A stochastic multi-scale analysis for MEMS stiction failure

Truong Vinh Hoang*, Ling Wu, Jean-Claude Golinval, Stéphane Paquay,
Maarten Arnst, Ludovic Noels

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Introduction

- **MEMS stiction failure**
  - Due to the dominance of surface adhesive forces
    - E.g., van der Waals forces and capillary forces
    - In humid condition, the capillary forces are dominant
  - Depends on the surface topologies
  - An uncertain phenomenon

Contact zone: Low humidity levels
Contact zone: High humidity levels

*Stiction failure in a MEMS sensor*  
( Jeremy A. Walraven Sandia National Laboratories. Albuquerque, NM USA)
Motivation

- Construct a numerical model
  - To predict the **crack length** $s$ and its uncertainties from the surface topology
  - At an **acceptable computational cost**

Initial configuration

Primary contact configuration

Condensing water formation

Failure

- The crack length $s$ characterizes the required energy to release the cantilever beam out of the failure configuration
Construct a **Stochastic Multi-scale Model (SMM)** for stiction problems

- **Multi-scale component of SMM**
  - Micro- to meso-scale model: evaluate the **meso-scale contact laws** from contacting topologies
  - Meso- to macro-scale model: use the **meso-scale contact laws** to predict the macro behaviors

- **Probabilistic component of SMM**
  - **Direct method (Full Monte Carlo method)**
    - Characterize the randomness of the micro-scale topology
    - Propagate the randomness through the multi-scale model
  - **Indirect method through “a stochastic model of the random meso-scale contact laws”** (*)
    - Implement “A stochastic model of the random meso-scale contact laws” to model the randomness of the meso-scale contact laws
      - Not a trivial task
    - Propagate the randomness of meso-scale contact laws (only) through the meso- to macro-scale model
      - Lower computational cost

(*) A Clément, C Soize, J. Yvonnet, *Uncertainty quantification in computational stochastic multi-scale analysis of nonlinear elastic materials*
Multi-scale component of SMM

- Meso-scale contact law: force-distance function modeling the interaction of two contacting bodies
  - The bridge between micro and macro-scales
  - The key ingredient of this research

1. Discretization
2. Contact modeling (*)
3. Finite Element model
   (n integration points)

- Meso-scale contact law: force-distance function modeling the interaction of two contacting bodies
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(*) Details of Contact modeling procedure

Micro-scale topologies $D_i$

Analytical contact models
- Meniscus
- Laplace pressure
- Asperity contact models
Probabilistic component of SMM: Direct method (Full Monte Carlo Method)

- Characterize the rough surface as a stationary Gaussian random field
1. Surface generator

Probabilistic component of **SMM**: Direct method (Full Monte Carlo Method)

- Propagate the randomness through the multi-scale model

(*) The axes have different scales: the $x$ and $y$ axes units are $\mu m$ and the $z$ axis one is nm
Propagating the randomness through the multi-scale model.

1. Surface generator
2. Contact modeling
3. Finite Element model

Topologies
Spectral Density
(different topologies)
• A time consuming process
• Requires a big memory
• Motivation for constructing the indirect method through a stochastic model of meso-scale contact laws
  • to represent the probability distribution of the meso-scale contact laws
Probabilistic component of **SMM**: Direct method (Full Monte Carlo Method)

1. Surface generator
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Reference random contact law fields

Reference distribution of contact length
Indirect method: Stochastic model of random contact laws

- The stochastic model of random contact laws $T$ represents the probability distribution of meso-scale contact laws
  - $T$: matching a random vector of a basic distribution, e.g., Gaussian one, to a random contact law

\[ \hat{F}(d) = T(d, \xi) \]

- Basic vector value random variable: e.g., Gaussian one

- Remark: The correlation of neighboring contact forces can be neglected
Stochastic model of random contact laws

- Reduced-order process
  - Fitting the adhesive contact laws using an analytical function (modified Morse potential) computed from the reduced parameters
  - Each contact law corresponds to a vector of reduced parameters and vice versa

