Min Max Generalization for Deterministic Batch Mode Reinforcement Learning: Relaxation Schemes

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Goal

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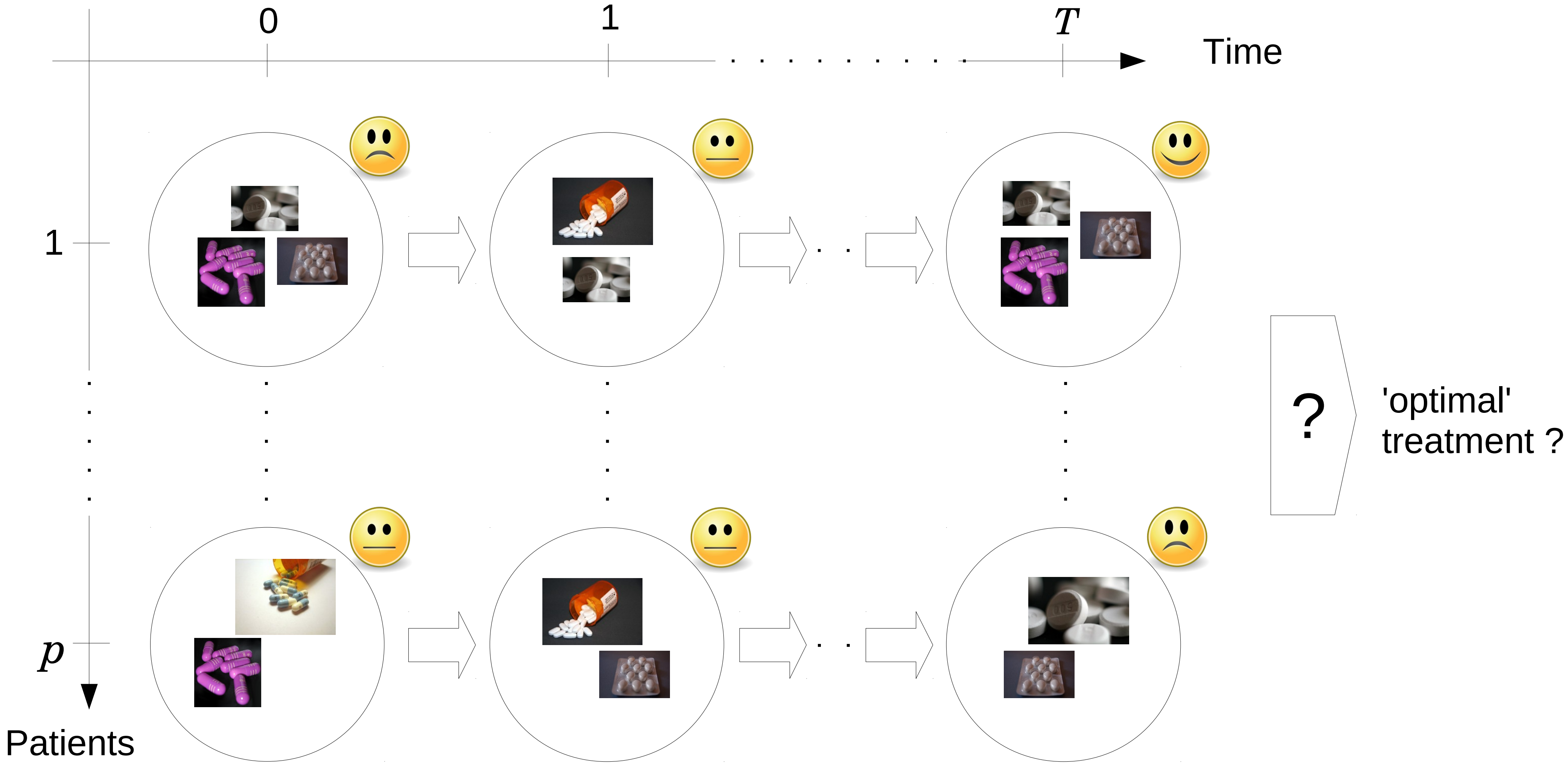
How to control a system so as to avoid the worst, given the knowledge of:

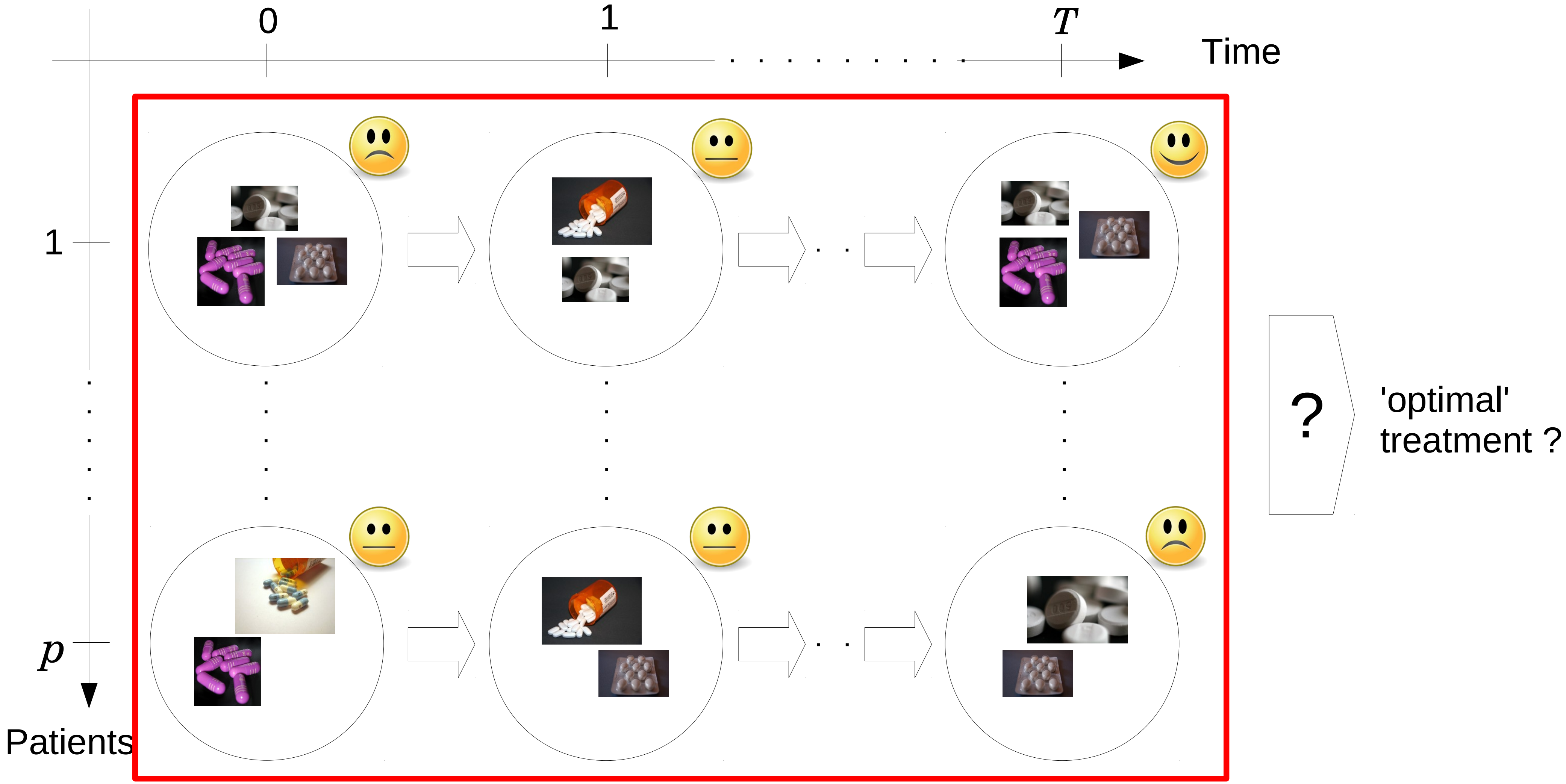
- A batch of (random) trajectories

- Maximal variations of the system, in the form of upper bounds on Lipschitz constants

A motivation: dynamic treatment regimes



A motivation: dynamic treatment regimes



Batch collection of trajectories of patients

Formalization

- Deterministic dynamics: $x_{t+1} = f(x_t, u_t) \quad t = 0, \ldots, T-1$
- Deterministic reward function: $r_t = \rho(x_t, u_t) \in \mathbb{R}$
- Fixed initial state: $x_0 \in \mathcal{X}$
- Continuous sate space, finite action space: $\mathcal{U} = \left\{u^{(1)}, \ldots, u^{(m)}\right\}$

  $\mathcal{X} \subset \mathbb{R}^d$

- Return of a sequence of actions:

$$J(u_0, \ldots, u_{T-1}) \triangleq \sum_{t=0}^{T-1} \rho(x_t, u_t)$$

- Optimal return:

$$J_T^* \triangleq \max_{(u_0, \ldots, u_{T-1}) \in \mathcal{U}^T} J(u_0, \ldots, u_{T-1})$$

The "batch" mode setting

Learning from trajectories

- System dynamics and reward function are **unnkown**
- For every action $u \in \mathcal{U}$, a **set of transitions** is known:

$$\mathcal{F}^{(u)} = \left\{ \left(x^{(u),k}, r^{(u),k}, y^{(u),k} \right) \right\}_{k=1}^{n^{(u)}}$$

$$y^{(u),k} = f\left(x^{(u),k}, u\right) \text{ and } r^{(u),k} = \rho\left(x^{(u),k}, u\right)$$

- Each set of transition is non-empty:

$$\forall u \in \mathcal{U}, \qquad n^{(u)} > 0$$

- Define:

$$\mathcal{F} = \mathcal{F}^{(1)} \cup \ldots \cup \mathcal{F}^{(m)}$$

Lipschitz continuity

Assumption about maximal variations

- We assume that the system dynamics and reward function are Lipschitz continuous:

$$\forall (x, x') \in \mathcal{X}^2, \forall u \in \mathcal{U},$$

$$\begin{aligned} \|f(x,u) - f(x',u)\| &\leq L_f \|x - x'\| \\ |\rho(x,u) - \rho(x',u)| &\leq L_\rho \|x - x'\| \end{aligned}$$

where $\|\cdot\|$ denotes the Euclidean norm over the state space

- We also assume that two constants L_f and L_ρ satisfying the above equations are **known**

Min max generalization

- One can define the sets of Lipschitz continuous functions compatible with the data:

$$\mathcal{L}_{\mathcal{F}}^{f} = \left\{ f' : \mathcal{X} \times \mathcal{U} \rightarrow \mathcal{X} \middle| \begin{cases} \forall x', x'' \in \mathcal{X}, \forall u \in \mathcal{U}, \\ \| f'(x', u) - f'(x'', u) \| \leq L_f \| x' - x'' \| , \\ \forall k, f'(x^{(u),k}, u) = f(x^{(u),k}, u) = y^{(u),k} \end{cases} \right\}$$

$$\mathcal{L}_{\mathcal{F}}^{\rho} = \left\{ \rho' : \mathcal{X} \times \mathcal{U} \rightarrow \mathbb{R} \middle| \begin{cases} \forall x', x'' \in \mathcal{X}, \forall u \in \mathcal{U} , \\ | \rho'(x', u) - \rho'(x'', u) | \leq L_\rho \| x' - x'' \| , \\ \forall k, \rho'(x^{(u),k}, u) = \rho(x^{(u),k}, u) = r^{(u),k} \end{cases} \right\}$$

and the return associated with a couple of fonctions taken in those two ensembles :

$$J_{(f', \rho')}(u_0, \ldots, u_{T-1}) = \sum_{t=0}^{T-1} \rho'(x'_t, u_t)$$

$$x'_{t+1} = f'(x'_t, u_t)$$

Min max generalization

- One can then define:

$$B^*(\mathcal{F}, u_0, \ldots, u_{T-1}) = \min_{(f', \rho') \in \mathcal{L}_{\mathcal{F}}^f \times \mathcal{L}_{\mathcal{F}}^\rho} \left\{ J_{(f', \rho')}(u_0, \ldots, u_{T-1}) \right\}$$

