

Dynamical flavor origin of \mathbb{Z}_N symmetries

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Discrete Abelian symmetries (\mathbb{Z}_N) are a common “artifact” of beyond the standard model physics models. They provide different avenues for constructing consistent scenarios for lepton and quark mixing patterns, radiative neutrino mass generation as well as dark matter stabilization. We argue that these symmetries can arise from the spontaneous breaking of the Abelian $U(1)$ factors contained in the global flavor symmetry transformations of the gauge invariant kinetic Lagrangian. This will be the case provided the ultra-violet completion responsible for the Yukawa structure involves scalar fields carrying non-trivial $U(1)$ charges. Guided by minimality criteria, we demonstrate the viability of this approach with two examples: first, we derive the “scotogenic” model Lagrangian, and second, we construct a setup where the spontaneous symmetry breaking pattern leads to a \mathbb{Z}_3 symmetry which enables dark matter stability as well as neutrino mass generation at the 2-loop order. This generic approach can be used to derive many other models, with residual \mathbb{Z}_N or $\mathbb{Z}_{N_1} \times \dots \times \mathbb{Z}_{N_k}$ symmetries, establishing an intriguing link between flavor symmetries, neutrino masses and dark matter.

I. INTRODUCTION

Addressing the flavor puzzle, that is to say pinning down the origin of the standard model (SM) fermion mass hierarchies and mixing patterns, has led to the formulation of different and ample number of theoretical ideas. Although seemingly unrelated, most of these approaches follow two conceptually distinct theoretical trends: (a) the underlying flavor theory involves new flavor symmetries under which different generation SM fermions (quarks and charged leptons) carry different charges. Mass hierarchies and mixing patterns are thus understood as a consequence of the different transformation properties of quarks and charged leptons, which in the SM Yukawa sector—being an effective realization of the fundamental flavor theory—are not manifest. Certainly, the Froggatt-Nielsen mechanism [1] provides the most representative example for this kind of approaches. (b) The other avenue consists in promoting the maximal global flavor symmetry of the SM gauge invariant kinetic Lagrangian ($G_F = U(3)^5 \times U(1)_H$, in shorthand notation) [2] to a fundamental flavor symmetry, something that calls for a G_F -invariant ultra-violet (UV) completion, endowed with scalar (flavon) fields capable of triggering spontaneous symmetry breaking (SSB) of G_F (or its subgroups) [3–16]. In this picture, therefore, mass (Yukawa) hierarchies and mixing patterns result from a high-scale dynamics rather than by a mismatch in SM fermion fields new quantum numbers.

Arguably, if a theory of flavor is indeed at work at a certain high-energy scale (Λ), it should provide as well the framework for other phenomenological puzzles to be addressed. In particular, one could envisage a more profound picture from where neutrino masses, the baryon asymmetry of the universe

and dark matter emerge. Thus, from this perspective, in principle unrelated phenomena would be just manifestations of a comprehensive scenario in which the flavor symmetry plays—directly or indirectly—an essential rôle in e.g. neutrino mass generation or dark matter stabilization.

In this letter we show that several aspects of such endeavor can be pursued without specifying the complete UV theory. Adopting approach (b), we consider the generation of residual discrete Abelian symmetries \mathbb{Z}_N whose origin can be traced back to the flavor symmetry of the gauge invariant kinetic Lagrangian. We rely on the Abelian part of the complete flavor symmetry, following well-established methods to induce residual symmetries from $U(1)$ -invariant theories [17, 18]. The feasibility of the approach is shown with two example models, both with an extended lepton sector and a second Higgs doublet: (i) a minimal scenario where the breaking of the flavor symmetry leads to the *scotogenic* model Lagrangian [19], and (ii) a different scenario leading to a remnant \mathbb{Z}_3 symmetry, which not only stabilizes dark matter, but also guarantees neutrino mass generation at the 2-loop level. These worked-examples are of course not unique, e.g. several other higher order remnant symmetries can be generated, including even direct products $\mathbb{Z}_{N_1} \times \dots \times \mathbb{Z}_{N_k}$.