- Randomness modeling process
  - Using Polynomial Chaos Expansion as the mean to represent the probability distribution of the reduced parameters

- Probabilistic Approximation

![Generated contact laws](image1)

![N observed contact laws from SMM using Monte Carlo method](image2)
Stochastic model of random contact laws: Reduced Order process

- Reduced-order process
  - Fitting the contact laws using an analytical function (modified Morse potential)

Observed meso-scale contact laws

- The logarithm is applied to enforce the positivity of $E_{\text{left}}$, $E_{\text{right}}$, $F_{\text{max}}$
Stochastic model of random contact laws: Randomness modeling process

- Using Hermite polynomial chaos expansion to construct the stochastic model:
  \[
  \begin{bmatrix}
  \log(E_{\text{left}}) \\
  \log(F_{\text{max}}) \\
  \log(E_{\text{right}}) \\
  d_{\text{max}} \\
  d_{\text{limit}}
  \end{bmatrix}
  \overset{d.}{=} \sum_{k=0}^{N_p} c_k \psi_k(\xi)
  \]

  Coefficients Vector \hspace{1cm} \overset{\text{Gaussian random vector}}{\rightarrow} \hspace{1cm} \text{Hermite Polynomial}

- The coefficients are found by solving Maximum Likelihood problem
  - Likelihood function is computed using multivariate kernel density estimation with Scott's data-based rule for the optimal bandwidth
  - The constraint of identical covariance

\[
CC^T = \text{Cov}(\Lambda) \quad \text{with} \quad C = [c_1 c_2 \ldots c_{N_p}]
\]

- The coefficient matrix can be rewritten as

\[
C = \left[\text{Cov}(\Lambda)^{-1/2}\right]^T S
\]

  where \(S\) is defined on the Stiefel manifold \(SS^T = I\)
Implementation (Summary)

- **Multi-scale component of SMM**
  - Using *analytical contact model for rough surfaces* (*) to solve the Micro- to meso-scale model
  - Using *Finite Element model of Euler-Bernoulli beam theory with a Newton-Raphson algorithm* for dealing with the nonlinearity of contact laws to solve the meso- to macro-scale model

- **Probabilistic component of SMM**
  - Using *Spectral Representation with Fast Fourier Transform implementation* for the simulation of the stationary Gaussian random field of topologies (**) 
  - Using *gradient-free optimization in which a line-search technique and the orthogonal directions obtained by Gram–Schmidt process* are applied to solve the maximum likelihood problem of PCE.

(*) TV Hoang et al., *A probabilistic model for predicting the uncertainties of the humid stiction phenomenon on hard materials*

(**) F Poirion, C Soize, *Numerical Methods and Mathematical Aspects For Simulation of Homogeneous and Inhomogeneous Gaussian Vector Fields*
Numerical results: Meso-scale contact laws

- Comparison of the distributions of reduced parameters of random meso-scale contact laws obtained
  - By full Monte Carlo method and
  - By the stochastic model of random meso-scale contact laws
Numerical results: Macro-scale stiction level

- Comparison of the distribution of crack lengths obtained by SMM with two different methodologies
  - Using direct method (full Monte Carlo method) as the reference and
  - Using indirect method through the stochastic model of random meso-scale contact laws
Numerical results: Macro-scale stiction level

- In case of **SMM** using stochastic model of random contact laws the crack lengths are shorter, the adhesive energies are higher.

- Due to the magnifying of the error resulting from the logarithm scaling

**Improvements:**

- Increase the order of PCE; or
- Adapt the probability distribution of the random variables.
Conclusions

- We construct a Stochastic Multi-scale Model (SMM) for stiction problems taking the surface topology into account by
  - Using multi-scale approach with the introduction of the meso-scale contact laws
  - Applying PCE to build a stochastic model of the random meso-scale contact laws
    - to reduce efficiently the computational cost
- The stochastic model of meso-scale contact laws needs to be improved
  - Increasing the order of PCE; or
  - Adapt the probability distribution of the random variables.
- Experimental validation
References

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Q&A

• Thank you for your attention