- And the solution of the min max generalization problem can be defined as follows:

$$(u_0, \ldots, u_{T-1}) \in \arg\max_{(u_0, \ldots, u_{T-1}) \in \mathcal{U}^T} B^*(\mathcal{F}, u_0, \ldots, u_{T-1})$$

Reformulation

- According to previous research [1], we know that computing the optimal bound for a given sequence of actions can be reformalized as follows:

$$(\mathcal{P}(\mathcal{F}, L_f, L_\rho, x_0, u_0, \ldots, u_{T-1})):$$

$$\min_{\substack{\hat{\mathbf{r}}_0 \quad \ldots \quad \hat{\mathbf{r}}_{T-1} \in \mathbb{R} \\ \hat{\mathbf{x}}_0 \quad \ldots \quad \hat{\mathbf{x}}_{T-1} \in \mathcal{X}}} \sum_{t=0}^{T-1} \hat{\mathbf{r}}_t,$$

subject to

(3.1)
$$\left|\hat{\mathbf{r}}_t - r^{(u_t),k_t}\right|^2 \leq L_\rho^2 \left\|\hat{\mathbf{x}}_t - x^{(u_t),k_t}\right\|^2 \ \forall (t, k_t) \in \{0, \ldots, T-1\} \times \left\{1, \ldots, n^{(u_t)}\right\},$$

(3.2)
$$\left\|\hat{\mathbf{x}}_{t+1} - y^{(u_t),k_t}\right\|^2 \leq L_f^2 \left\|\hat{\mathbf{x}}_t - x^{(u_t),k_t}\right\|^2 \ \forall (t, k_t) \in \{0, \ldots, T-1\} \times \left\{1, \ldots, n^{(u_t)}\right\},$$

(3.3)
$$|\hat{\mathbf{r}}_t - \hat{\mathbf{r}}_{t'}|^2 \leq L_\rho^2 \|\hat{\mathbf{x}}_t - \hat{\mathbf{x}}_{t'}\|^2 \ \forall t, t' \in \{0, \ldots, T-1 | u_t = u_{t'}\},$$

(3.4)
$$\|\hat{\mathbf{x}}_{t+1} - \hat{\mathbf{x}}_{t'+1}\|^2 \leq L_f^2 \|\hat{\mathbf{x}}_t - \hat{\mathbf{x}}_{t'}\|^2 \ \forall t, t' \in \{0, \ldots, T-2 | u_t = u_{t'}\},$$

(3.5)
$$\hat{\mathbf{x}}_0 = x_0.$$

[1] "**Towards Min Max Generalization in Reinforcement Learning**". R. Fonteneau, S.A. Murphy, L. Wehenkel and D. Ernst. Agents and Artificial Intelligence: International Conference, ICAART 2010, Valencia, Spain, January 2010, Revised Selected Papers. Series: Communications in Computed and Information Science (CCIS), Volume 129, pp. 61-77. Editors: J. Filipe, A. Fred, and B.Sharp. Springer, Heidelberg, 2011.

Reformulation

- According to previous research [1], we know that computing the optimal bound for a given sequence of actions can be reformalized as follows:

$(\mathcal{P}(\mathcal{F}, L_f, L_\rho, x_0, u_0, \ldots, u_{T-1}))$:

$$\min_{\substack{\hat{\mathbf{r}}_0 \ \ldots \ \hat{\mathbf{r}}_{T-1} \in \mathbb{R} \\ \hat{\mathbf{x}}_0 \ \ldots \ \hat{\mathbf{x}}_{T-1} \in \mathcal{X}}} \sum_{t=0}^{T-1} \hat{\mathbf{r}}_t,$$

subject to

(3.1)

$$\left|\hat{\mathbf{r}}_t - r^{(u_t),k_t}\right|^2 \leq L_\rho^2 \left\|\hat{\mathbf{x}}_t - x^{(u_t),k_t}\right\|^2 \ \forall (t, k_t) \in \{0, \ldots, T-1\} \times \left\{1, \ldots, n^{(u_t)}\right\},$$

(3.2)

$$\left\|\hat{\mathbf{x}}_{t+1} - y^{(u_t),k_t}\right\|^2 \leq L_f^2 \left\|\hat{\mathbf{x}}_t - x^{(u_t),k_t}\right\|^2 \ \forall (t, k_t) \in \{0, \ldots, T-1\} \times \left\{1, \ldots, n^{(u_t)}\right\},$$

(3.3) $\quad |\hat{\mathbf{r}}_t - \hat{\mathbf{r}}_{t'}|^2 \leq L_\rho^2 \|\hat{\mathbf{x}}_t - \hat{\mathbf{x}}_{t'}\|^2 \ \forall t, t' \in \{0, \ldots, T-1 | u_t = u_{t'}\},$

(3.4)

$$\|\hat{\mathbf{x}}_{t+1} - \hat{\mathbf{x}}_{t'+1}\|^2 \leq L_f^2 \|\hat{\mathbf{x}}_t - \hat{\mathbf{x}}_{t'}\|^2 \ \forall t, t' \in \{0, \ldots, T-2 | u_t = u_{t'}\},$$

(3.5) $\quad \hat{\mathbf{x}}_0 = x_0.$

[1] proposes a lower bound on the optimal bound (computed independently from this reformulation)

Here, we directly target this problem in order to find a bound tighter than [1]

[1] "**Towards Min Max Generalization in Reinforcement Learning**". R. Fonteneau, S.A. Murphy, L. Wehenkel and D. Ernst. Agents and Artificial Intelligence: International Conference, ICAART 2010, Valencia, Spain, January 2010, Revised Selected Papers. Series: Communications in Computed and Information Science (CCIS), Volume 129, pp. 61-77. Editors: J. Filipe, A. Fred, and B.Sharp. Springer, Heidelberg, 2011.