SSB of $U(1)$ symmetries and their connection with discrete \mathbb{Z}_N symmetries are not at all new subjects, see e.g. [20–27]. We find however a pivotal conceptual difference between these approaches and what we here aim at discussing: the $U(1)$ factors are “sourced” by the same symmetry G_F that dictates fermion mass hierarchies and mixing patterns. Thus, in scenarios where the remnant discrete \mathbb{Z}_N symmetry plays a rôle in e.g. dark matter stabilization and/or neutrino mass generation, this approach can be—conceivably—understood as a first step towards the establishment of a common comprehensive framework for flavor, neutrino and dark matter physics¹.

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¹ Dark matter stabilization in the minimal flavor violating [28] context has been considered in Ref. [29]. Rather than making use of the Abelian sym-

II. THE STANDARD MODEL FLAVOR SYMMETRY AND ITS REMNANT \mathbb{Z}_N SYMMETRIES

As has been previously anticipated, the group of global symmetry transformations of the gauge invariant kinetic terms of the SM quark doublet (q_L) and singlets (u_R, d_R), lepton doublet (ℓ_L) and singlet (e_R), and Higgs doublet (H) is given by [2]

$$G_F = \left[\prod_a SU(3)_a \times U(1)_a \right] \times U(1)_H \quad (1)$$

where $a = \{q_L, u_R, d_R, \ell_L, e_R\}$. The SM Yukawa interactions, however, explicitly break this symmetry leaving behind just five global $U(1)$ factors:

$$G_{SM} = U(1)_B \times \left[\prod_{\alpha=\{e,\mu,\tau\}} U(1)_{L_\alpha} \right] \times U(1)_Y, \quad (2)$$

readily identifiable with conservation of baryon (B) and lepton flavor (L_α) numbers (SM accidental symmetries) and hypercharge (Y), which according to Tab. I are given by the following linear combinations of the $U(1)_a$ charges:

$$B = \frac{1}{3} (Q_{q_L} + Q_{u_R} + Q_{d_R}), \quad L_\alpha = (Q_{\ell_L} + Q_{e_R})_\alpha, \quad (3)$$

$$Y = \frac{1}{6} (Q_{q_L} + 4Q_{u_R} - 2Q_{d_R}) + \frac{1}{2} (Q_H - Q_{\ell_L} - 2Q_{e_R}). \quad (4)$$

It is worth stressing at this point that, since massive neutrinos is an experimental fact, $U(1)_{L_\alpha}$ is actually broken. If neutrinos have Dirac masses, one can have total lepton number $U(1)_L$ conservation. On the other hand, if neutrinos have Majorana masses, even $U(1)_L$ is broken. Formal invariance of the full Lagrangian under G_F can however be recovered, provided the Yukawa couplings are promoted to complex scalar fields, i.e. flavon fields with definitive G_F transformations (see Tab. I) but singlets under the SM gauge symmetry. At the ‘‘fundamental’’ level, this means that at some unknown—but certainly large—energy scale, G_F is an exact symmetry of the UV Lagrangian. The flavor symmetry is then spontaneously broken by the vacuum expectation values (vevs) of new heavy scalar degrees of freedom with suitable G_F transformation properties. Thus, in that picture the SM Yukawa Lagrangian (which emerges once below the characteristic UV energy scale Λ the heavy degrees of freedom are integrated out) is an effective manifestation of the flavored UV theory, namely

$$-\mathcal{L}_{SM} = \overline{q_L} \frac{\langle Y_u \rangle}{\Lambda} u_R \tilde{H} + \overline{q_L} \frac{\langle Y_d \rangle}{\Lambda} d_R H + \overline{\ell_L} \frac{\langle Y_e \rangle}{\Lambda} e_R H + \text{H.c.}, \quad (5)$$

where $\tilde{H} = \varepsilon H^*$ with $\varepsilon = i\tau_2$ and τ_2 the second Pauli matrix. The Lagrangian, written in this way, assumes that $\langle Y_X \rangle$ triggers not only SSB of the non-Abelian sector of G_F but also of

