

# Electric current crowding in nanostructured conductors

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*BELGIUM*

Université  
de Liège



LABORATORY OF PHYSICS OF  
NANOSTRUCTURED MATERIALS

# Collaborators

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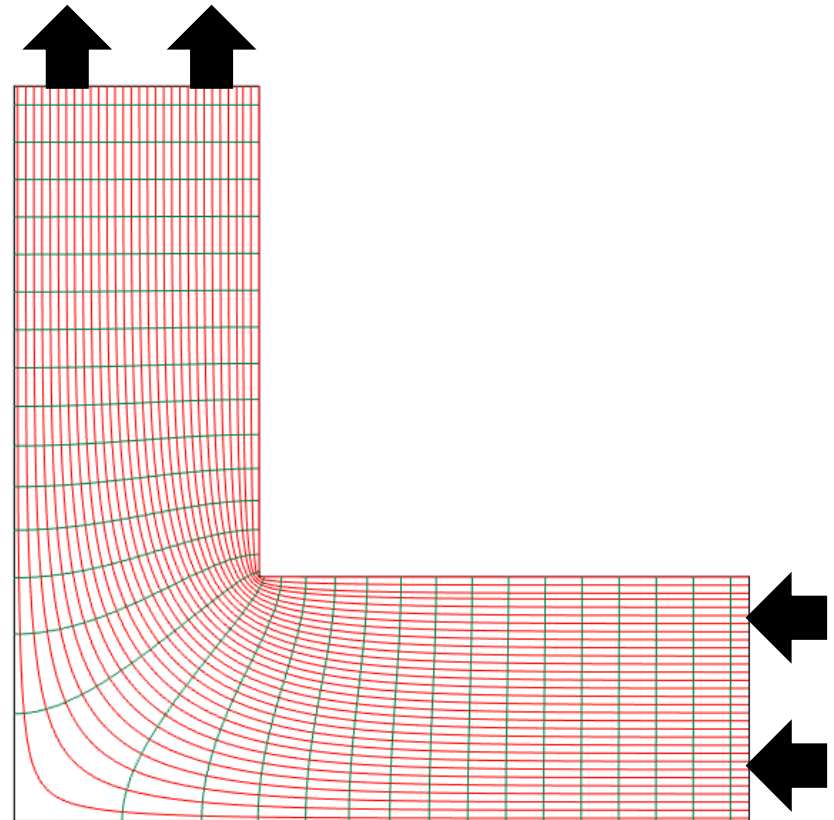
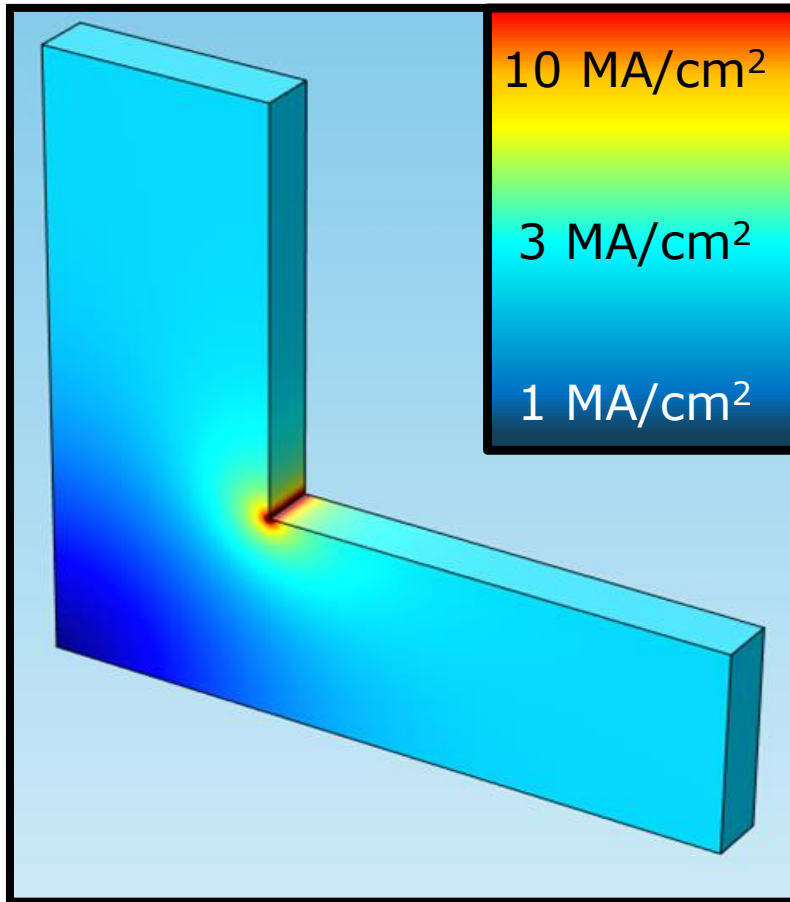
M. Motta, F. Colauto, W. Ortiz (Sao Carlos, BR)

J.I. Vestgarden, T.H. Johansen (Oslo, NO)

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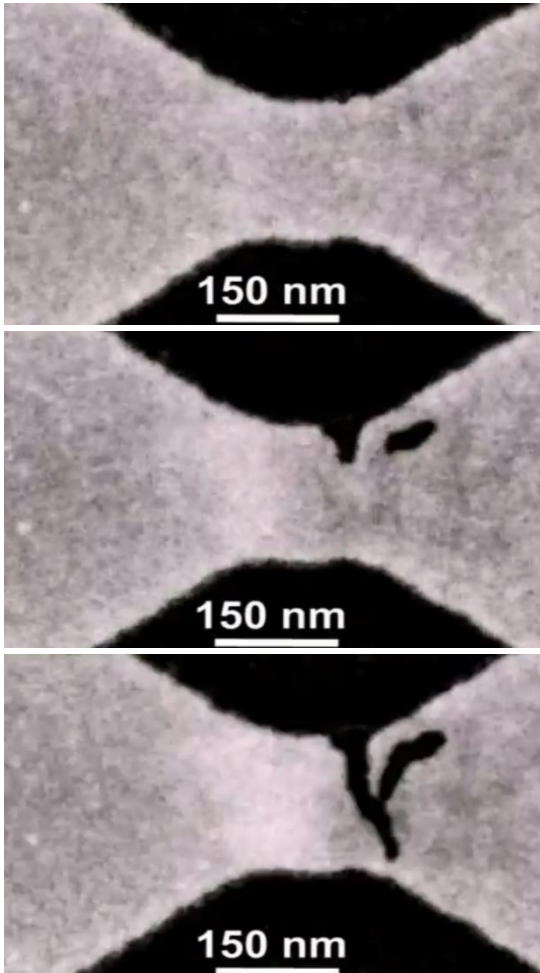
C. Cirillo, C. Attanassio (Salerno, IT)

# What is current crowding ?

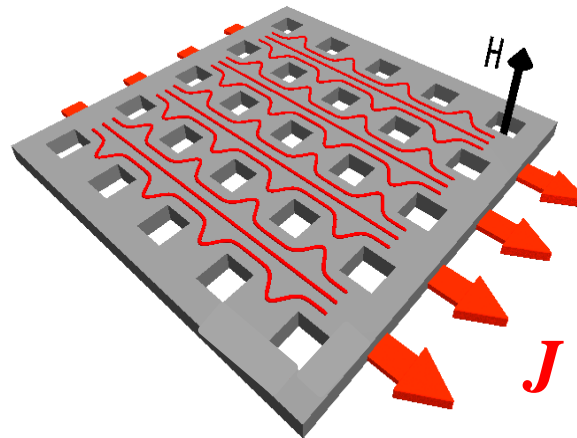


# Why is it important ?

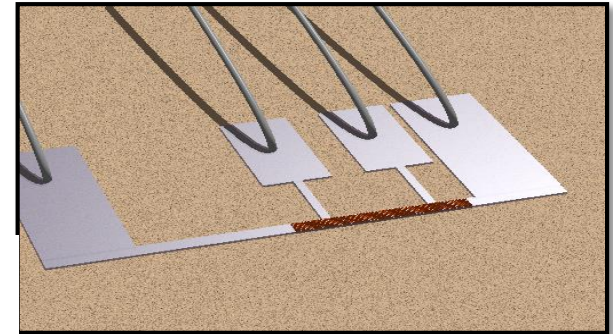
Electromigration



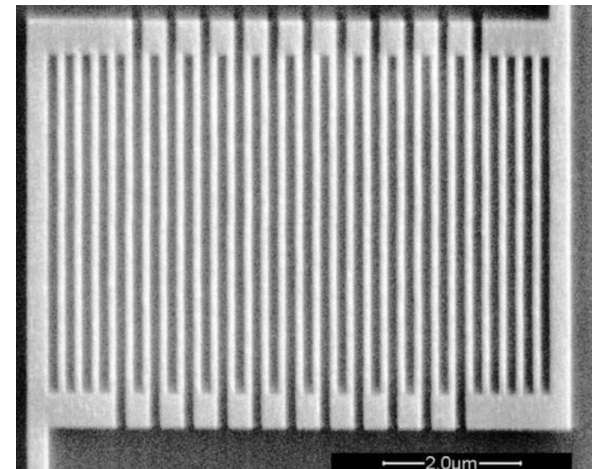
Nanostructured superconductors



Kelvin probe bridges



Single photon detectors



# Outline

- CURRENT CROWDING IN NORMAL METALS
- CURRENT CROWDING IN SUPERCONDUCTORS
  - ↗ SHARP BENDS
  - ↗ SURFACE INDENTATIONS
  - ↗ MAGNETIC FLUX AVALANCHES
- NANOSTRUCTURING VIA CURRENT CROWDING
- CONCLUSION