Small simplification

- One can show that type (3.3) constraints are redundant:

LEMMA 4.1. *Consider* $(\hat{\mathbf{r}}^*, \hat{\mathbf{x}}^*) \in \mathbb{R}^T \times \mathcal{X}^T$ *an optimal solution to* $\bar{\mathcal{P}}(\mathcal{F}, u_0, \ldots, u_{T-1})$. *Then, for all* t, t' *such that* $u_t = u_{t'}$,

$$\left|\hat{\mathbf{r}}_t^* - \hat{\mathbf{r}}_{t'}^*\right|^2 \leq L_\rho^2 \left\|\hat{\mathbf{x}}_t^* - \hat{\mathbf{x}}_{t'}^*\right\|^2 .$$

- We can deduce the solution for time t=0 :

LEMMA 4.2. *The solution of the problem* $(\mathcal{P}'(\mathcal{F}, u_0))$ *is*

$$\hat{\mathbf{r}}_0^* = \max_{k_0 \in \left\{1, \ldots, n^{(u_0)}\right\}} r^{(u_0), k_0} - L_\rho \left\| x_0 - x^{(u_0), k_0} \right\| .$$

New problem

$(\mathcal{P}''(\mathcal{F}, u_0, \ldots, u_{T-1})):$

$$\min_{\substack{\hat{\mathbf{r}}_1 \quad \ldots \quad \hat{\mathbf{r}}_{T-1} \in \mathbb{R} \\ \hat{\mathbf{x}}_0 \quad \ldots \quad \hat{\mathbf{x}}_{T-1} \in \mathcal{X}}} \sum_{t=1}^{T-1} \hat{\mathbf{r}}_t,$$

subject to

(5.1)

$$\left|\hat{\mathbf{r}}_t - r^{(u_t),k_t}\right|^2 \leq L_\rho^2 \left\|\hat{\mathbf{x}}_t - x^{(u_t),k_t}\right\|^2 \ \forall (t, k_t) \in \{1, \ldots, T-1\} \times \left\{1, \ldots, n^{(u_t)}\right\},$$

(5.2)

$$\left\|\hat{\mathbf{x}}_{t+1} - y^{(u_t),k_t}\right\|^2 \leq L_f^2 \left\|\hat{\mathbf{x}}_t - x^{(u_t),k_t}\right\|^2 \ \forall (t, k_t) \in \{0, \ldots, T-1\} \times \left\{1, \ldots, n^{(u_t)}\right\},$$

(5.3)

$$\|\hat{\mathbf{x}}_{t+1} - \hat{\mathbf{x}}_{t'+1}\|^2 \leq L_f^2 \|\hat{\mathbf{x}}_t - \hat{\mathbf{x}}_{t'}\|^2 \ \forall t, t' \in \{0, \ldots, T-2 | u_t = u_{t'}\},$$

(5.4) $\quad \hat{\mathbf{x}}_0 = x_0 \ .$

Complexity

- One can show that such a problem is **NP-hard**
- We propose relaxation schemes of **polynomial complexity**
- We want those relaxation schemes to preserve the philosophy of the original problem, i.e., to provide lower bounds
- We propose two types of relaxations:
  - The Intertwined Trust-Region (ITR) relaxation scheme
  - The Lagrangian relaxation scheme
- We show that those relaxations are more efficient than previous solution given in [1]

Relaxation schemes

(I) Intertwined trust-region

- First approach: remove constraints until the problem becomes polynomial

$(\mathcal{P}''(\mathcal{F}, u_0, \ldots, u_{T-1}))$:

$$\min_{\substack{\hat{\mathbf{r}}_1 \quad \ldots \quad \hat{\mathbf{r}}_{T-1} \in \mathbb{R} \\ \hat{\mathbf{x}}_0 \quad \ldots \quad \hat{\mathbf{x}}_{T-1} \in \mathcal{X}}} \quad \sum_{t=1}^{T-1} \hat{\mathbf{r}}_t,$$

subject to

Only one constraint

(5.1)

$$\left|\hat{\mathbf{r}}_t - r^{(u_t),k_t}\right|^2 \leq L_\rho^2 \left\|\hat{\mathbf{x}}_t - x^{(u_t),k_t}\right\|^2 \ \forall (t, k_t) \in \{1, \ldots, T-1\} \times \left\{1, \ldots, n^{(u_t)}\right\},$$

(5.2)

$$\left\|\hat{\mathbf{x}}_{t+1} - y^{(u_t),k_t}\right\|^2 \leq L_f^2 \left\|\hat{\mathbf{x}}_t - x^{(u_t),k_t}\right\|^2 \ \forall (t, k_t) \in \{0, \ldots, T-1\} \times \left\{1, \ldots, n^{(u_t)}\right\},$$

(5.3)

$$\|\hat{\mathbf{x}}_{t+1} - \hat{\mathbf{x}}_{t'+1}\|^2 \leq L_f^2 \|\hat{\mathbf{x}}_t - \hat{\mathbf{x}}_{t'}\|^2 \ \forall t, t' \in \{0, \ldots, T-2 | u_t = u_{t'}\},$$

