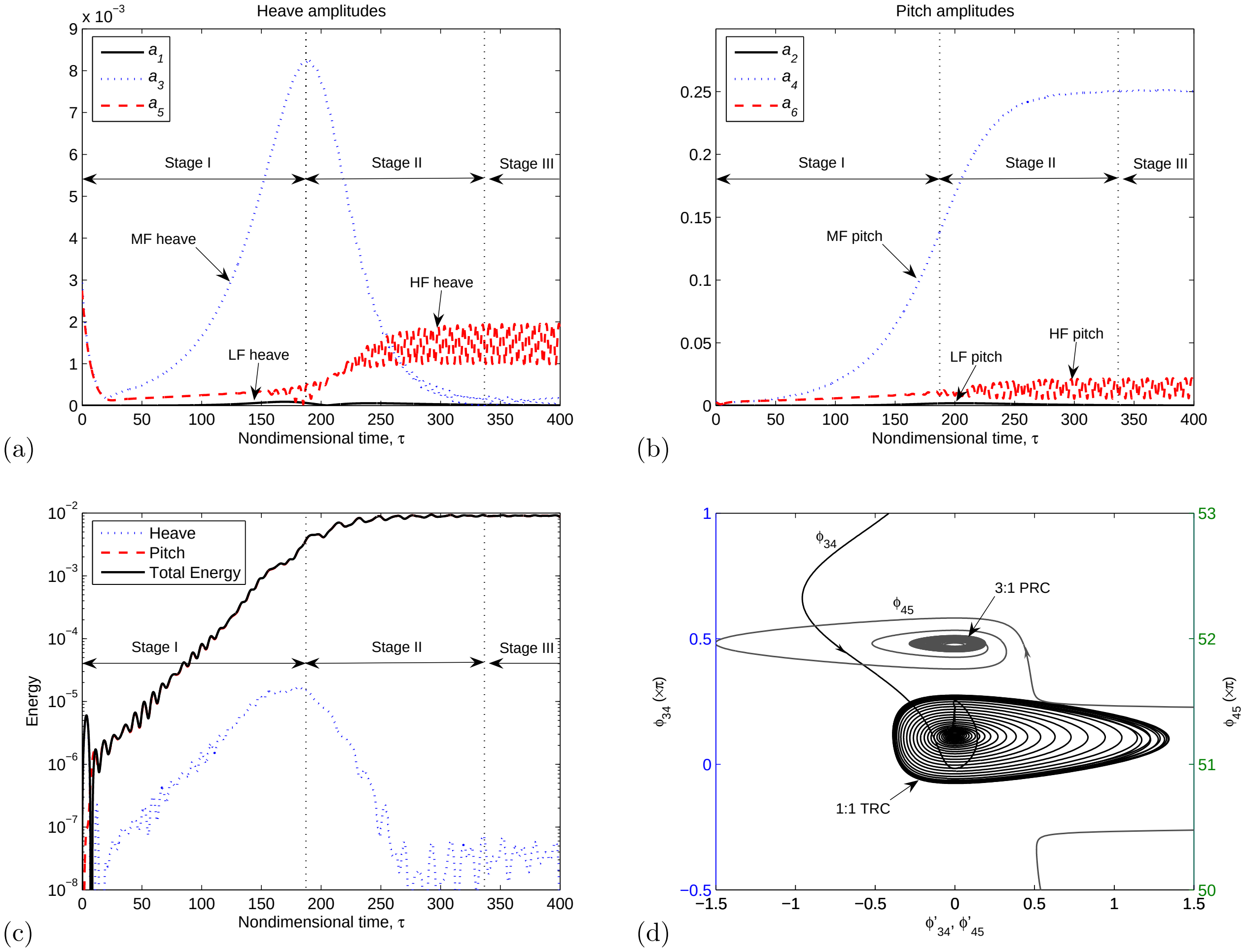
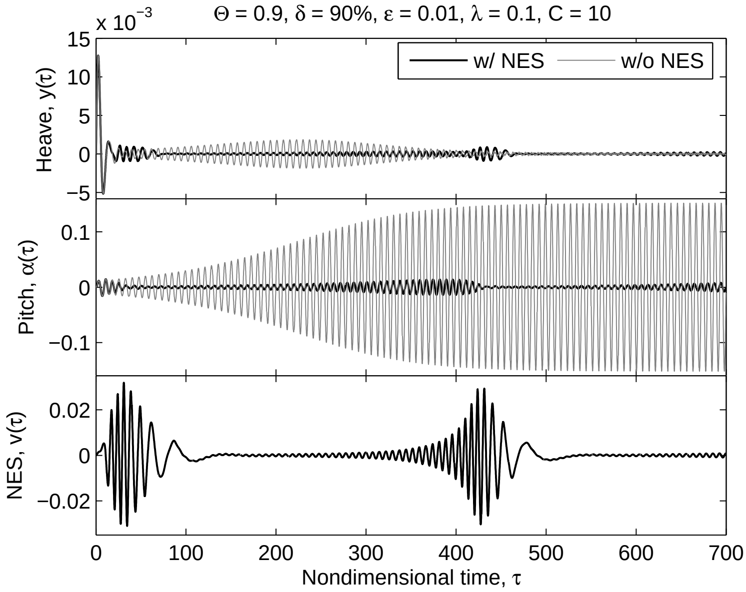