# Pre-history: normal conductors

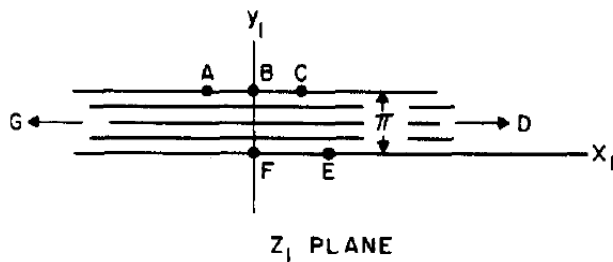
JOURNAL OF APPLIED PHYSICS

VOLUME 34, NUMBER 1

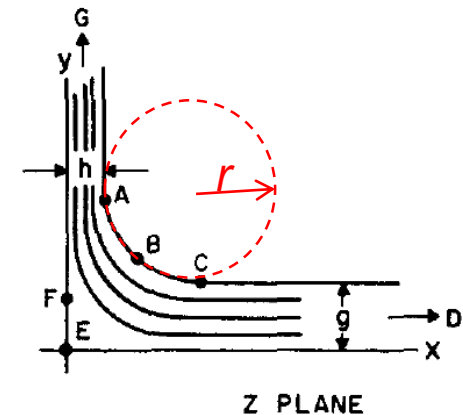
JANUARY 1963

## Right-Angle Bends in Thin Strip Conductors

F. B. HAGEDORN AND P. M. HALL  
*Bell Telephone Laboratories, Inc., Murray Hill, New Jersey*  
 (Received 25 June 1962)



conformal  
mapping

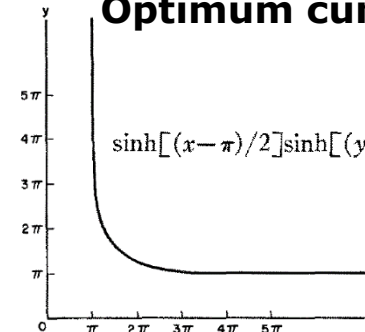


$$i_{ABC} \approx \left( \frac{g}{r} \right)^{1/3} i_0$$

$i_0$  is the asymptotic  
current density in the  
leg

The perturbations of  
the current crowding  
propagate about three  
strips widths into the  
legs

### Optimum curvature



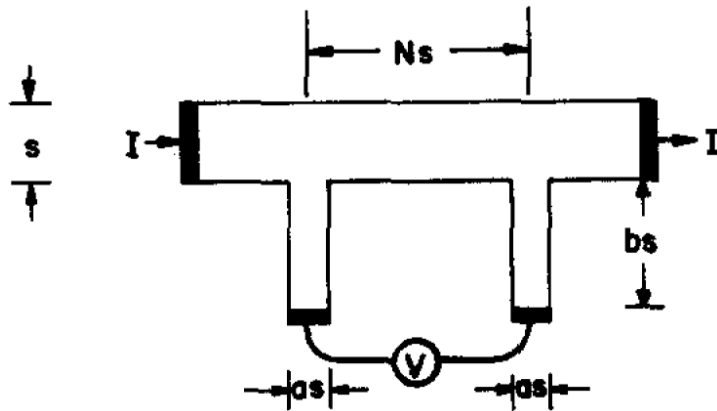
# Pre-history: normal conductors

## RESISTANCE CALCULATIONS FOR THIN FILM PATTERNS

P. M. HALL

*Thin Solid Films*, 1 (1967/68) 277-295

*Bell Telephone Laboratories, Inc., Allentown, Pa. (U.S.A.)*



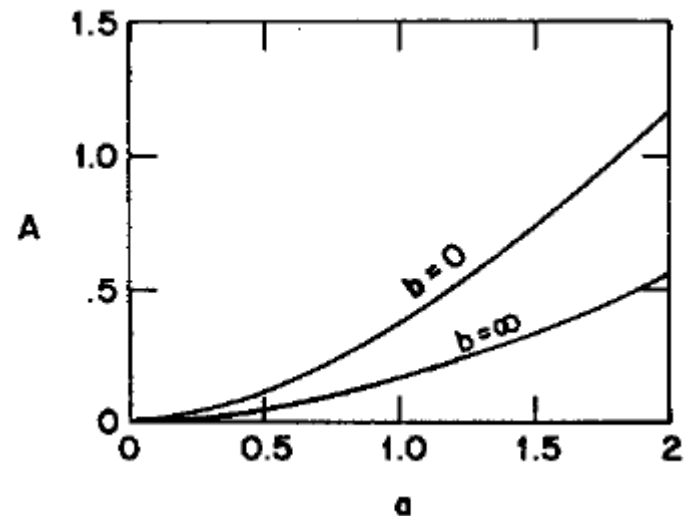
if  $b \gg a$  and  $N \gg 1$

$$R = \frac{\rho}{t} \left[ N + \frac{2}{\pi} \ln \left( \frac{a^2}{4} + 1 \right) - \frac{2a}{\pi} \tan^{-1} \left( \frac{a}{2} \right) \right] = \frac{\rho}{t} [N - A]$$

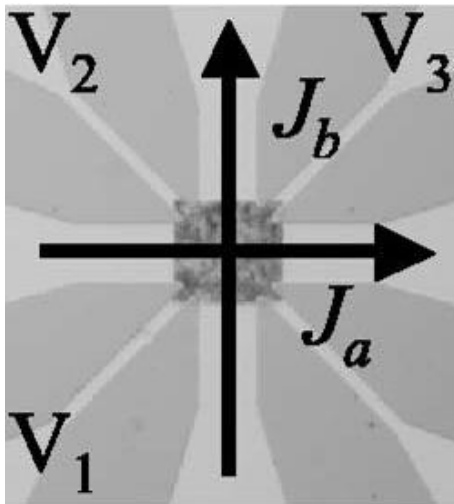
***a** as small as possible*

*and*

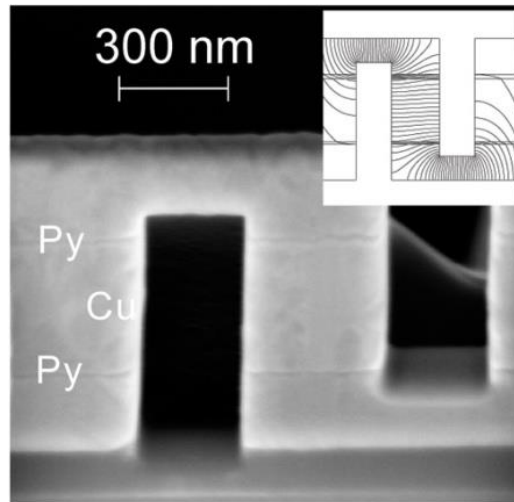
***b** and **N** as large as possible*



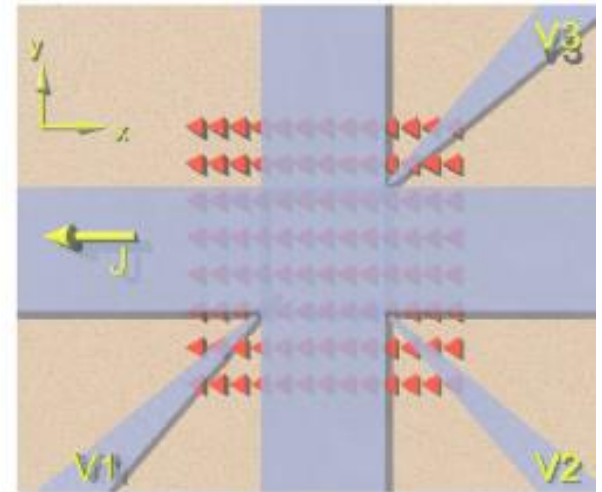
# History: superconductors



Villegas *et al.* (2005)  
Phys. Rev. B **72**, 064507



A Palau *et al.* (2007)  
Phys. Rev. Lett. **98**, 117003



Silhanek *et al.* (2008)  
Appl. Phys. Lett. **92**, 176101

*...substantial deformation of the current-voltage characteristic when the voltage pads are attached close to the vertices.*



# Superconductors (vortex nucleation)

PHYSICAL REVIEW B 84, 174510 (2011)

## Geometry-dependent critical currents in superconducting nanocircuits

John R. Clem

*Ames Laboratory and Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011-3160, USA*

Karl K. Berggren

*Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA and Kavli Institute of Nanoscience,  
Delft University of Technology, Lorentzweg 1, NL-2628CJ Delft, The Netherlands*

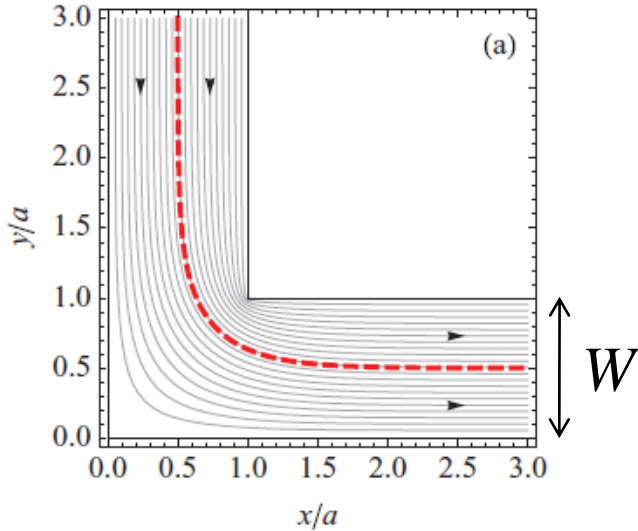
$$\xi \ll W \ll 2\lambda^2 / d$$

**Definition of  $J_c$** ...current at which a nucleating vortex surmounts the Gibbs-free-energy barrier at the wire edge and then is driven entirely across the strip

$$J_c = R J_0 \quad R < 1$$

$J_0$  the critical current of a superconducting strip

# Comparison superconductors vs metals



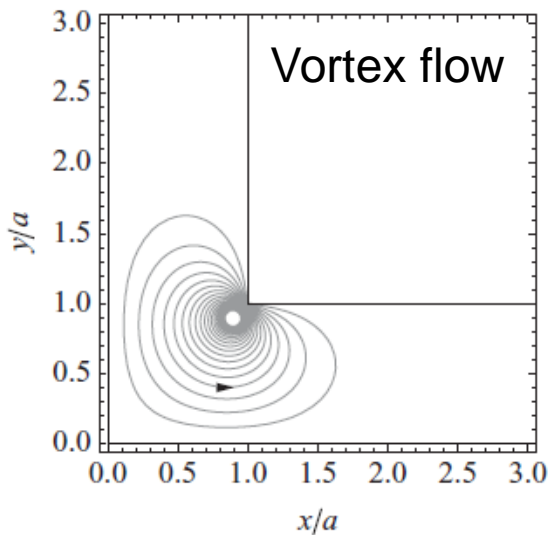
$$J_c = R J_0$$

$$R \xrightarrow{r \rightarrow 0} \frac{3}{2} \left( \frac{\pi \xi}{4W} \right)^{1/3}$$