(5.4) $$\hat{\mathbf{x}}_0 = x_0 \ .$$

(I) Intertwined trust-region

- We get the ITR problem:

$$\left(\mathcal{P}''_{ITR}(\mathcal{F}, u_0, \ldots, u_{T-1}, \bar{k}_0, \ldots, \bar{k}_{T-1})\right):$$

$$\min_{\substack{\hat{\mathbf{r}}_1, \ldots, \hat{\mathbf{r}}_{T-1} \in \mathbb{R} \\ \hat{\mathbf{x}}_0, \ldots, \hat{\mathbf{x}}_{T-1} \in \mathcal{X}}} \quad \sum_{t=1}^{T-1} \hat{\mathbf{r}}_t$$

subject to

$$(5.5) \qquad \left| \hat{\mathbf{r}}_t - r^{(u_t), \bar{k}_t} \right|^2 \leq L_\rho^2 \left\| \hat{\mathbf{x}}_t - x^{(u_t), \bar{k}_t} \right\|^2, \qquad t \in \{1, \ldots, T-1\},$$

$$(5.6) \qquad \left\| \hat{\mathbf{x}}_t - y^{(u_{t-1}), \bar{k}_{t-1}} \right\|^2 \leq L_f^2 \left\| \hat{\mathbf{x}}_{t-1} - x^{(u_{t-1}), \bar{k}_{t-1}} \right\|^2, \qquad t \in \{1, \ldots, T-1\},$$

$$(5.7) \qquad \hat{\mathbf{x}}_0 = x_0.$$

- A closed-form solution of this problem can be obtained

(I) Intertwined trust-region

THEOREM 5.4. *The solution to* $(\mathcal{P}''_{ITR}(\mathcal{F}, u_0, \ldots, u_{T-1}, \bar{k}_0, \ldots, \bar{k}_{T-1}))$ *is given by*

$$B''_{ITR}(\mathcal{F}, u_0, \ldots, u_{T-1}, \bar{k}_0, \ldots, \bar{k}_{T-1}) = \sum_{t=1}^{T-1} \hat{\mathbf{r}}_t^*,$$

where

$$\begin{aligned}
\hat{\mathbf{r}}_t^* &= r^{(u_t),\bar{k}_t} - L_\rho \left\| \hat{\mathbf{x}}_t^*(\bar{k}_0, \ldots, \bar{k}_t) - x^{(u_t),\bar{k}_t} \right\|, \\
\hat{\mathbf{x}}_t^*(\bar{k}_0, \ldots, \bar{k}_t) &\doteq y^{(u_{t-1}),\bar{k}_{t-1}} \\
&\quad + L_f \frac{\left\| \hat{\mathbf{x}}_{t-1}^*(\bar{k}_0, \ldots, \bar{k}_{t-1}) - x^{(u_{t-1}),\bar{k}_{t-1}} \right\|}{\left\| y^{(u_{t-1}),\bar{k}_{t-1}} - x^{(u_t),\bar{k}_t} \right\|} \left(y^{(u_{t-1}),\bar{k}_{t-1}} - x^{(u_t),\bar{k}_t} \right)
\end{aligned}$$

if $y^{(u_{t-1}),\bar{k}_{t-1}} \neq x^{(u_t),\bar{k}_t}$

and, if $y^{(u_{t-1}),\bar{k}_{t-1}} = x^{(u_t),\bar{k}_t}$, $\hat{\mathbf{x}}_t^*(\bar{k}_0, \ldots, \bar{k}_t)$ *can be any point of the sphere centered in* $y^{(u_{t-1}),\bar{k}_{t-1}} = x^{(u_t),\bar{k}_t}$ *with radius* $L_f \|\hat{\mathbf{x}}_{t-1}^*(\bar{k}_0, \ldots, \bar{k}_{t-1}) - x^{(u_{t-1}),\bar{k}_{t-1}}\|$.

(I) Intertwined trust-region

- The ITR problem can be solved for any selection of constraints
- One can thus define a maximal ITR bound :

DEFINITION 5.5 (ITR bound $B_{ITR}(\mathcal{F}, u_0, \ldots, u_{T-1})$).

$$B_{ITR}(\mathcal{F}, u_0, \ldots, u_{T-1}) \triangleq \hat{\mathbf{r}}_0^* + \max_{\substack{\bar{k}_{T-1} \in \{1, \ldots, n^{(u_{T-1})}\} \\ \ldots \\ \bar{k}_0 \in \{1, \ldots, n^{(u_0)}\}}} B''_{ITR}(\mathcal{F}, u_0, \ldots, u_{T-1}, \bar{k}_0, \ldots, \bar{k}_{T-1})$$

(II) Lagrangian relaxation

$(\mathcal{P}''_{LD}(\mathcal{F}, u_0, \ldots, u_{T-1}))$:

$$\max_{\substack{\nu_{t,t'} \in \mathbb{R} \\ \lambda_{t,k_t} \in \mathbb{R} \\ \mu_{t,k_t} \in \mathbb{R}}} \quad \min_{\substack{\hat{\mathbf{r}}_1, \ldots, \hat{\mathbf{r}}_{T-1} \in \mathbb{R} \\ \hat{\mathbf{x}}_1, \ldots, \hat{\mathbf{x}}_{T-1} \in \mathcal{X}}}$$

$$\hat{\mathbf{r}}_1 + \cdots + \hat{\mathbf{r}}_{T-1}$$

$$+ \sum_{(t,k_t) \in \{1,\ldots,T-1\} \times \{1,\ldots,n^{(u_t)}\}} \mu_{t,k_t} \left(\left| \hat{\mathbf{r}}_t - r^{(u_t),k_t} \right|^2 - L_\rho^2 \left\| \hat{\mathbf{x}}_t - x^{(u_t),k_t} \right\|^2 \right)$$

$$+ \sum_{(t,k_t) \in \{1,\ldots,T-1\} \times \{1,\ldots,n^{(u_t)}\}} \lambda_{t,k_t} \left(\left\| \hat{\mathbf{x}}_{t+1} - y^{(u_t),k_t} \right\|^2 - L_f^2 \left\| \hat{\mathbf{x}}_t - x^{(u_t),k_t} \right\|^2 \right)$$

$$+ \sum_{t,t' \in \{0,\ldots,T-2 | u_t = u_{t'}\}} \nu_{t,t'} \left(\left\| \hat{\mathbf{x}}_{t+1} - \hat{\mathbf{x}}_{t'+1} \right\|^2 - L_f^2 \left\| \hat{\mathbf{x}}_t - \hat{\mathbf{x}}_{t'} \right\|^2 \right).$$