	$\prod_a SU(3)_a$	Q_{q_L}	Q_{u_R}	Q_{d_R}	Q_{ℓ_L}	Q_{e_R}	Q_H
q_L	$3 \times 1 \times 1 \times 1 \times 1$	1	0	0	0	0	0
u_R	$1 \times 3 \times 1 \times 1 \times 1$	0	1	0	0	0	0
d_R	$1 \times 1 \times 3 \times 1 \times 1$	0	0	1	0	0	0
ℓ_L	$1 \times 1 \times 1 \times 3 \times 1$	0	0	0	1	0	0
e_R	$1 \times 1 \times 1 \times 1 \times 3$	0	0	0	0	1	0
H	$1 \times 1 \times 1 \times 1 \times 1$	0	0	0	0	0	1
Y_u	$3 \times \bar{3} \times 1 \times 1 \times 1$	1	-1	0	0	0	1
Y_d	$3 \times 1 \times \bar{3} \times 1 \times 1$	1	0	-1	0	0	-1
Y_e	$1 \times 1 \times 1 \times 3 \times \bar{3}$	0	0	0	1	-1	-1

TABLE I. Flavor transformation properties of the SM fermion and Higgs fields as well as of the scalar fields $Y_{u,d,e}$. Q_a and Q_H stand for the charges of the different $U(1)$ factors.

the Abelian sector, the $U(1)$ factors. This choice is to some extent arbitrary, and indeed more interesting possibilities do exist, see for example [10].

The picture described so far is however incomplete since neutrino masses must be generated. Here we will assume that neutrinos are Majorana fermions, thus breaking lepton number. Although one could well extend the SM Lagrangian to include the dimension five lepton number breaking operator $\ell\ell HH$ [31], we stick to the *standard* picture involving *three* right-handed neutrinos ν_R . This implies enlarging G_F to $G_F \times U(1)_{\nu_R} \times SU(3)_{\nu_R}$ [15, 32], and introducing additional flavon fields to give rise to the observed mass spectrum and mixing in the lepton sector through SSB. Furthermore, it requires extending the definition of lepton number in Eq. (3) to $L_\alpha = (Q_{\ell_L} + Q_{e_R} + Q_{\nu_R})_\alpha$. In this context, $U(1)_{\nu_R}$ breaking has been shown to have deep implications, allowing the detachment of the right-handed neutrino mass and lepton number-breaking scales [33], or rendering right-handed neutrino production viable, and in some cases implying large charged lepton flavor-violating effects [34].

Certainly the dynamics of the non-Abelian sector of G_F plays a rôle. Actually, in full generality, one should expect the UV completion to involve not only SSB of the non-Abelian structure via ‘‘Yukawa’’ fields, but also of the Abelian one through flavon fields. Thus, the question is then whether that dynamics leaves traces beyond those that we have discussed. This might be indeed the case, provided the flavored UV completion involves scalar fields with suitable charges under some of the $U(1)$ global factors (a single one suffices). Let us discuss this in more detail. Consider a simple model of two self-interacting scalar fields ($\sigma_{1,2}$) subject to the following global $U(1)$ transformations [17, 18]:

$$\sigma_1 \rightarrow e^{iN\alpha} \sigma_1 \quad \text{and} \quad \sigma_2 \rightarrow e^{-i\alpha} \sigma_2. \quad (6)$$

The $U(1)$ -invariant renormalizable as well as non-renormalizable Lagrangian describing such system is given by:

$$\mathcal{L} = \mu_i^2 \sigma_i^* \sigma_i + \lambda_{ij} (\sigma_i^* \sigma_i) (\sigma_j^* \sigma_j) + \frac{\lambda_{\sigma_M}}{\Lambda^{M(N+1)-4}} \sigma_1^M (\sigma_2^N)^M, \quad (7)$$

metries in the flavor symmetry group, this approach relies on the non-Abelian part. Dark matter stabilization through flavor discrete symmetries was investigated in [30].

with μ_i dimension one and $\lambda_{ij}, \lambda_{\sigma M}$ dimensionless couplings. If σ_1 acquires a vev the resulting Lagrangian will involve a collection of terms $(\sigma_2^N)^M$, thus being \mathbb{Z}_N invariant, namely

$$\sigma_2 \rightarrow \eta_N^n \sigma_2 \quad \text{with} \quad \eta_N = e^{2\pi i/N} \quad (n = 0, 1, \dots, N-1). \quad (8)$$

Although discussed in a rather simple context, this idea can be extended to realistic models involving fermion fields and therefore Yukawa terms. There is however something that one should bear in mind. SSB of the global flavor symmetry, including its $U(1)$ factors, imply the presence of massless Nambu-Goldstone bosons for which a large variety of phenomenological constraints exist, including rare decays, cosmological and astrophysical data. One solution is that of gauging part of or the full flavor symmetry, which of course implies the presence of new gauge bosons and calls for gauge anomaly cancellation which requires the introduction of new fermions [7, 8]. If one insists on a global symmetry, phenomenological consistency can be achieved provided SSB takes place at a rather high energy scale, which will suppress couplings to fermions [35, 36]. However, scalar couplings with the Higgs sector at the renormalizable level can potentially affect the Higgs phenomenology (e.g. giving rise to Higgs invisible decays).