Clem-Berggren  
(superconductor)

$$R \xrightarrow{r \rightarrow 0} \left( \frac{r}{W} \right)^{1/3}$$

Hagedorn-Hall  
(normal metal)

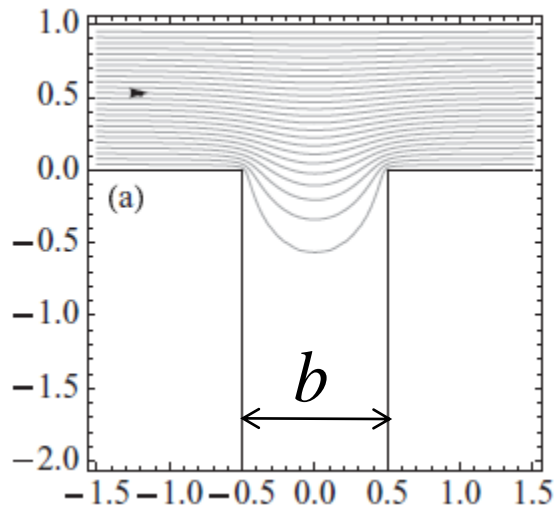


$\lambda$  does not play a role  
The critical current of a right-angle bend is finite

There is an optimum curvature which permits to avoid current crowding. The minimum radius being  $1.27 W$

# CC in voltage and current leads

## Voltage Contact

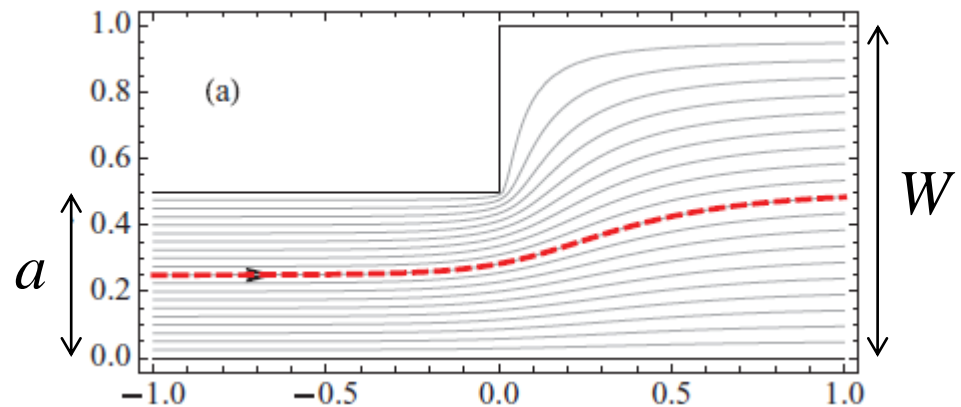


$$C = \frac{3}{2} \left( \frac{\pi \xi}{b} \right)^{1/3} \quad \text{if } \xi < b \ll W$$

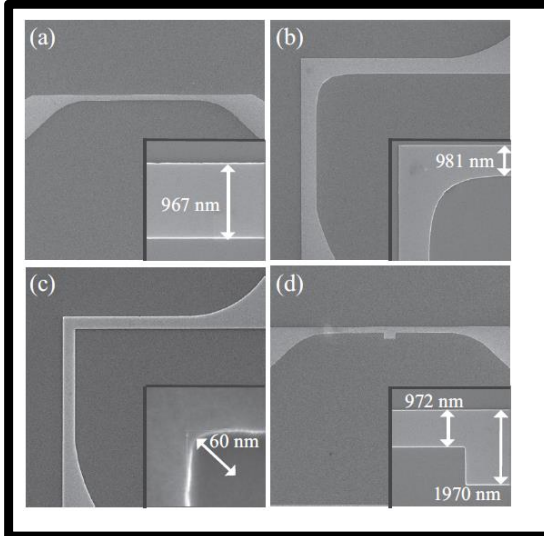
$$C \approx 1 \quad \text{if } b < \xi$$

$$C = \frac{3}{2} \left( \frac{\pi W^2 \xi}{(W^2 - a^2) a} \right)^{1/3}$$

## Current Contact



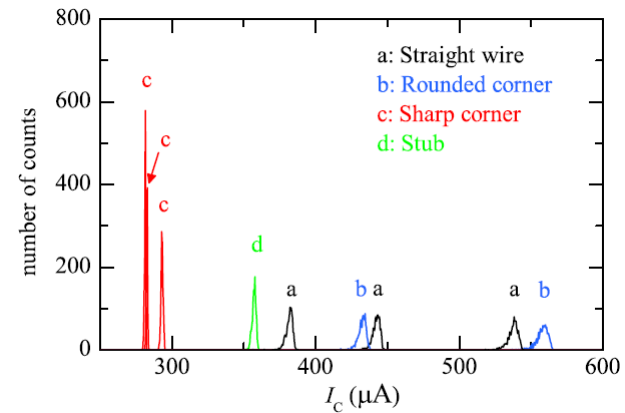
# Supporting experimental evidence



H. L. Hortensius *et al.* Appl. Phys. Lett. **100**, 182602 (2012)

NbTiN

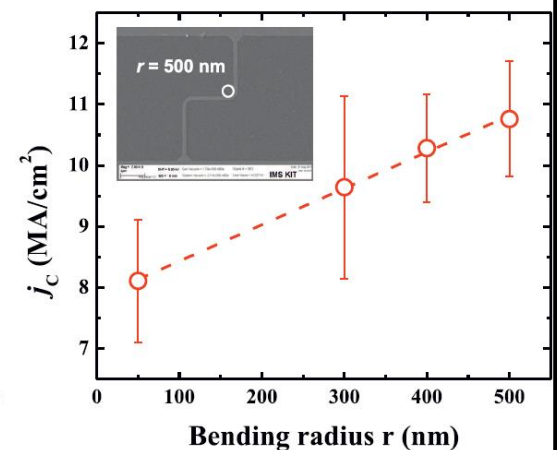
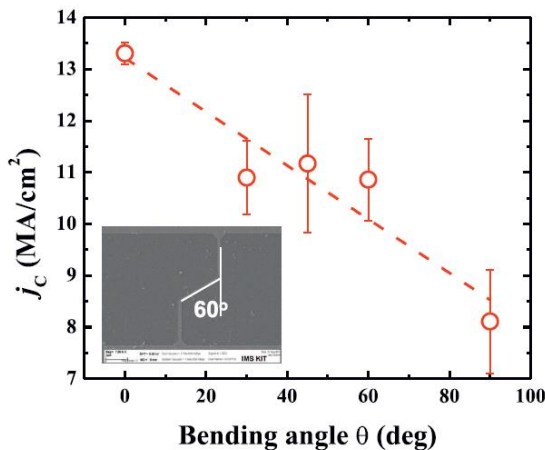
$\xi \sim 7 \text{ nm}$   
 $\Lambda \sim 20 \mu\text{m}$   
 $W \sim 1 \mu\text{m}$



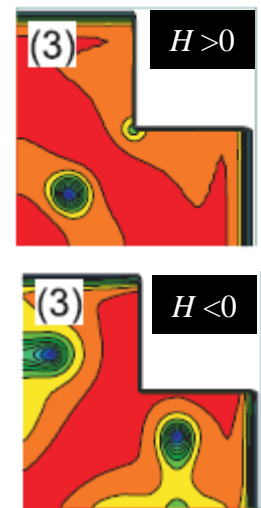
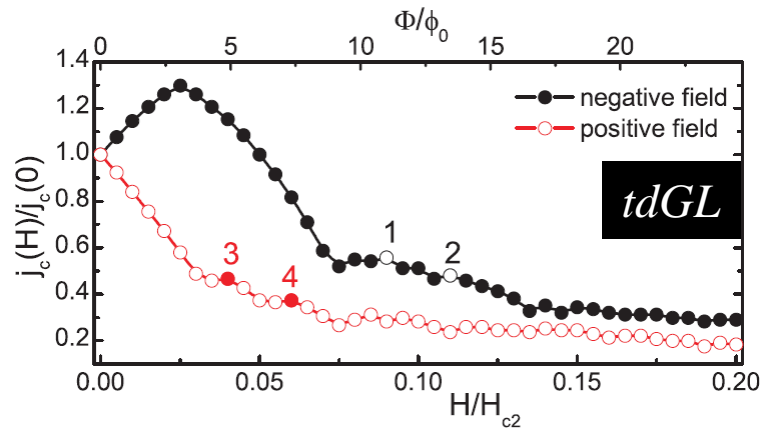
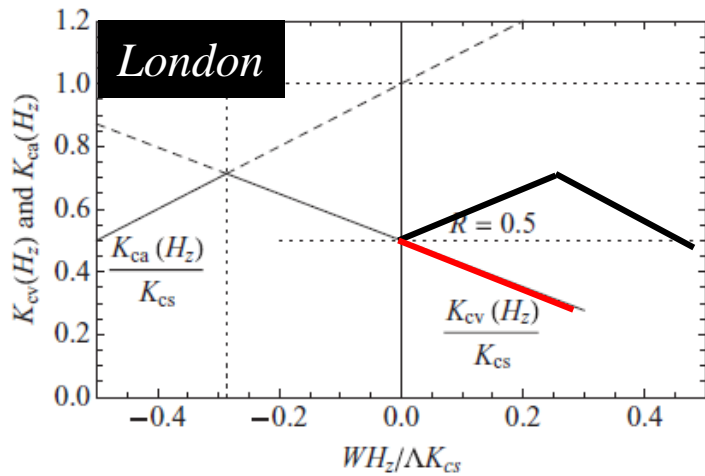
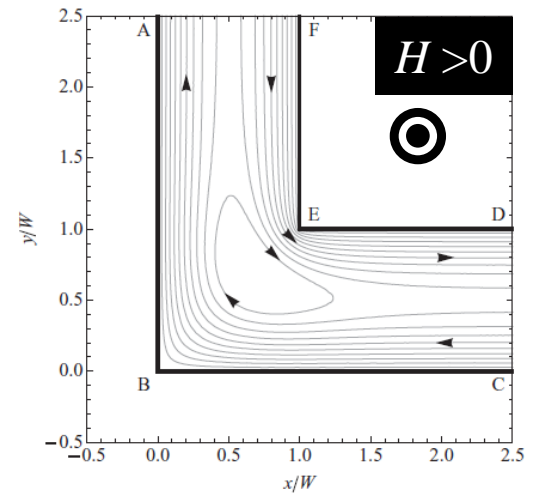
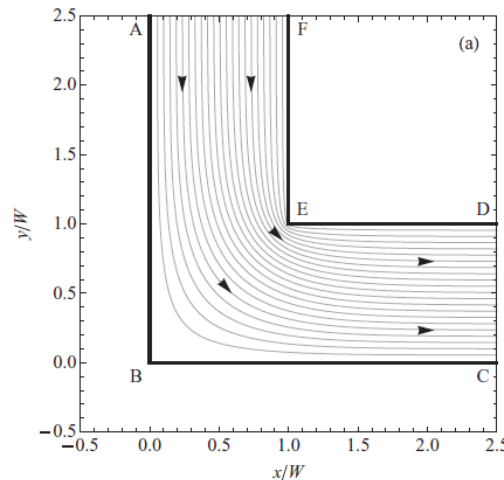
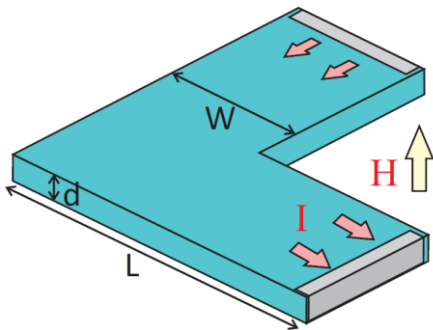
D. Henrich *et al.*, Phys. Rev. B **86**, 144504 (2012)