Figure 4. Transient dynamics when LCOs are developed ($\Theta = 0.95$): (a) and (b) time responses of each heave and pitch components; (c) instantaneous energy exchange between modes; and (d) phase interactions between MF heave and MF pitch modes (left; 1:1 TRC), and between HF heave and MF pitch modes (right; 3:1 PRC).

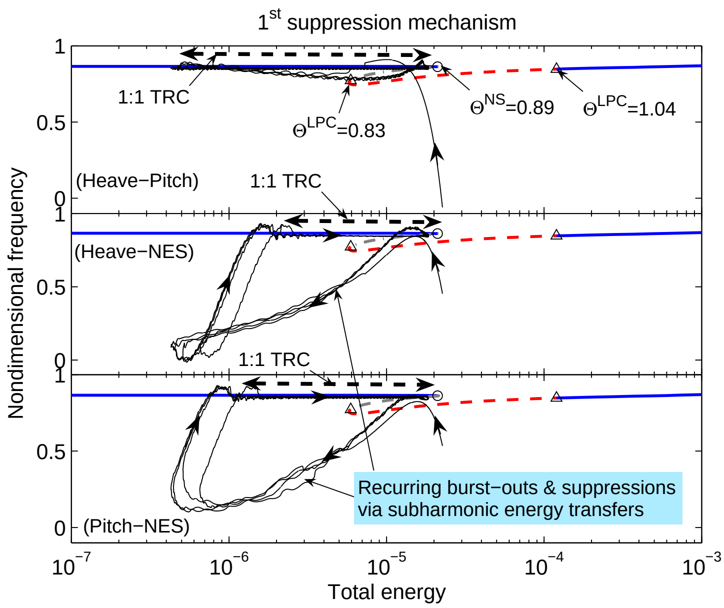
with only a small amount being transferred to the heave mode, so that both the NES and the heave mode reach their maximum amplitude modulations; this is followed by suppression of the burst–out, and this process is repeated until at a later stage complete suppression of the aeroelastic instability is reached. The beating–like (quasiperiodic) modal interactions observed during the recurrent burst-outs turn out to be associated with high–codimension bifurcations (i.e., Neimark–Sacker bifurcations[14] of a periodic solution) and to be critical for determining domains of robust suppression.

- **The second suppression mechanism** (Fig. 6) *Intermediate suppression of LCOs* is the typical behavior in this case, and is commonly observed when there occurs partial LCO suppression. The initial action of the NES is the same as in the first suppression mechanism. Targeted energy transfer to the NES then follows under conditions of 1:1 TRC, followed by conditions of 1:1 PRC where both heave and pitch modes attain constant (but nonzero) steady–state amplitudes. We note that the heave mode can grow larger than its response in the corresponding system with no NES attached (exhibiting an LCO), at the expense of suppressing the pitch mode. We also note that, in contrast to the first suppression mechanism, the action of the NES is nonrecurring in this case, as it acts at the early phase of the motion stabilizing the wing and suppressing the LCO.

