Deformations of an elastic pipe submitted to gravity and internal fluid flow

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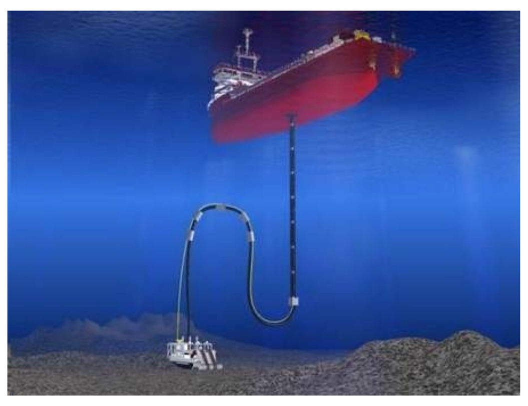
Context

Elastic pipes conveying a fluid flow are used for:

Fire fighting

Watering



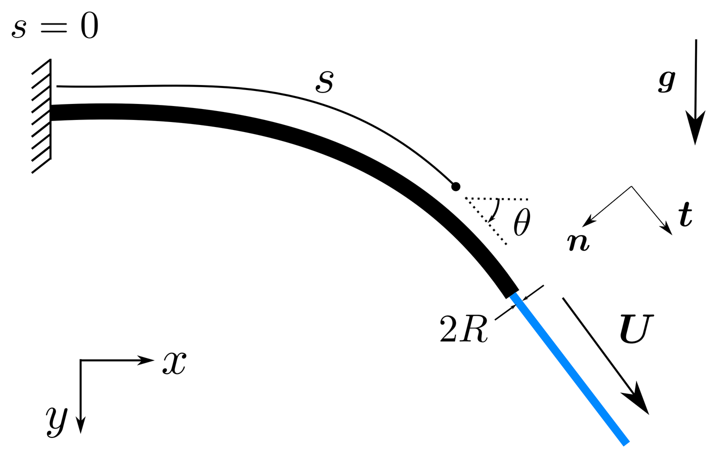
Deep ocean mining

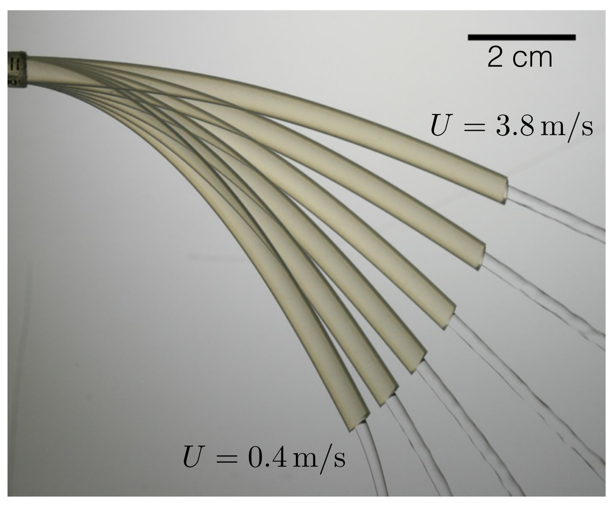
Nuclear plants cooling

How an internal fluid flow interacts with a compliant pipe?

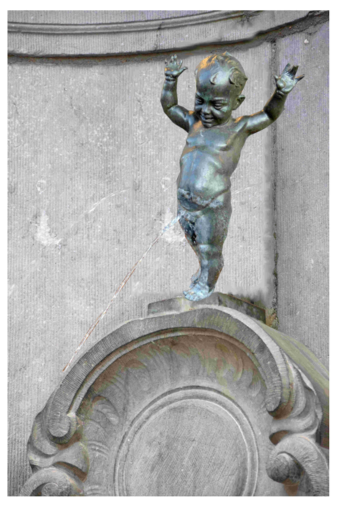
Experiment

To answer this question, we study the static shape of an elastic pipe clamped horizontally at one end, the other end being free, which conveys a water flow.