NbN

$\xi \sim 5 \text{ nm}$   
 $\Lambda \gg W$   
 $W \sim 0,3 \mu\text{m}$

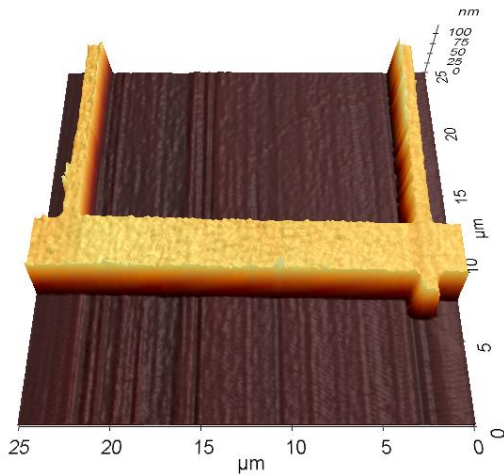


# Field dependence

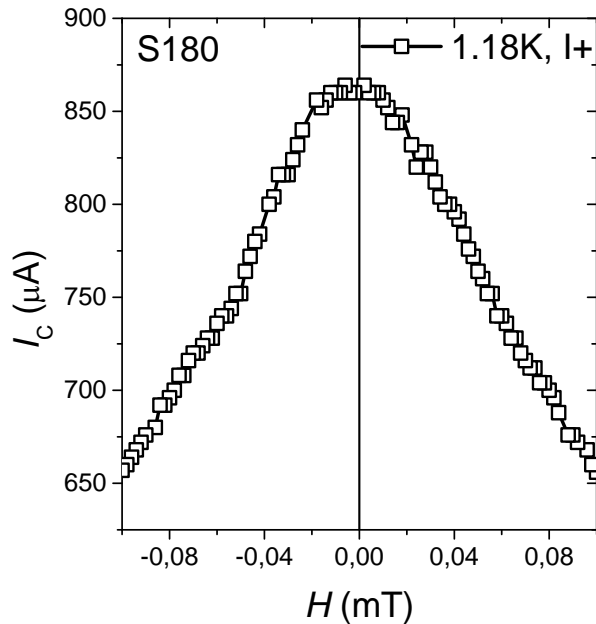
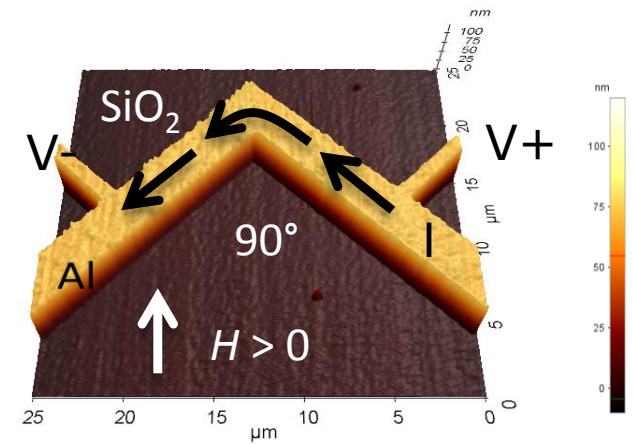


Compensation effect between the field induced stream-lines and the externally applied current at the current crowding point

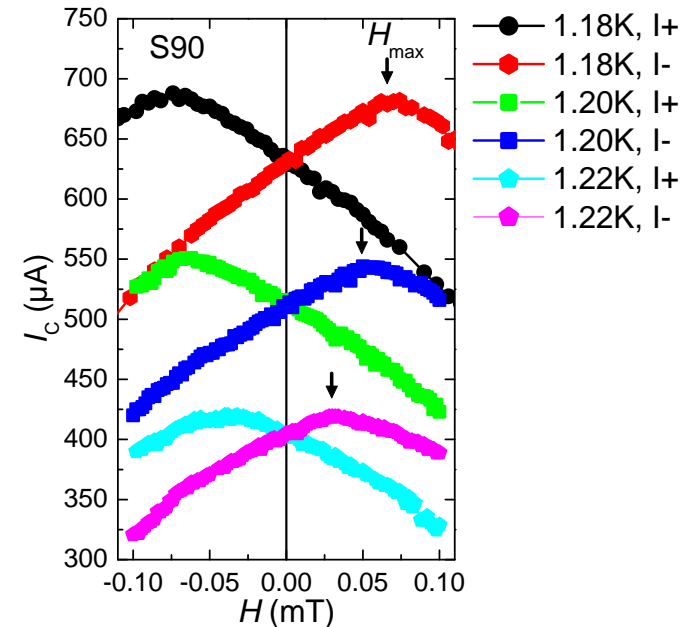
# Experimental confirmation



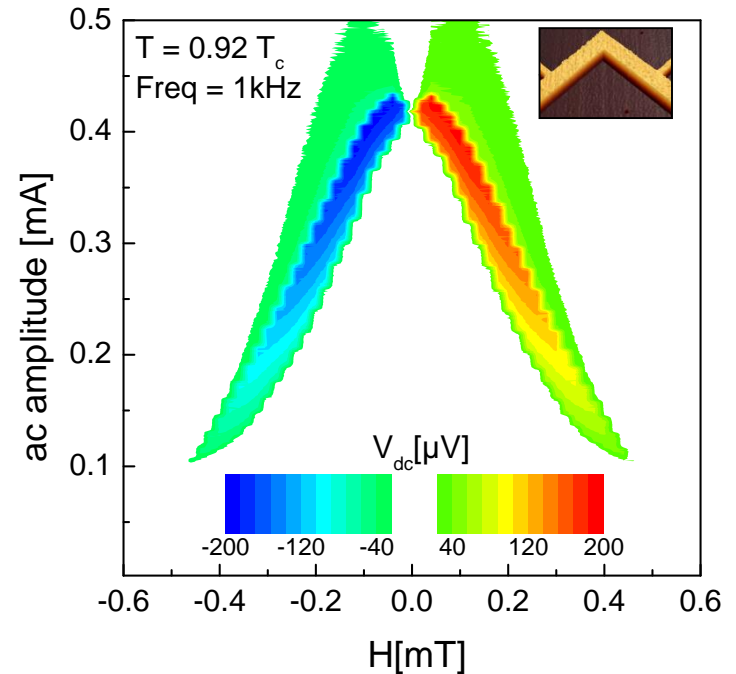
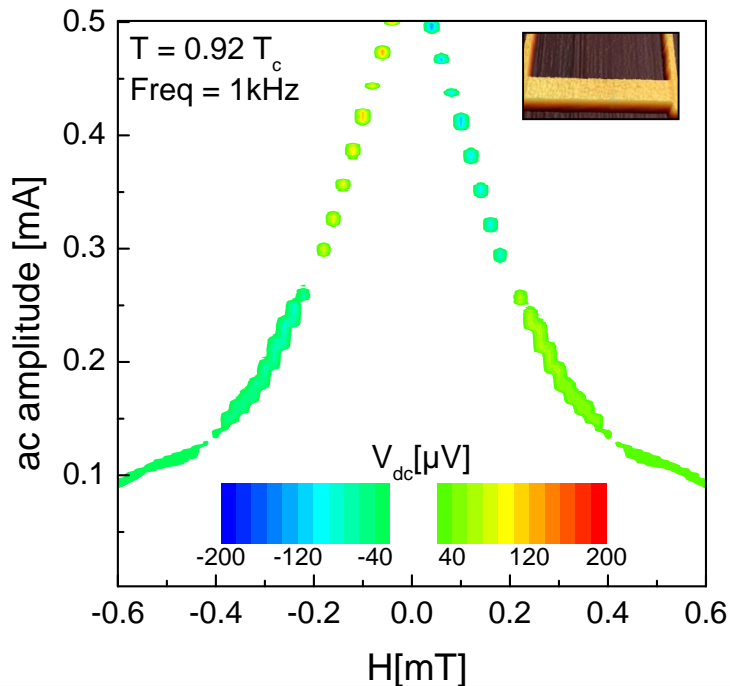
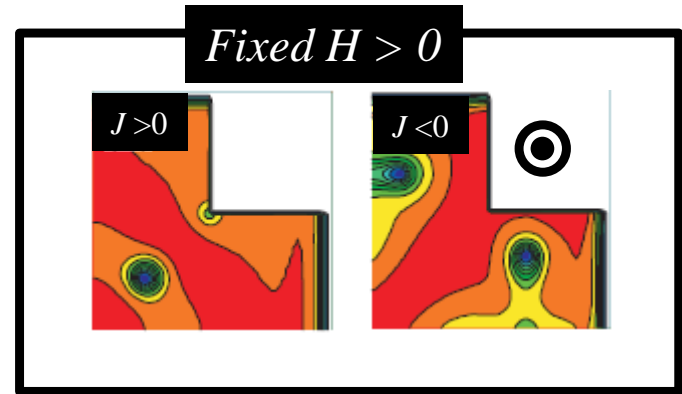
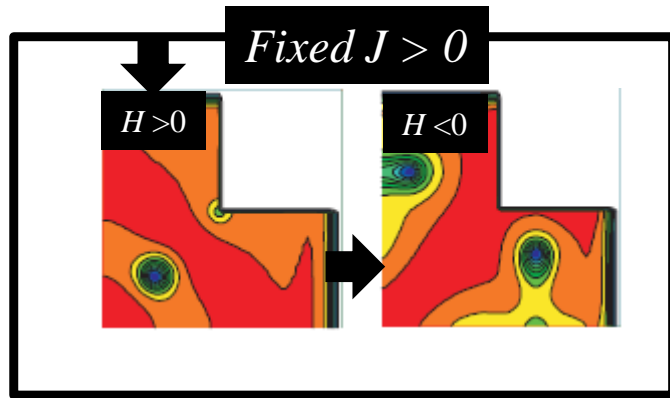
Al  
 $\xi(0) \sim 120 \text{ nm}$   
 $\Lambda(1,22 \text{ K}) \sim 8,3 \text{ }\mu\text{m}$   
 $W \sim 3,3 \text{ }\mu\text{m}$