III. EMERGING \mathbb{Z}_N -BASED MODELS: BENCHMARK EXAMPLES

Guided by the aforementioned considerations, we now turn to the discussion of some specific \mathbb{Z}_N -based realizations. Since we consider only the (extended) lepton sector, we assume that all beyond the SM (BSM) fields transform trivially under the quark flavor symmetry, $U(3)_{q_L} \times U(3)_{u_R} \times U(3)_{d_R}$, and we only specify their transformation properties under $(SU(3)_{\ell_L}, SU(3)_{e_R}, SU(3)_{\nu_R})_{(U(1)_{\ell_L}, U(1)_{e_R}, U(1)_{\nu_R}, U(1)_H)}$. In particular, we focus on fields which give rise to light neutrino masses. For instance, the SM lepton and Higgs doublet are written respectively as $\ell_L = (3, 1, 1)_{(1,0,0,0)}$ and $H = (1, 1, 1)_{(0,0,0,1)}$. For definiteness, we consider models extended with a second Higgs doublet $\Phi = (1, 1, 1)_{(0,0,-1,-1)}$ and three right-handed neutrinos $\nu_R = (1, 1, 3)_{(0,0,1,0)}$ while the flavon fields which we will introduce below are singlets under the SM gauge symmetry. It is worth stressing that the approach discussed here is not limited to scenarios with right-handed neutrinos. In principle, it can be implemented in a large variety of BSM scenarios, where the inclusion of additional degrees of freedom leads to an enlargement of the flavor symmetry and thus to additional $U(1)$ factors whose SSB result in remnant discrete symmetries. This includes models with larger symmetry groups and models with a non-trivial embedding of the quark sector.

Model I: With a setup defined as above, one can envisage different flavor-invariant Lagrangians depending on the number of available gauge singlet flavon fields. Assuming a minimal content, subject to flavor transformation properties given by $Y_e = (3, \bar{3}, 1)_{(1,-1,0,-1)}$, $Y_\nu = (3, 1, \bar{3})_{(1,0,0,1)}$ and

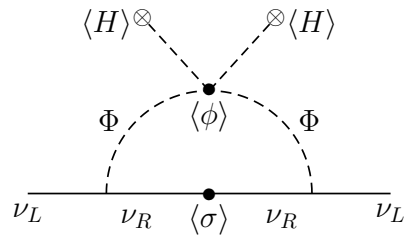


FIG. 1. Feynman diagram responsible for the neutrino mass matrix in the \mathbb{Z}_2 -based model.

$\sigma = (1, 1, \bar{6})_{(0,0,-2,0)}$, the following lepton sector Yukawa Lagrangian can be written:

$$\mathcal{L} = \frac{\lambda_e}{\Lambda} \bar{\ell}_L Y_e e_R H + \frac{\lambda_\nu}{\Lambda} \bar{\ell}_L Y_\nu \nu_R \Phi + \lambda_\sigma \bar{\nu}_R^\sigma \sigma \nu_R + \text{H.c.} \quad (9)$$

Here, one can also choose σ to be an $SU(3)_{\nu_R}$ triplet. Such a choice will certainly affect the Majorana mass spectrum, but will not have any impact in our conclusions. Note that one cannot write charged lepton Yukawa couplings involving Φ , nor type-I seesaw Yukawa couplings involving H . Their presence would require extra ‘‘Yukawa’’ fields, which are absent as demanded by our minimality criteria. The flavor-invariant and renormalizable scalar potential consist of three pieces:

$$V = V_{\text{SM}} + V(H, \Phi) + V(H, \Phi, \sigma), \quad (10)$$

where V_{SM} has an obvious meaning and $V(H, \Phi, \sigma)$ involves quadratic and quartic σ terms as well as mixed $H - \sigma$ and $\Phi - \sigma$ terms. Of particular interest for neutrino mass generation is the $V(H, \Phi)$ piece, which we explicitly write:

$$V(H, \Phi) = M_\Phi^2 |\Phi|^2 + \lambda_\Phi |\Phi|^4 + \lambda_{H\Phi} |H|^2 |\Phi|^2 + \lambda_a |H^T \epsilon \Phi|^2. \quad (11)$$