- **The third suppression mechanism** (Fig. 7) In this mechanism energy transfers from the wing to the NES are caused by nonlinear modal interactions during 1:1 RCs. Both heave and pitch modes as well as the NES exhibit exponentially decaying responses resulting in *complete elimination of LCOs*. In general, higher NES masses are required for complete elimination of LCOs for increasing reduced speeds.

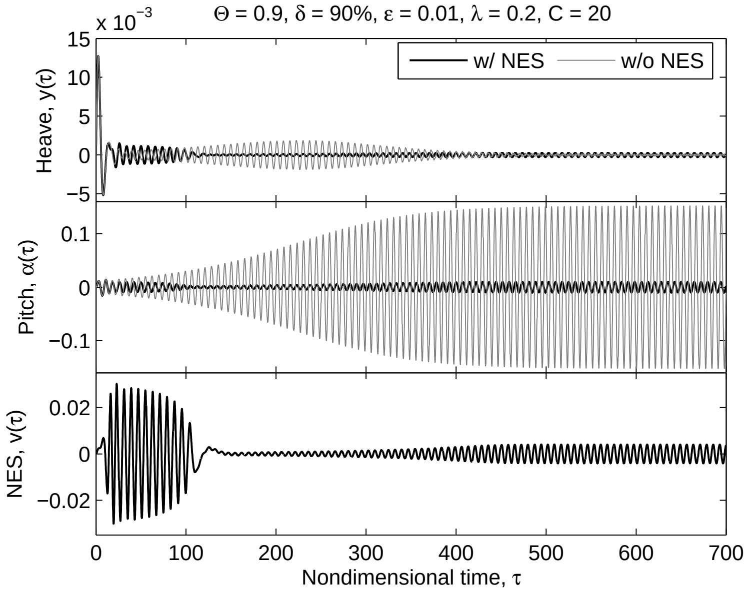


(a)

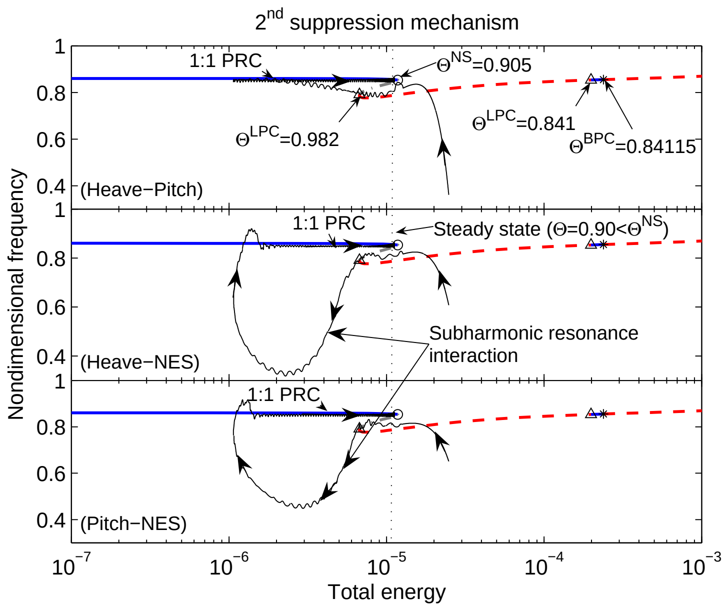


(b)

Figure 5. The first suppression mechanism when $\Theta = 0.9$, $\delta = 90\%$, $\epsilon = 1\%$, $\lambda = 0.1$, and $C = 10$. All zero initial conditions except $y'(0) = 0.01$ are used: (a) Time response and (b) dynamics on the energy plot.