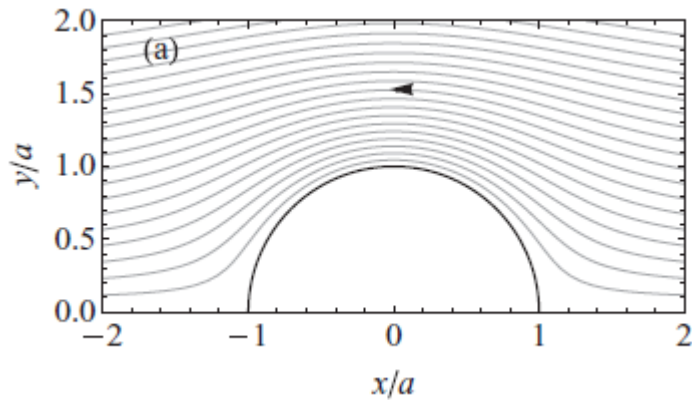
$$H_{\max} \propto \frac{1}{\xi(T)}$$



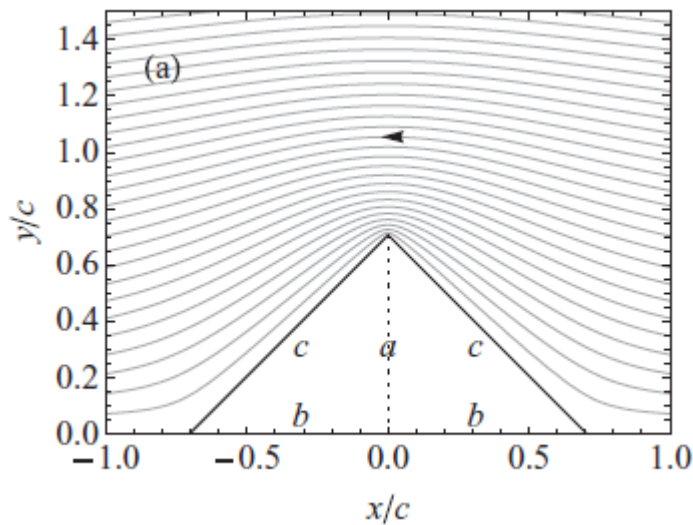
# Rectified motion of vortices



# Surface indentations



$$C \approx \frac{1}{2}$$

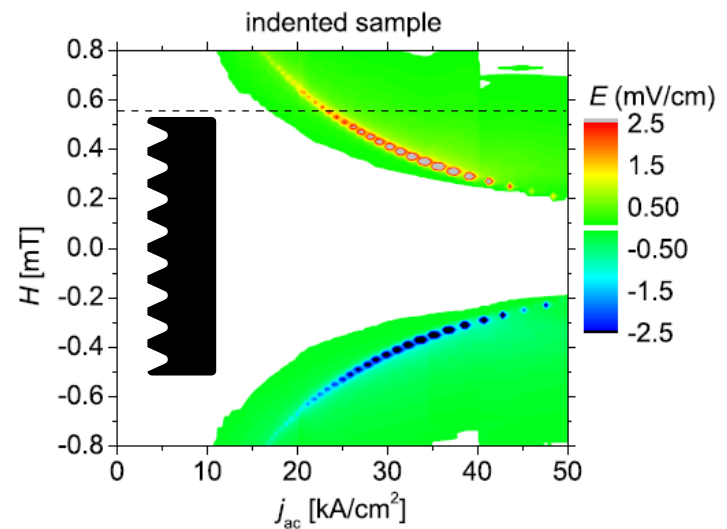
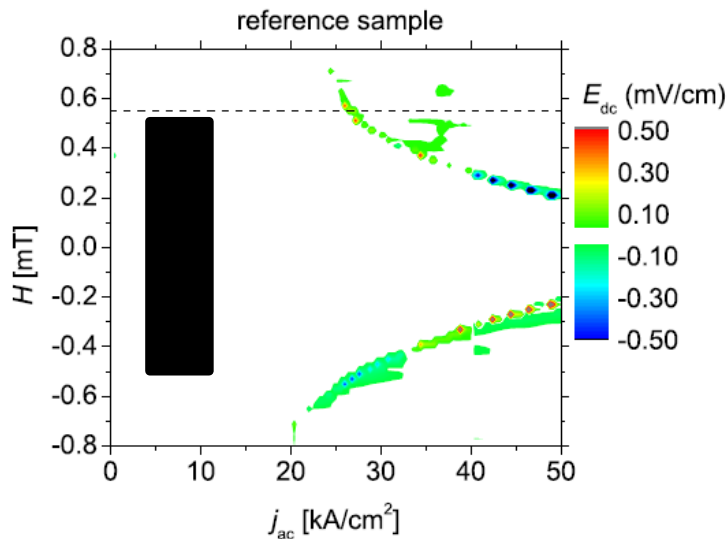
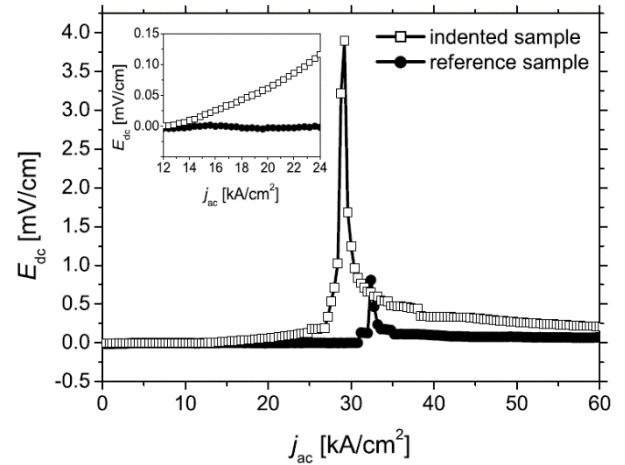
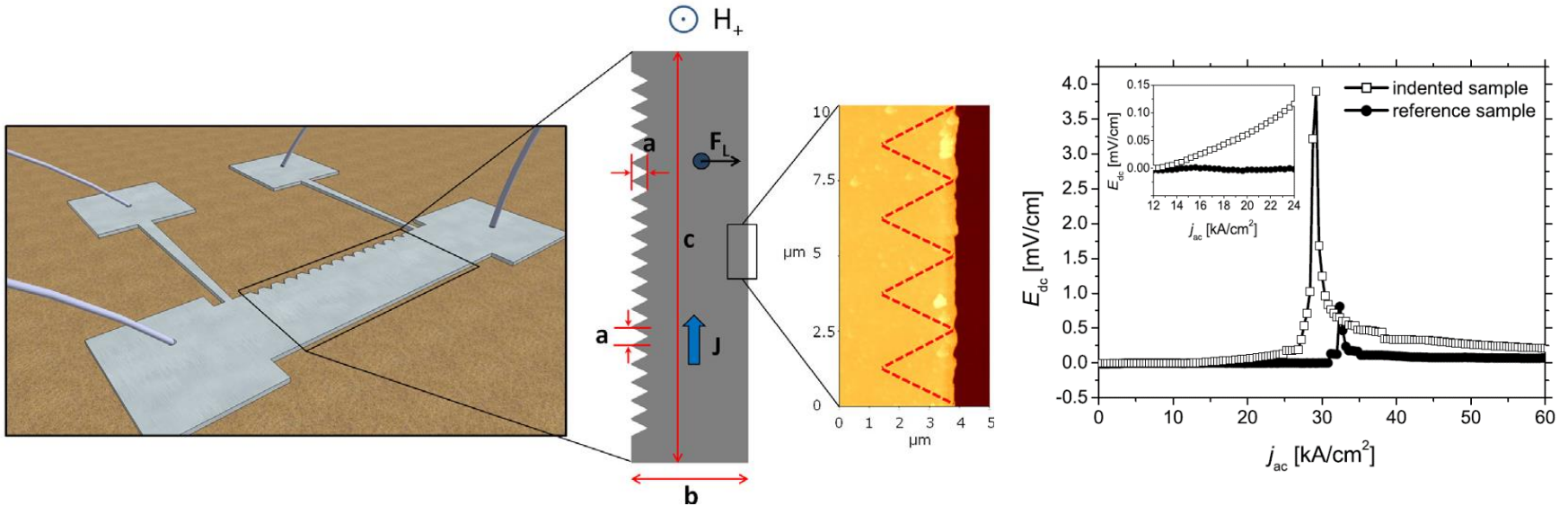


$$C \approx \left( \frac{\xi}{a} \right)^{1/3} \quad \text{if} \quad \theta = 90^\circ$$