As long as $V(H, \Phi, \sigma)$ allows for a $U(1)_{\nu_R} \times SU(3)_{\nu_R}$ non-invariant ground state, the $U(1)_{\nu_R} \times SU(3)_{\nu_R}$ symmetry will be spontaneously broken to \mathbb{Z}_2 via $\langle \sigma \rangle \neq 0$ ². The \mathbb{Z}_2 Abelian discrete symmetry is, therefore, a residual symmetry resulting from the SSB of the global $U(1)_{\nu_R}$ factor. At this symmetry breaking stage, \mathbb{Z}_2 is an exact symmetry of the full Lagrangian under which the SM fields are even while the BSM fields ν_R and Φ are odd. It will remain so—even after electroweak symmetry breaking—provided M_Φ^2 and λ_a in (11) are positive, case in which $\langle \Phi \rangle = 0$ ³.

Under these conditions, the setup of eqs. (9), (10) and (11) is $U(1)_L$ invariant (even though right-handed neutrino Majorana mass terms are generated after SSB of $U(1)_{\nu_R} \times$

² Depending on the vev structure of $\langle \sigma \rangle$ one could also have an additional remnant symmetry e.g. if $\langle \sigma \rangle \propto I_{3 \times 3}$, one will be left with a residual $O(3)_{\nu_R}$.

³ Whether the desired vacuum i.e. $\langle H \rangle, \langle \sigma \rangle \neq 0$ and $\langle \Phi \rangle = 0$ is stable under radiative corrections clearly depends on the parameters of the Lagrangian. A dedicated study of the viable parameter space is beyond the scope of this work. Likewise in the rest of the paper, we will simply assume the configuration of the desired vacuum.

$SU(3)_{\nu_R}$). Since light Majorana neutrino masses demand lepton number violation, new terms are then required. The simplest choice is to include a new flavon $\phi = (1, 1, 1)_{(0,0,2,0)}$ which enables extending the scalar potential in (10) with a non-renormalizable term ⁴:

$$V(H, \Phi) \supset \frac{\lambda_\phi}{\Lambda} \phi (H^T \varepsilon \Phi)^2 + \text{H.c.} . \quad (12)$$

After ϕ acquires a vev, this term will induce an effective coupling $\lambda_5 (H^T \varepsilon \Phi)^2$, with $\lambda_5 = \lambda_\phi \langle \phi \rangle / \Lambda$. The presence of this new term implies unavoidably $U(1)_L$ breaking (by two units). The setup therefore generates light Majorana neutrino masses at the 1-loop level via the exchange of right-handed neutrinos and the neutral CP-even and CP-odd components of Φ ($\text{Re}(\Phi^0)$ and $\text{Im}(\Phi^0)$, respectively), as shown in Fig. 1 and exactly as in the scotogenic model [19]. Note that λ_5 being an effective coupling, it is expected to be small ($\langle \phi \rangle \ll \Lambda$), which in turn implies a small mass splitting between $\text{Re}(\Phi^0)$ and $\text{Im}(\Phi^0)$.

Model II: We now go a step further and consider the case of a remnant higher order cyclic symmetry, where not only the would-be Yukawa couplings are effective terms, but also the $\bar{\nu}_R^c \nu_R$ coupling. This requires the introduction of three flavon fields, $\rho = (1, 1, \bar{6})_{(0,0,-3,0)}$, $\phi_1 = (1, 1, 1)_{(0,0,1,0)}$ and $\phi_2 = (1, 1, 1)_{(0,0,-3,0)}$. In addition, we also include three generations of the new fermionic field N_L , the left-handed Dirac partner of ν_R , with exactly the same transformation properties under all gauge and flavor symmetries ⁵. Under these considerations, the Yukawa sector consists of the first two non-renormalizable terms in (9) and the terms