- Polynomial complexity

Tightness of the bounds

- Comparison with the relaxation proposed in [1] :

DEFINITION 5.8 (CGRL bound $B_{CGRL}(\mathcal{F}, u_0, \ldots, u_{T-1})$).

$$
\begin{aligned}
&B_{CGRL}(\mathcal{F}, u_0, \ldots, u_{T-1}) \\
&\triangleq \max_{\substack{\bar{k}_{T-1} \in \{1, \ldots, n^{(u_{T-1})}\} \\ \ldots \\ \bar{k}_0 \in \{1, \ldots, n^{(u_0)}\}}} r^{(u_0),\bar{k}_0} - L_\rho \left(1 + L_f + L_f^2 + \cdots + L_f^{T-2}\right) \left\| x^{(u_0),\bar{k}_0} - x_0 \right\| \\
&\qquad + \cdots + \\
&\qquad + r^{(u_{T-2}),\bar{k}_{T-2}} - L_\rho \left(1 + L_f\right) \left\| y^{(u_{T-3}),\bar{k}_{T-3}} - x^{(u_{T-2}),\bar{k}_{T-2}} \right\| \\
&\qquad + r^{(u_{T-1}),\bar{k}_{T-1}} - L_\rho \left\| y^{(u_{T-2}),\bar{k}_{T-2}} - x^{(u_{T-1}),\bar{k}_{T-1}} \right\| .
\end{aligned}
$$

Tightness of the bounds

- ITR versus [1] :

THEOREM 5.9.

$$B_{CGRL}(\mathcal{F}, u_0, \ldots, u_{T-1}) \leq B_{ITR}(\mathcal{F}, u_0, \ldots, u_{T-1})$$

Sketch of proof :

- Compute the ITR relaxation with the constraints used by the CGRL bound

Tightness of the bounds

- Lagrangian relaxation versus ITR :

THEOREM 5.17.

$$B_{ITR}(\mathcal{F}, u_0, \ldots, u_{T-1}) \leq B_{LD}(\mathcal{F}, u_0, \ldots, u_{T-1})$$

Sketch of proof:

  - Strong duality holds for the Lagrangian relaxation of the ITR problem

Tightness of the bounds

- Synthesis:

$$\begin{aligned} B_{CGRL}(\mathcal{F}, u_0, \ldots, u_{T-1}) &\leq B_{ITR}(\mathcal{F}, u_0, \ldots, u_{T-1}) \\ &\leq B_{LD}(\mathcal{F}, u_0, \ldots, u_{T-1}) \\ &\leq B^*(\mathcal{F}, u_0, \ldots, u_{T-1}) \\ &\leq J(u_0, \ldots, u_{T-1}) \ . \end{aligned}$$

- All these bounds converge to the actual return of sequences of actions when the dispersion decreases towards zero

Illustration

- Dynamics: $\forall (x,u) \in \mathcal{X} \times \mathcal{U}, \qquad f(x,u) = x + 3.1416 \times u \times 1_d$
- Reward function: $\forall (x,u) \in \mathcal{X} \times \mathcal{U}, \qquad \rho(x,u) = \sum_{i=1}^{d} x(i)$
- Initial state: $x_0 = 0.5772 \times 1_d$
- Decision space: $\mathcal{U} = \{0, 0.1\}$
- Grid :

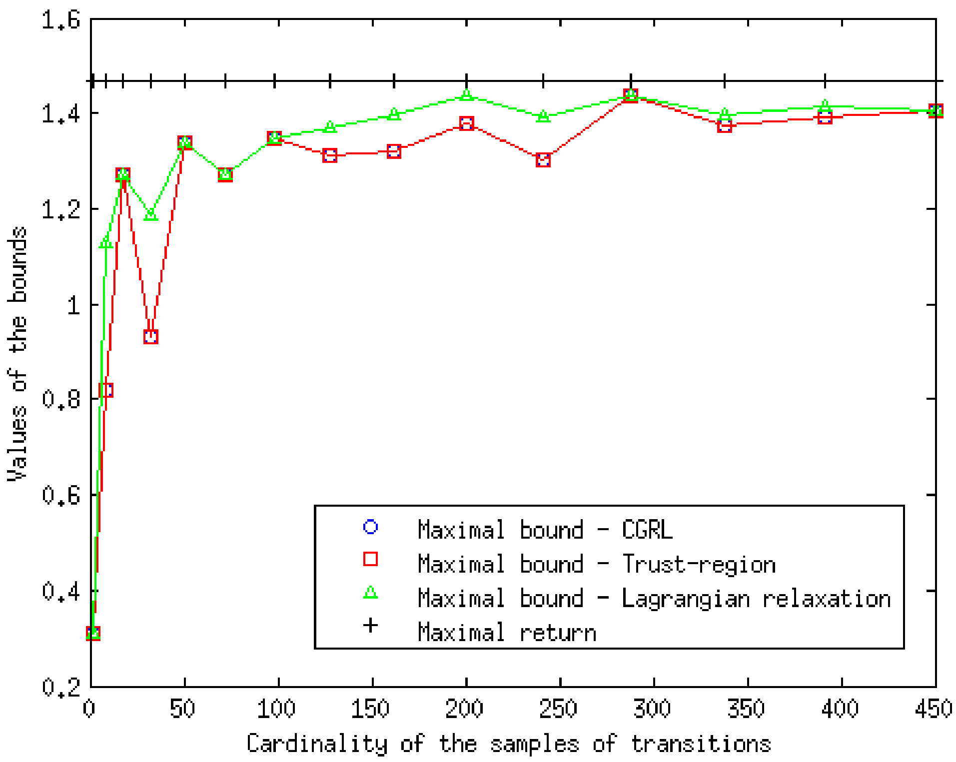
$$\forall u \in \mathcal{U}, \mathcal{F}_{c_i}^{(u)} = \left\{ \left(\left[\frac{i_1}{i}; \frac{i_2}{i} \right], u, \rho\left(\left[\frac{i_1}{i}; \frac{i_2}{i} \right], u \right), f\left(\left[\frac{i_1}{i}; \frac{i_2}{i} \right], u \right) \right) \middle| (i_1, i_2) \in \{1, \ldots, i\}^2 \right\}$$