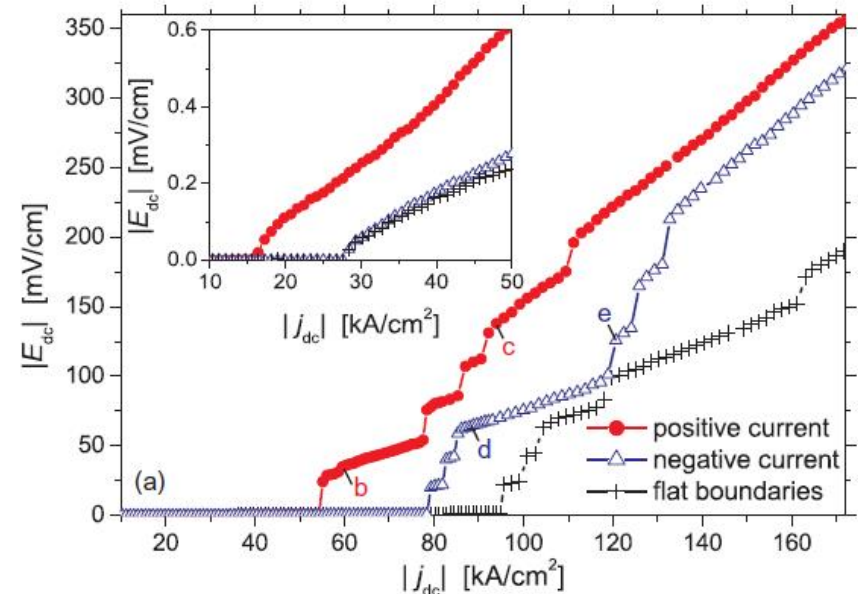
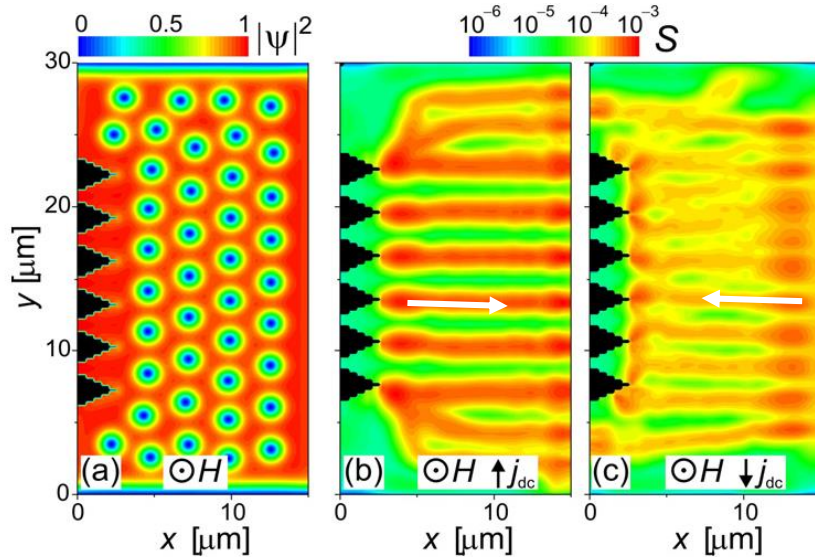
Current crowding is more important for the triangular indentation



# Surface indentations

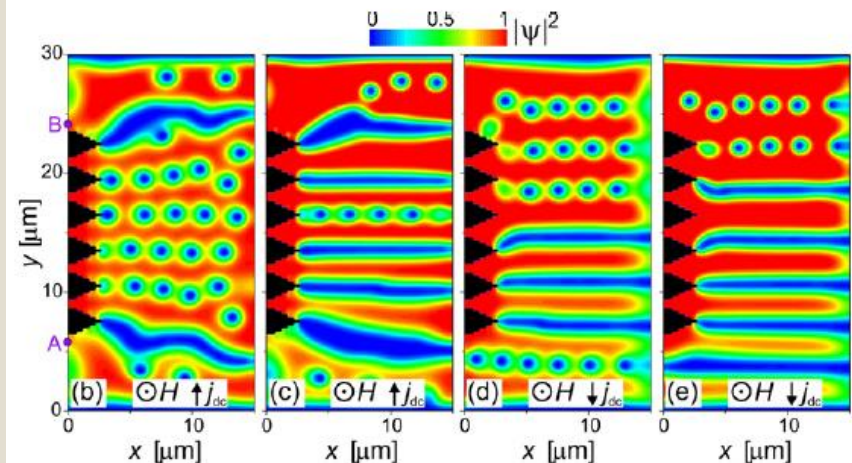


# Surface indentations

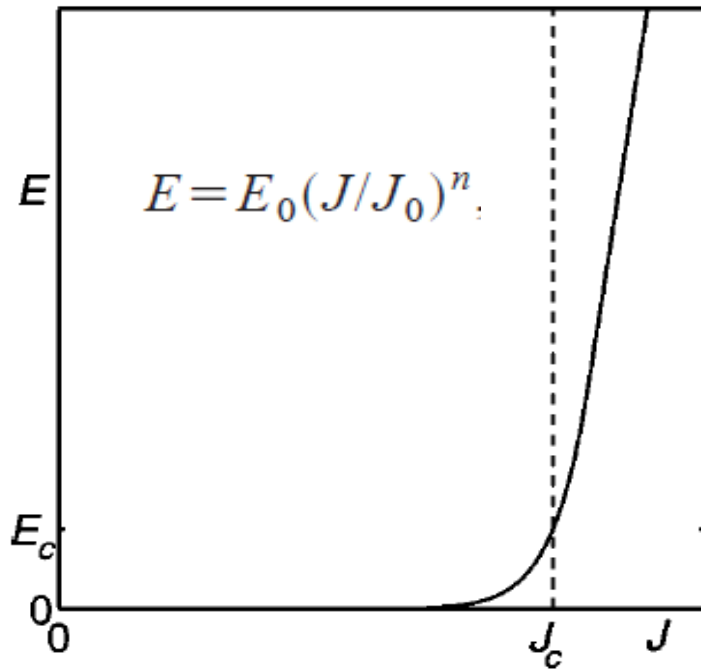


The onset of the resistive regime is mainly determined by the properties of the 'inlet' boundary of the strip.

The effect due to patterning of the 'outlet' boundary facilitates the formation of PSLs

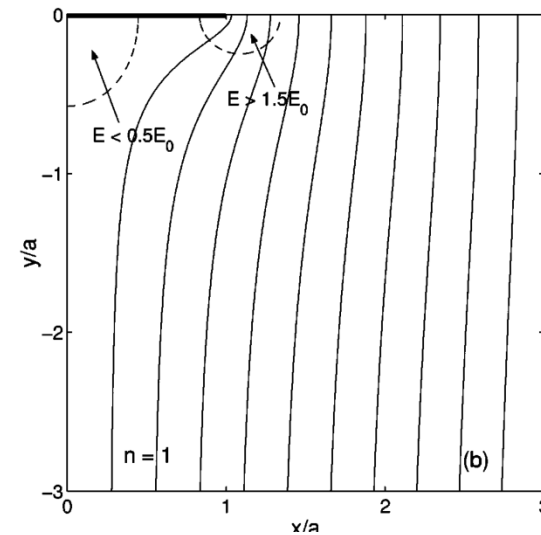
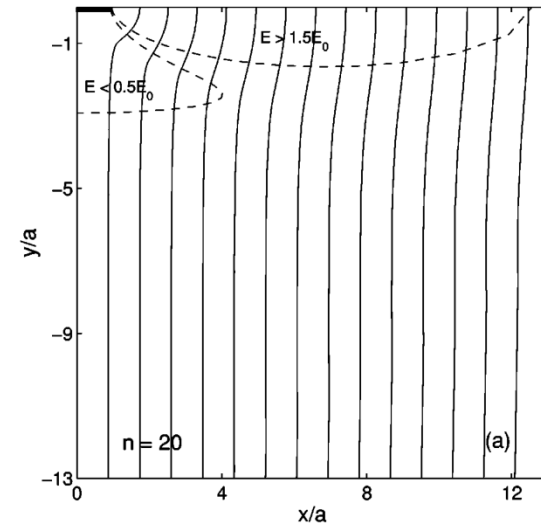


# High field behavior

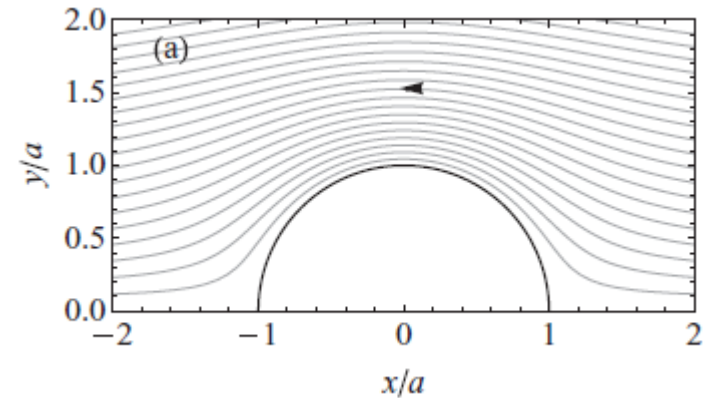
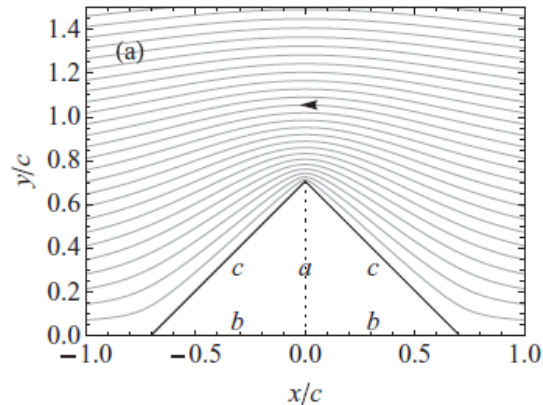