$$\mathcal{L} \supset \frac{\lambda_{\phi\rho}}{\Lambda} \phi_1 \bar{N}^c \rho N + M_N \bar{N} N + \text{H.c.} , \quad (13)$$

where $N = N_L + \nu_R$. Flavor invariance allows, of course, for scalar terms which are readily derivable, and so we do not write them explicitly. We just highlight the existence of two terms of particular relevance for neutrino mass generation:

$$V \supset \lambda_{12} \phi_1^3 \phi_2 + \mu_\Phi H^T \varepsilon \Phi \phi_1 + \text{H.c.} . \quad (14)$$

We will assume that the complete scalar potential has a minimum characterized by $\langle \Phi \rangle = \langle \phi_1 \rangle = 0$, $\langle \rho \rangle \neq 0$ and $\langle \phi_2 \rangle \neq 0$. Thus, in this case, $U(1)_{\nu_R} \times SU(3)_{\nu_R}$ gets spontaneously broken, leaving a remnant \mathbb{Z}_3 symmetry under which $N \rightarrow e^{2\pi i/3} N$, $\Phi \rightarrow e^{4\pi i/3} \Phi$, $\phi_1 \rightarrow e^{2\pi i/3} \phi_1$ and the remaining fields transform trivially. As in the previous example, this symmetry is generated from $U(1)_{\nu_R}$ SSB.

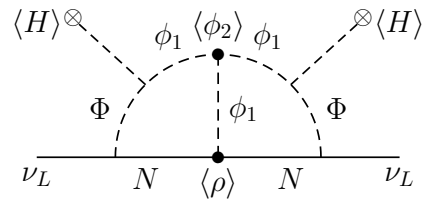


FIG. 2. Feynman diagram responsible for the neutrino mass matrix in the \mathbb{Z}_3 -based model.

In the present setup $U(1)_L$ is violated, inducing Majorana neutrino masses at the 2-loop order, as depicted in Fig. 2. In the same vein of the \mathbb{Z}_2 case, the remnant \mathbb{Z}_3 symmetry allows for dark matter stabilization, which can be either of fermionic or scalar nature, namely N , Φ^0 or ϕ_1 . However, in contrast to the usual \mathbb{Z}_2 -based dark matter scenarios, this dark matter particle will have semi-annihilation processes [37, 38].

IV. CONCLUSIONS

We have pointed out that the same dynamical flavor symmetry that governs SM fermions mass hierarchies and mixing patterns, might be as well at the origin of Abelian discrete symmetries, \mathbb{Z}_N . Spontaneous symmetry breaking of the Abelian sector yields such symmetries, provided the flavored UV completion involves flavon fields with suitable charges. These symmetries, which quite often are *ad hoc* “artifacts”, are employed for Majorana neutrino mass generation and dark matter stabilization, among others. Thus, we have suggested that the discrete symmetry generated in this way, provides a non-trivial link between the *theory* of flavor and the origin of neutrino masses and dark matter. We have shown the feasibility of this approach by constructing \mathbb{Z}_2 - and \mathbb{Z}_3 -based models, the former resembling the well-known scotogenic model, while the latter a new realization with quite a few interesting phenomenological implications.

In summary, we argued that discrete \mathbb{Z}_N symmetries have a dynamical flavor origin, and we have illustrated how this approach can be implemented. Finally, we would like to stress that this picture offers several interesting theoretical as well as phenomenological avenues which are worth exploring.

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⁴ This term can be generated for example by introducing a new heavy scalar $S = (1, 1, 1)_{(0,0,1,0)}$ with mass m_S which couples to H , Φ and ϕ as follows $\mu_S S H^T \epsilon \Phi$ and $\mu_{S\phi} S^* \phi$. Assuming $\mu_S \sim \mu_{S\phi} \sim m_S \equiv \Lambda$ to be much larger than the rest of the mass scales in the scalar potential, we can integrate out

the S field and obtain the desired operator.

⁵ As we will see in the following, without N_L , ν_R would be a massless dark matter particle, something ruled out by cosmological data. Moreover, this is actually required for gauge anomaly cancellation if we promote e.g. $U(3)_{\nu_R}$ to a local symmetry.