- 100 samples of transitions drawn uniformly at random

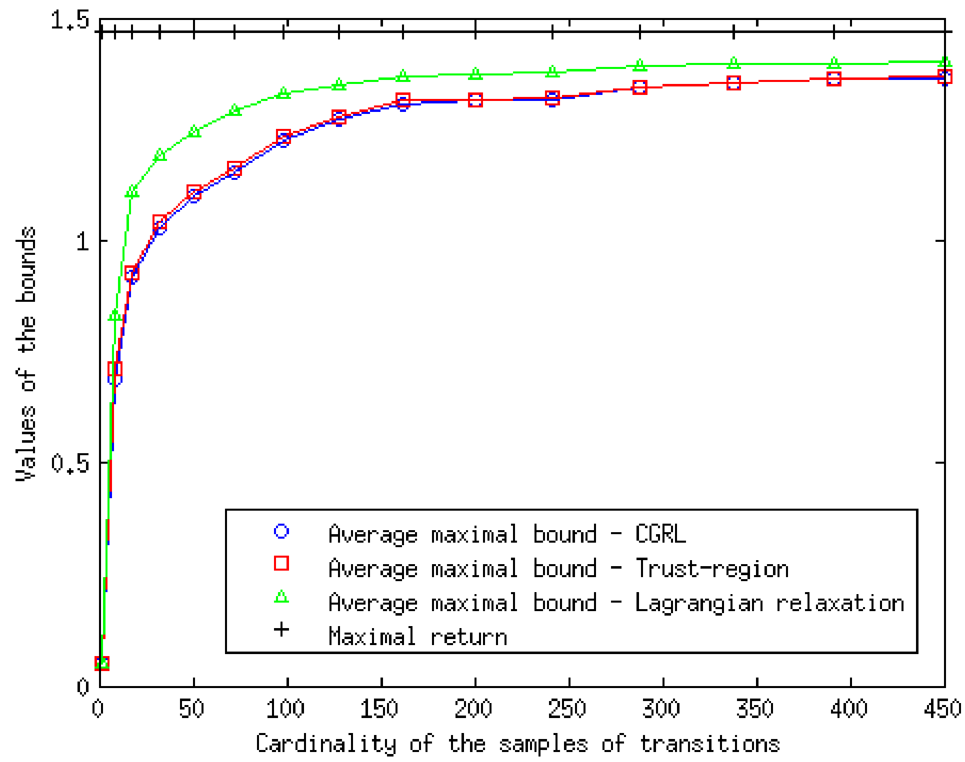
Illustration

Maximal bounds

Grid



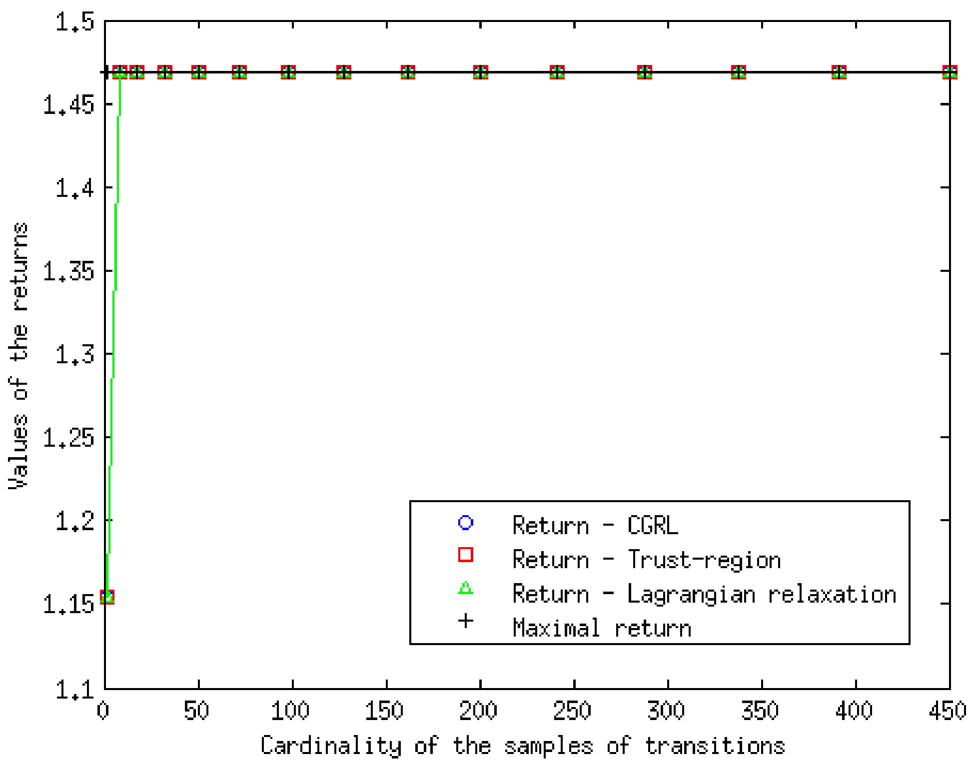
Empirical average over random samples