$$L_{\perp} \sim an$$

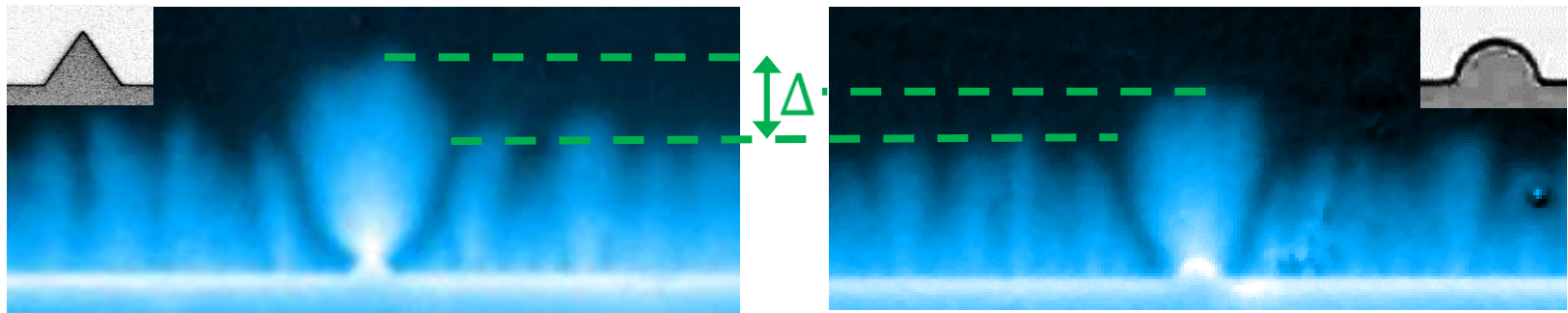
$$L_{\parallel} \sim a\sqrt{n} \gg a$$



# Surface indentations (many vortices)

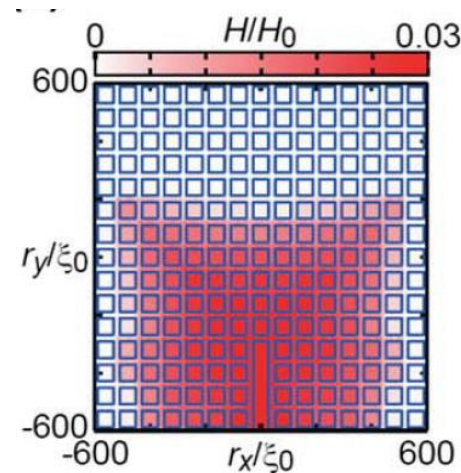
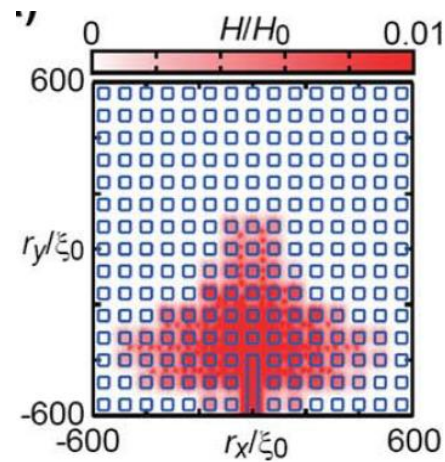
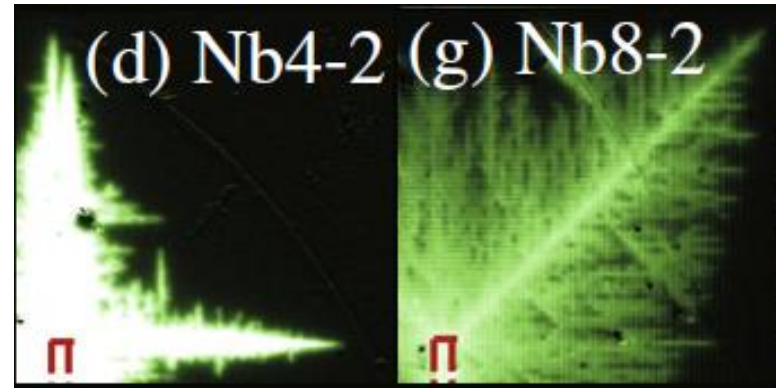
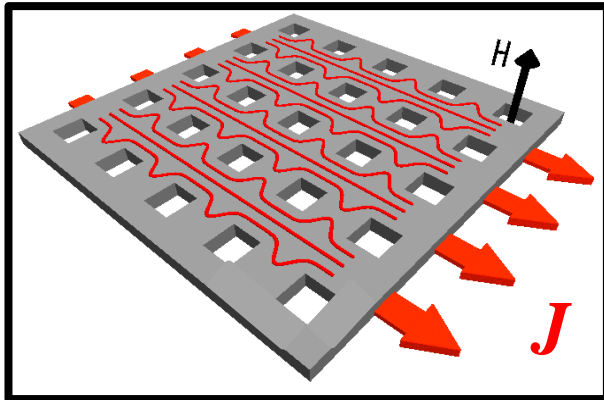


J. I. Vestgård *et al.*, PRB 76, 174509 (2007) → Meissner currents concentrate in front of the indentation where their density reaches  $j_c$  and hence lead to even deeper flux penetration. This is why the flux front near the indentation advances faster than in the rest of the film.



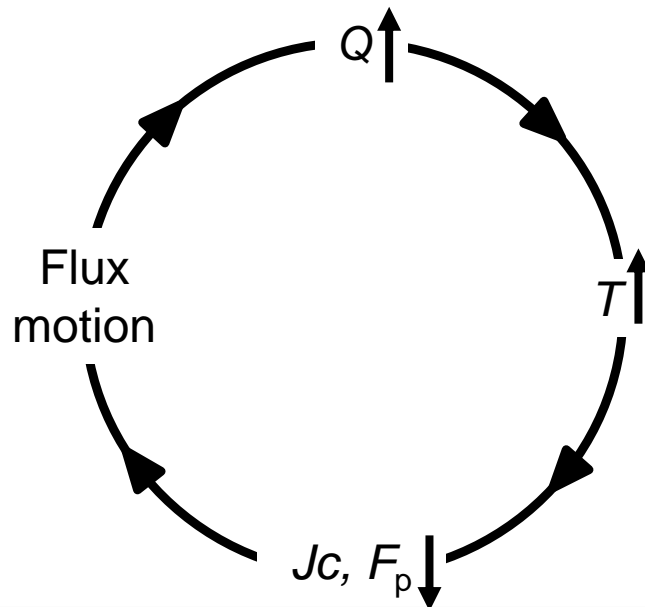
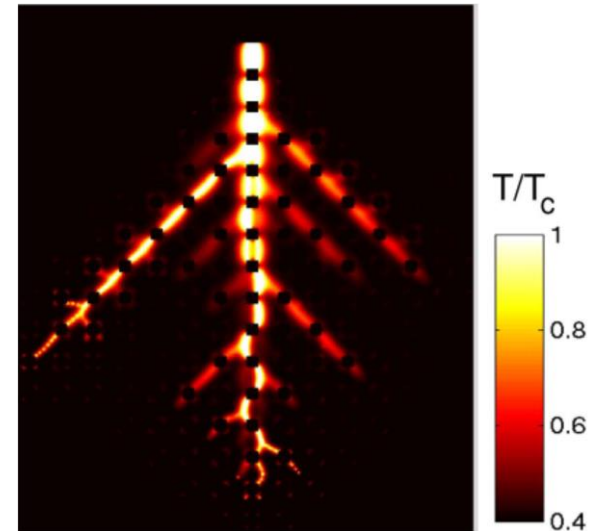
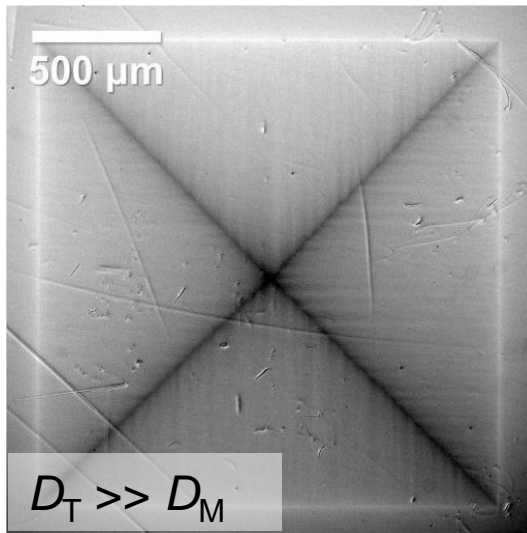
Nb,  $H=2$  mT,  $T=4$ K

# CC in nanostructured superconductors



Nakai & Machida *Physica C* **470** 1148 (2010)