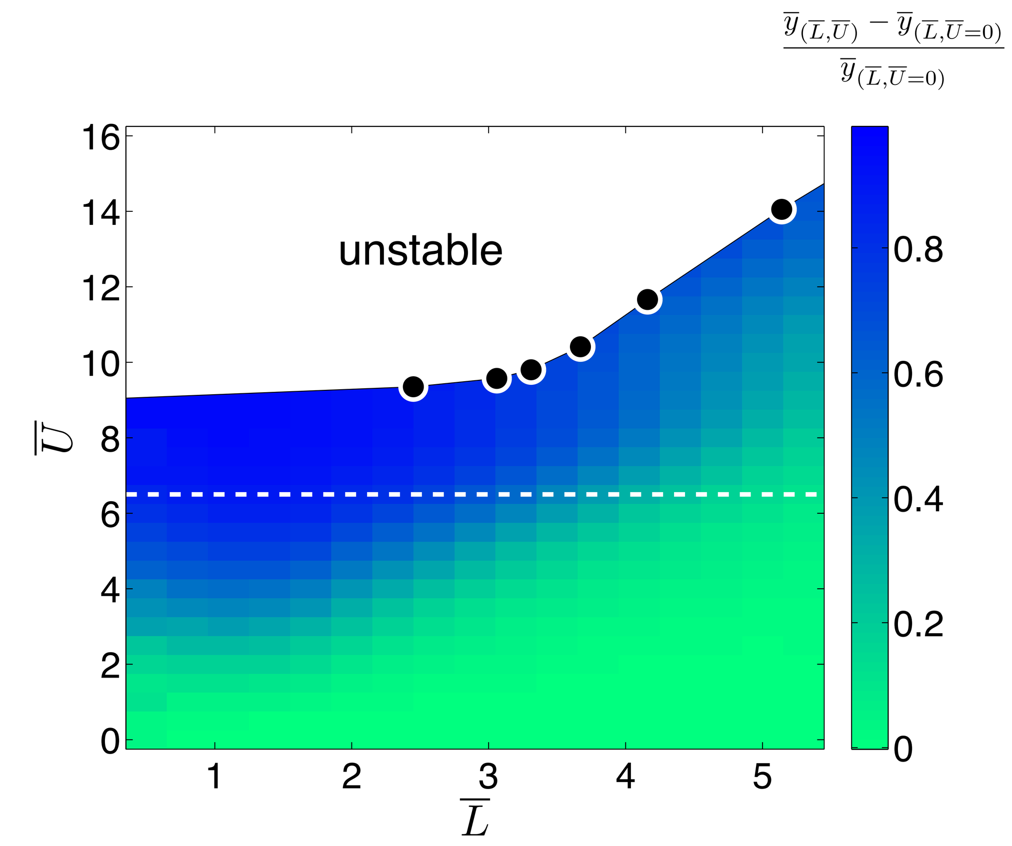
Larger is the fluid velocity U, more horizontal is the pipe.



It can be useful to pee without hands!

Free end stability

Above a critical fluid velocity, the free end of the pipe becomes unstable and starts to oscillate. We measured the velocity threshold for different pipe lengths (black dots in the following graph).



Païdoussis *et al.* performed a linear stability analysis of this problem and showed that without gravity, the critical normalized fluid velocity only depends on the fluid-solid linear mass ratio ρ_f/ρ_p.

In our experiments, $\rho_f/\rho_p=0.27$. In these conditions, Païdoussis *et al.* predicted a critical normalized velocity of $\bar{U}=6.5$ (indicated by a white dotted line in the previous graph). Experimental critical velocities are larger than this value meaning that the gravity stabilizes the free end of the pipe.

Model

The Euler-Bernoulli beam approximation and the conservation of the fluid momentum provide the deformation of the structure along the normal and the tangential directions:

$$EI\frac{d^3\theta}{ds^3}+(\rho_f U^2-T)\frac{d\theta}{ds}-\rho_p g\cos\theta=0$$

$$\frac{dT}{ds}+\rho_p g\sin\theta=0$$

with the initial conditions:

$$\theta(s=0)=0 \qquad \frac{d\theta}{ds}(s=L)=0 \qquad \frac{d^2\theta}{ds^2}(s=L)=0 \qquad T(s=L)=0$$

where:

EI	flexural rigidity of the pipe
ρ_f	linear mass of the liquid in the pipe
ρ_p	linear mass of the pipe filled with fluid
$\boldsymbol{T}=T\boldsymbol{t}$	tension along the pipe

The previous equations are normalized by the following characteristic length, velocity and force:

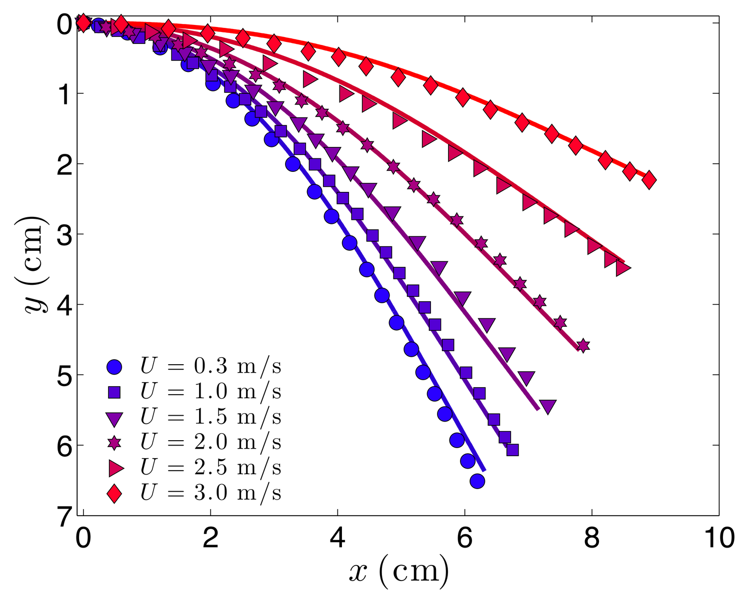
$$l=\left(\frac{EI}{\rho_p g}\right)^{1/3} \qquad u=\frac{1}{L}\left(\frac{EI}{\rho_f}\right)^{1/2} \qquad f=\rho_f u^2$$

which yields:

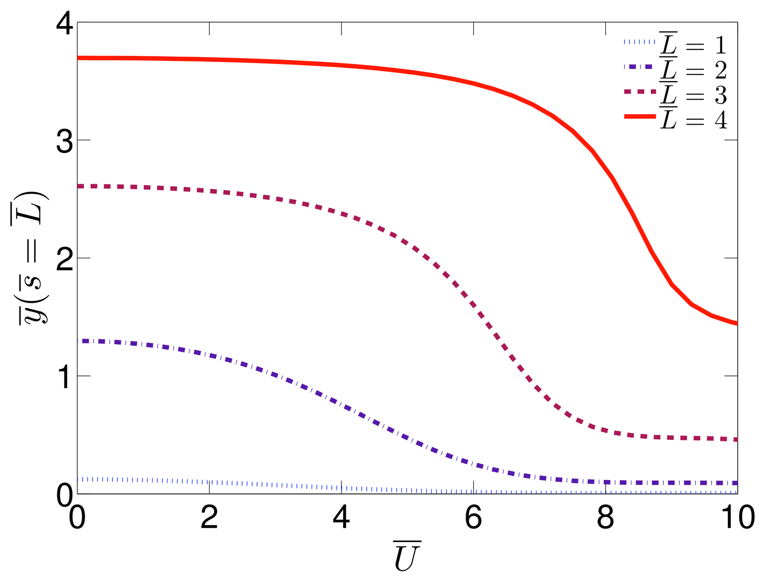
$$\frac{d^3\theta}{d\bar{s}^3}+\left(\frac{\bar{U}^2-\bar{T}}{\bar{L}^2}\right)\frac{d\theta}{d\bar{s}}-\cos\theta=0$$

$$\frac{d\bar{T}}{d\bar{s}}+\bar{L}^2\sin\theta=0$$

The set of equation is solved numerically for the parameters corresponding to the experiments and compared with them:



The model provides the normalized free end position of the pipe as a function of its normalized length and the normalized fluid velocity:



The optimal conditions to use this system as a flow meter can be determined.

Applications

Water jetpacks use the reaction of a fluid jet to lift a man in the air. According to this study, the water flow in a pipe curved upwards produces a force orientated in the opposite direction which reduces the total lift force of water jetpacks. Such an effect has been observed by constructors who know that the lift force of a compliant pipe can be up to 15% lower than the one of a straight and rigid pipe. Finally, one may expect that the flexural rigidity of the pipe is a compromise between maneuverability and lift reduction.

Bibliography

Païdoussis, M. P., & Issid, N. T. (1974). Dynamic stability of pipes conveying fluid. Journal of sound and vibration, 33(3), 267-294.
Darbois Texier, B., & Dorbolo, S. (2015). Deformations of an elastic pipe submitted to gravity and internal fluid flow. Journal of Fluids and Structures.

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