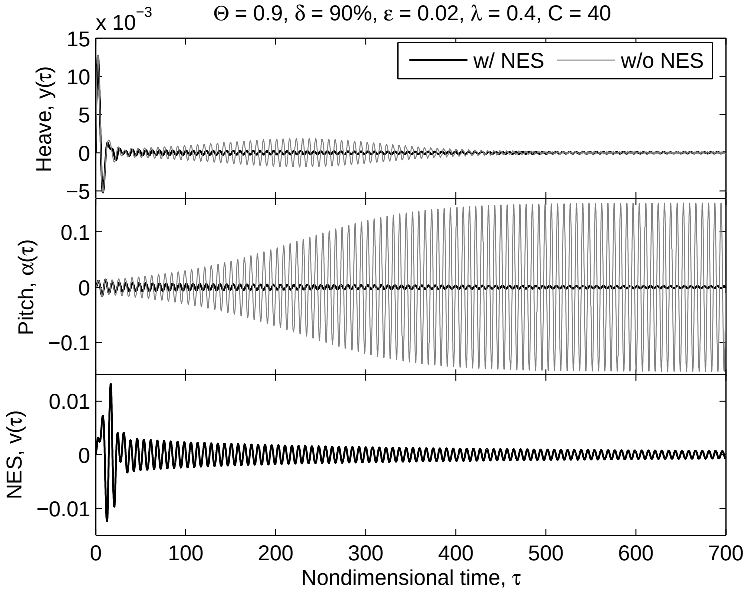


(a)

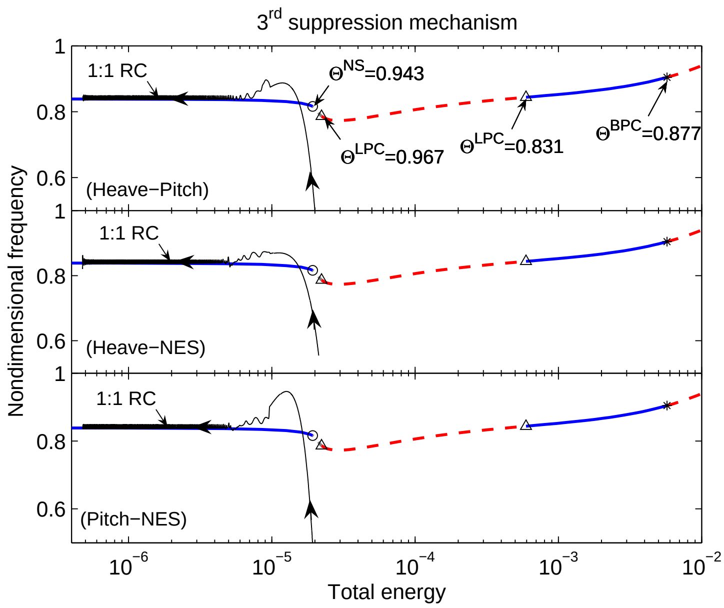


(b)

Figure 6. The second suppression mechanism when $\Theta = 0.9$, $\delta = 90\%$, $\epsilon = 1\%$, $\lambda = 0.2$, and $C = 20$. All zero initial conditions except $y'(0) = 0.01$ are used: (a) Time response and (b) dynamics on the energy plot.



(a)



(b)

Figure 7. The third suppression mechanism when $\Theta = 0.9$, $\delta = 90\%$, $\epsilon = 1\%$, $\lambda = 0.4$, and $C = 40$. All zero initial conditions except $y'(0) = 0.01$ are used: (a) Time response and (b) dynamics on the energy plot.

- **No suppression** This happens when the NES does not act as efficient absorber of the energy input extracted from a fluid due to aeroelastic interaction. In some cases, the steady–state amplitudes of LCOs grow even larger than those of the corresponding system with no NES attached. Depending on the parameter values, the steady state may possess a superharmonic frequency relation between modes. Similarly to the behavior observed in the LCO triggering mechanism in Lee et al.,[13] in the case of no suppression, there exists a transition from 1:1 to 3:1 locking of frequency ratios between the heave and pitch modes; this implies the occurrence of a 1:1 TRC followed by a transition to a 3:1 steady–state PRC between the heave and pitch modes. This observation suggests that, in order to suppress the aeroelastic instabilities, the NES must interact with both heave and pitch modes in such a way as to prevent direct energy transfer from the flow to the wing modes through subharmonic and superharmonic resonance captures (similar conclusions were drawn in Lee et al.,[11] where LCO suppressions of a van der Pol oscillator were studied).