# Magnetic flux avalanches



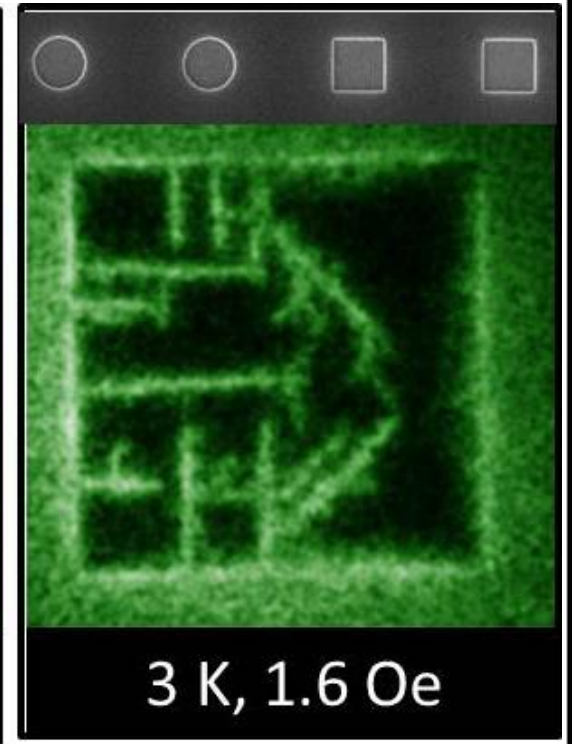
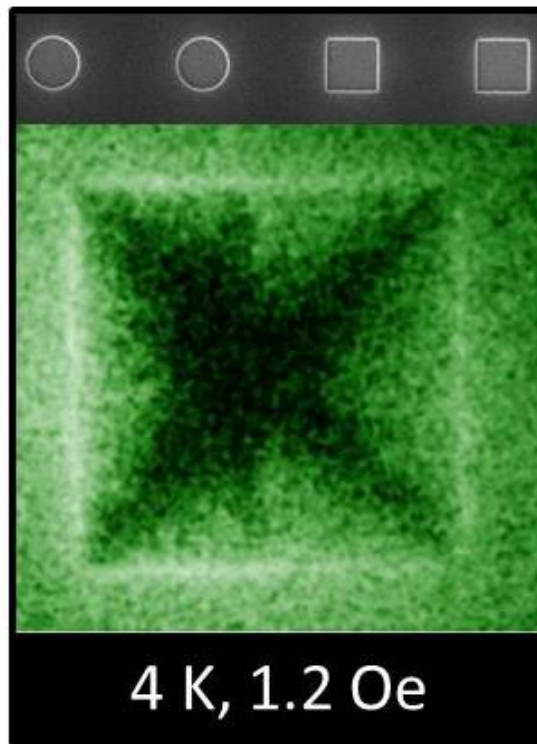
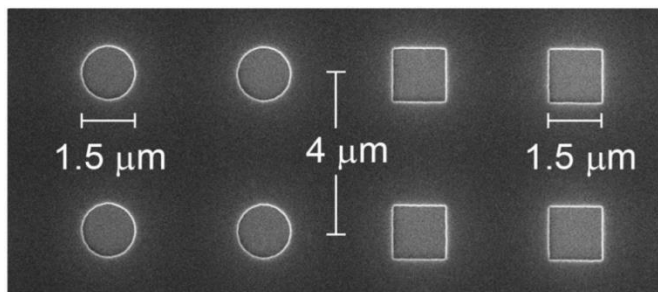
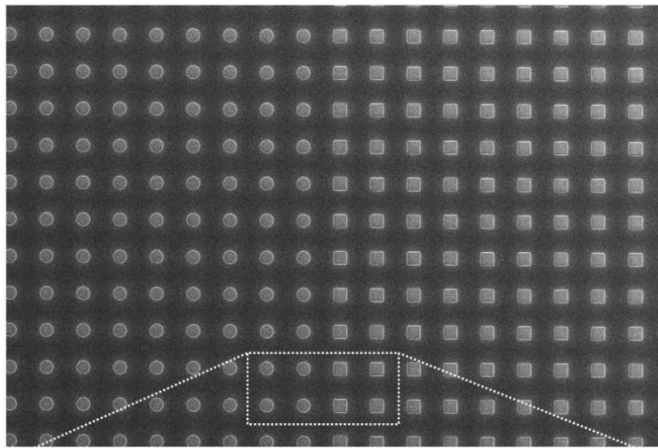
Adiabatic conditions,  $\Delta T = Q/C(T)$

$v > 10 \text{ km/s} > \text{sound velocity } 3 \text{ km/s}$

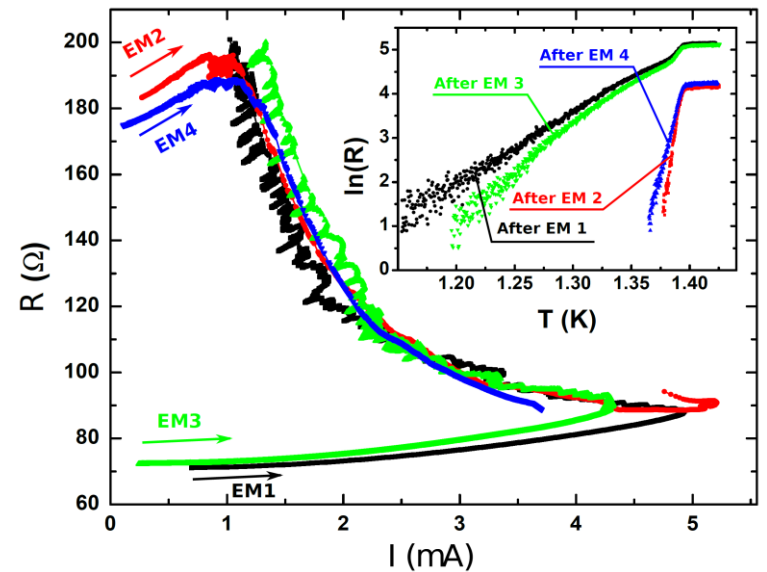
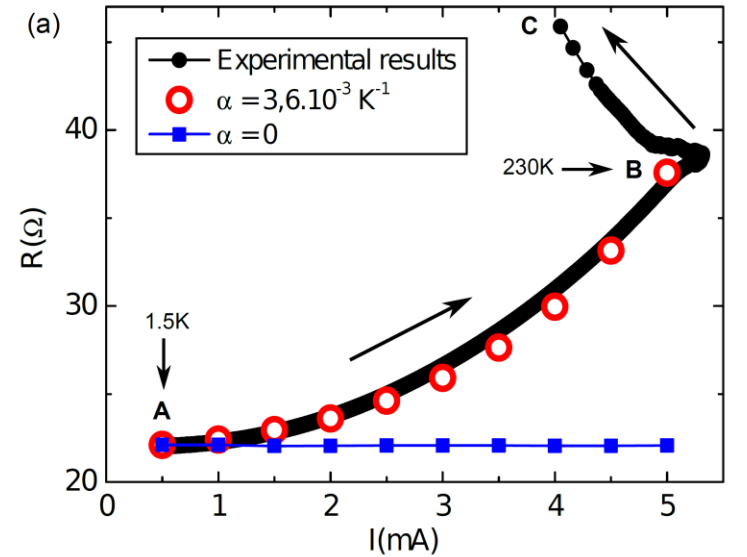
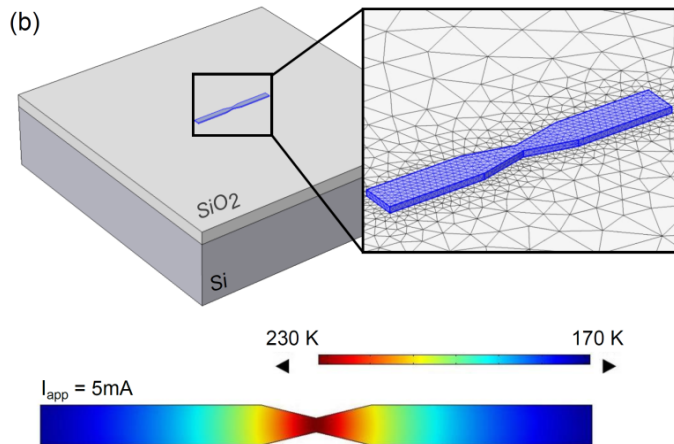
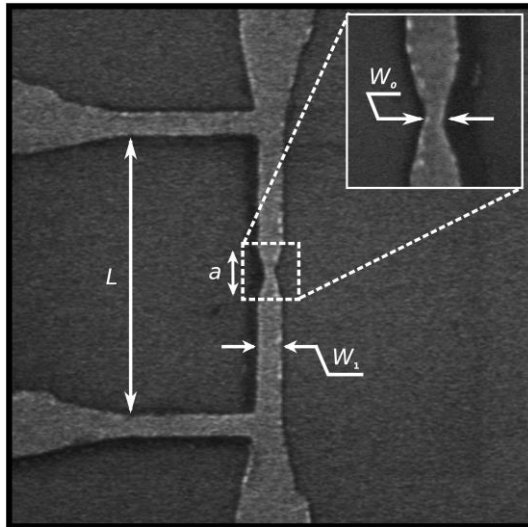
$v_{\text{Abrikosov}} \ll 1 \text{ km/s}$

$v_{\text{kinematics}} \sim 1-10 \text{ km/s}$

# Magnetic flux avalanches



# Electromigration



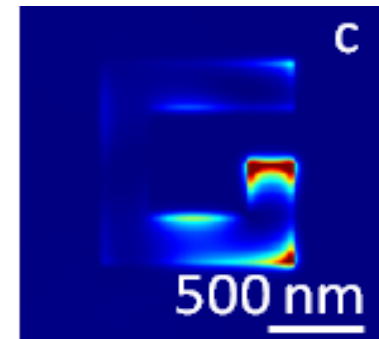
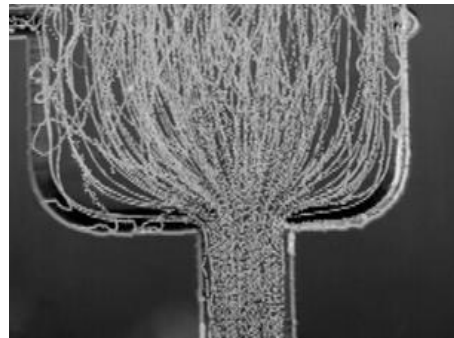
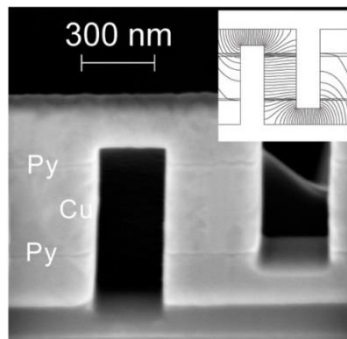
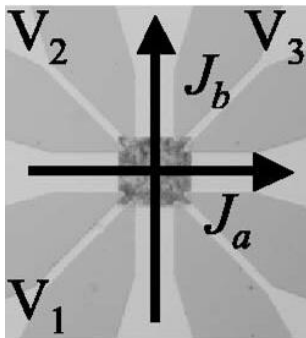


# Conclusion

In the same way that magnetic field lines lead to demagnetization effects, deformation of current stream lines lead to current crowding.

This effect have important consequences on

- the resistance calculation in normal metals
- $V(I)$  characteristics in superconductors
- unwanted ratchet signal
- hot spots (joule heating)
- reduction of the critical current



**fnr**s  
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