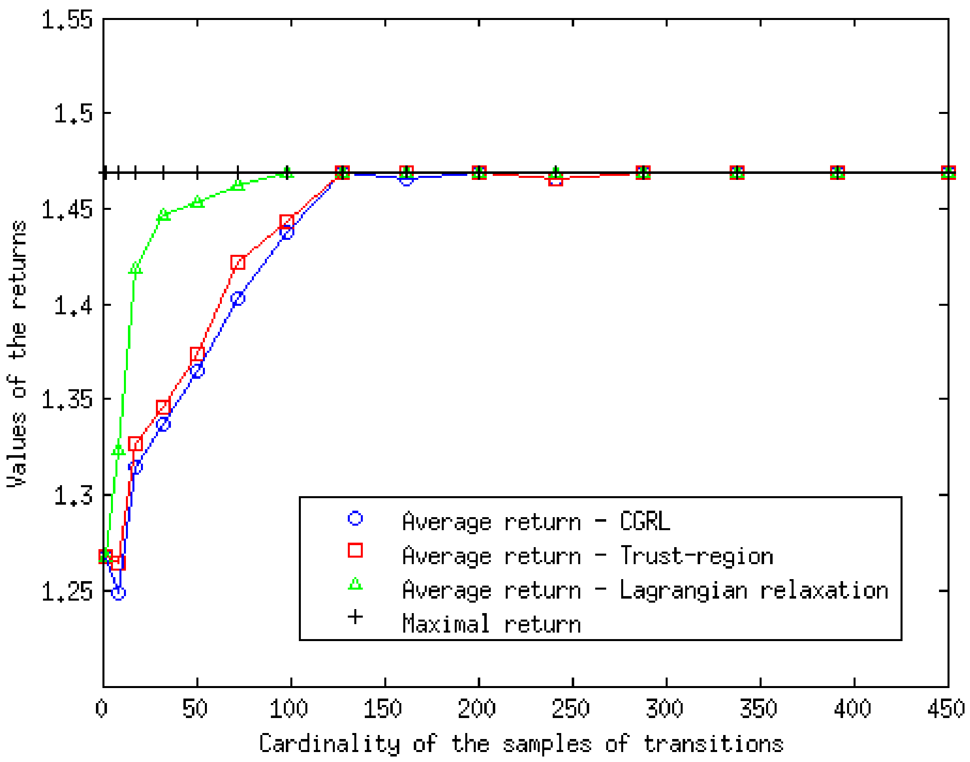
Illustration

Returns of sequences

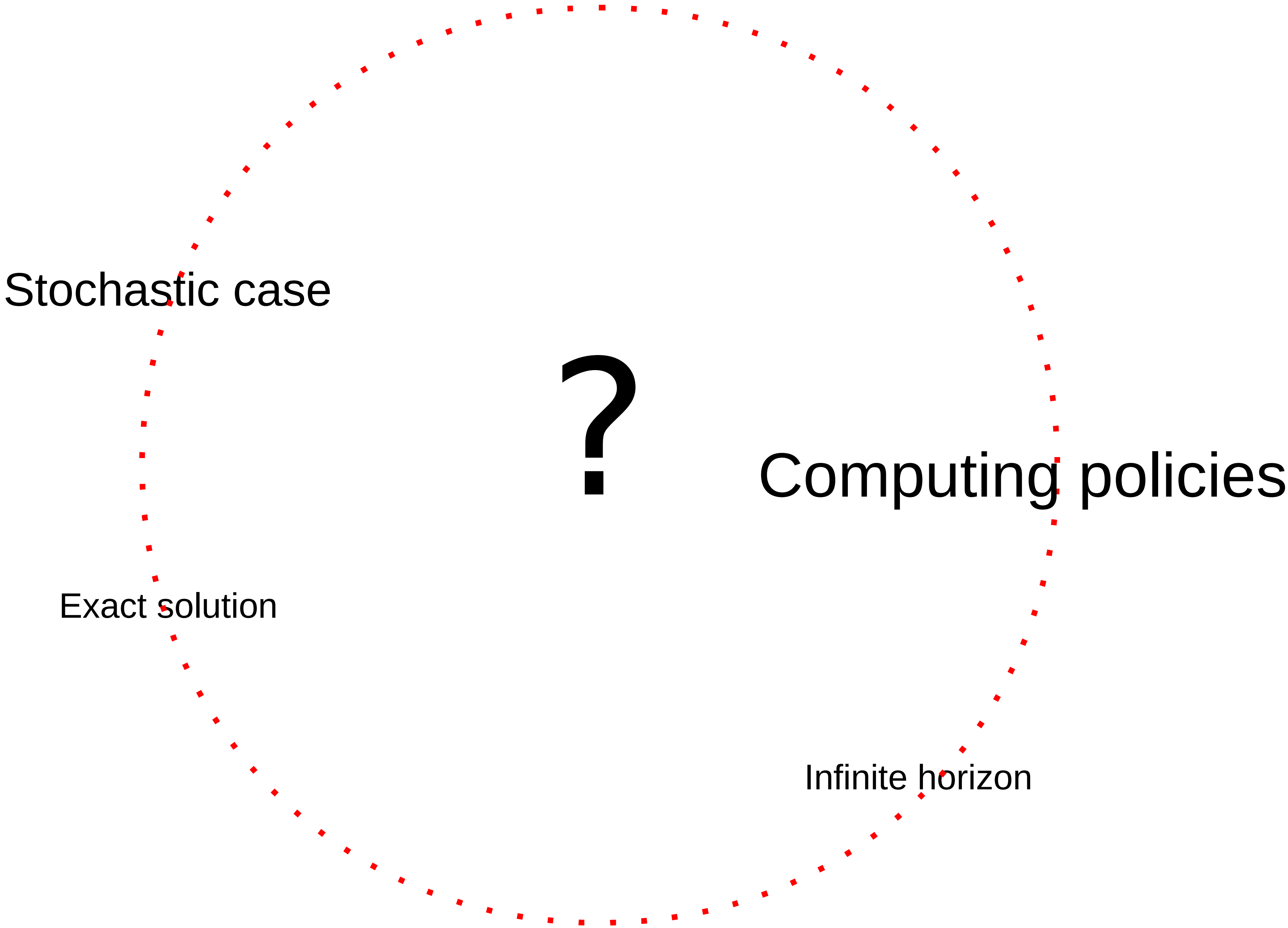
Grid



Empirical average over random samples



Future work



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