III. Application to LCO of a Wind-Tunnel Model

To demonstrate the efficacy of nonlinear energy pumping in improving the stability of a self-excited aerodynamic system, an NES was designed at the University of Illinois specifically for use with an existing wind-tunnel setup, the nonlinear aerodynamic test apparatus (NATA),[15,16,17,18,19] at Texas A&M University.

For the first proof-of-concept experiments with an NES in an aerodynamic application, the design goals were similar to what would be desired of flight hardware, tempered by the realities of the laboratory environment and the scale of the test program. It was desired to design a light-weight, passive, self-contained attachment that would significantly improve the dynamic response of the NATA under typical operating conditions. "Light-weight" here must be construed as small with respect to the total translational mass m_t of the NATA. When the structure supporting the wing section was taken into account, it was found that $m_t = 12$ kg. To make the best use of available hardware, it was convenient to fix the mass of the NES at $m_s = 1.2$ kg, corresponding to a mass ratio, in plunge, of $m_s/m_t = 10\%$. Because of the manner in which the wing is supported in the NATA, it was possible to regard the NES as interacting directly with only the plunge degree of freedom of the wing. An important implication of successful LCO suppression in a system constrained in this way is immediate validation of the LCO triggering theory expounded above.

With the mass fixed, preliminary design of the NES was reduced to the specification of the linear viscous damping coefficient c_s and the essentially nonlinear spring stiffness k_s coupling the NES mass m_s to the NATA plunge displacement $h(t)$. The ranges of values of these two parameters that could be readily produced with the existing NES had been established in earlier experiments on other (non-aerodynamic) NES applications, and data were available relating nonlinear stiffness and coupling efficiency at low structural frequencies.[20] On this basis, preliminary values of the damping and stiffness were selected, then refined through a series of numerical simulations. The NES dynamics are governed by

$$m_s \ddot{v} = -f_s \tag{6}$$

where

$$f_s = c_s \left(\dot{v} - \dot{h} \right) + k_s |v - h|^{2.8} \operatorname{sgn}(v - h) \tag{7}$$

represents the force f_s exerted on the NATA (in plunge) by the NES. The exponent, 2.8, is typical of the values identified for the (theoretically purely cubic) nonlinear coupling spring.

The results of these simulations indicated that good performance could be achieved over a range of damping values and with nonlinear coupling stiffnesses toward the low end of those previously realized. While the viscous damping coefficient c_s does affect the rate and amount of energy pumping in the combined system, the simulations were relatively insensitive to this parameter and so a value of 0.40 kg/s, typical of damping levels identified in earlier experiments, was chosen. Additional simulations showed values of k_s in the range $[1.0,\ 2.0] \times 10^6$ N/m$^{2.8}$ to produce an effective NES, with larger values in this range to be preferred for practical reasons, such as smaller relative displacements during testing.

The values for a preliminary NES design are collected in Table 1. During testing, additional nonlinear coupling stiffnesses and dashpot coefficients were tried; these were measured in place. Simulation results obtained with these values at various speeds and with different magnitudes of the initial plunge displacement indicated this NES should exhibit all three LCO suppression mechanisms discussed in the preceding section: complete suppression; reduction in LCO amplitude, and repeated "bursting". All of these were found in the experimental program reported in Hill et al.[5]

Table 1. NES parameters.

$m_s = 1.2$ kg	NES Mass
$c_s = 0.4$ kg/s	Viscous Damping Coefficient
$k_s = 1.6 \times 10^6$ N/m$^{2.8}$	Spring Stiffness

IV. Conclusion

A nonlinear energy sink has been found to suppress limit cycle oscillation of a structurally nonlinear aeroelastic system. An understanding of the triggering mechanism, in which a transient heave response precedes the development of pitching, allowed the application of energy pumping for the reduction or elimination of LCO. Three suppression mechanisms, repeated bursting, amplitude reduction, and complete elimination of the LCO, were found to be possible. Theory and simulation, summarized herein, provided guidance in the design of a device with many of the qualities that will eventually be necessary in flight hardware, including relatively low mass and robust performance. Experimental results, presented separately, bear out predictions of improved dynamic response of the system following addition of the NES, and demonstrate all three suppression mechanisms.

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