Exact and Heuristic Solution Methods for a VRP with Time Windows and Variable Service Start Time

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- Max allotted time for each route.

A classic solution method for the rich VRP: Branch-and-Price

- At each node of the branch-and-bound tree, the linear relaxation of the set-covering formulation is solved via column generation.

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- The pricing sub-problem is an elementary shortest path problem with resource constraints (ESPPRC).

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- At each node of the branch-and-bound tree, the linear relaxation of the set-covering formulation is solved via column generation.
- The pricing sub-problem is an elementary shortest path problem with resource constraints (ESPPRC).
- If the underlying graph may have negative cost cycles, the ESPPRC is strongly NP-Hard[1].

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Exact Dynamic Programming for the ESPPRC [2]

- Each state associated to vertex i represents a path from the source s to i.

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- Extension of a state from i to j corresponds to adding the arc (i, j) to a path from s to i.
- We terminate when all states have been extended in all feasible ways.

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- To enforce elementarity, we introduce a dummy unitary resource El_k, which is consumed when vertex k is visited.

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Dominance rules

State $(C', R', (\mathrm{EI}_k)'_{k\in V}, i)$ dominates $(C'', R'', (\mathrm{EI}_k)''_{k\in V}, i)$ iff

$$C' \leq C''$$
$$R' \leq R''$$
$$(\mathrm{EI}_k)'_{k\in V} \leq (\mathrm{EI}_k)''_{k\in V}$$

and at least one of the equalities is strict.

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- There is a single vehicle available at any time for a duration $\mathcal{S}$.
- The total cost of a path P depends on total distance D_P and total travel time T_P.
- We aim to find the service start time T_s and path P that minimize the total cost:

$$C_P(T_s) = \alpha D_P + \beta T_P(T_s) - \sum_{i \in P} \eta_i.$$

Resources for DP

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- a dummy resource $(\mathrm{El}_k)^i_{k \in V}$, requiring $(\mathrm{El}_k)^i \in [0, 1]$, $\forall k \in V$.
- A DP state for vertex i in our scenario is therefore

$$(C_i, T_i, S_i, \mathrm{Del}_i, (\mathrm{El}_k)^i_{k \in V}).$$

Dominance rules: issue with time dependency

- T_i, S_i, and the total cost of the subpath s-i C_i clearly depend on the starting time T_s.

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- We must therefore take into account an infinite number of Pareto-optimal states.
- We can't apply directly normal dominance rules.

Time functions

- We will consider a path on a network with n vertices:

$$P : s = 0 \to \cdots \to i - 1 \to i \to \cdots \to n + 1 = t$$

.

- Let us define the adjusted travel time: $\bar{t}_{i-1,i} := t_{i-1,i} + s_{i-1}$

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- The latest feasible start time from the source: $l_i := \min_{1 \leq j \leq i}\{b_j - \theta_j\}$
- The earliest feasible service start time at vertex i: $\tilde{a}_i := \max\{a_i, \tilde{a}_{i-1} + \bar{t}_{i-1,i}\}$.

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Description of the service start time function

For all i, if $\tilde{a}_i < l_i + \theta_i$,

$$T_i(T_s) = \begin{cases} \tilde{a}_i, & \text{if } T_s \leq \tilde{a}_i - \theta_i, \\ T_s + \theta_i, & \text{if } \tilde{a}_i - \theta_i \leq T_s \leq l_i; \end{cases}$$

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- They are piecewise linear functions from which the other time-dependent functions derive directly.

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\begin{aligned}
-l_i &= -\min\{l_{i-1}, b_i - \theta_i\} \\
\tilde{a}_i &= \max\{a_i, \tilde{a}_{i-1} + \overline{t}_{i-1,i}\} \\
A_i &= \max\{A_{i-1} + \beta\overline{t}_{i-1,i}, \beta(\tilde{a}_i - l_i)\} \\
\delta_i &= \delta_{i-1} + \alpha c_{i-1,i} - \eta_{i-1} \\
\mathrm{Del}_i &= \mathrm{Del}_{i-1} + d_i \\
\mathrm{El}_k^i &= \begin{cases} \mathrm{El}_k^{i-1} + 1 & \text{if} k = i \\ \mathrm{El}_k^{i-1} & \text{otherwise} \end{cases} \qquad \forall k \in V
\end{aligned}
$$

where $\theta_i = \theta_{i-1} + \overline{t}_{i-1,i}$ and A_i is the minimum value of the function $T_i(T_s)$.

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- It suffices to invert the time windows with a constant M and change direction of the arcs, then use monodirectional DP:

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- The latest feasible start time from the depot becomes the earliest feasible arrival time at the depot: $M - l_i^b = e_i$.

Path concatenation

- To see if we obtain a feasible path this needs to be true:

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Path concatenation

- To see if we obtain a feasible path this needs to be true:

$$\begin{aligned} \tilde{a}_i &\leq \tilde{b}_i \\ \mathrm{Del}_i + \mathrm{Del}_i^b - d_i &\leq Q \\ \mathrm{El}_k^i + \mathrm{El}_k^{ib} &\leq 1, \forall k \in V \setminus \{i\} \\ T_P &\leq S, \end{aligned}$$

  where T_P is the total travel time of path P obtained by concatenation.

- We need a *concatenation theorem*[3] to compute the actual total travel time T_P - we can't sum the partial times directly.

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DP improvements[4]: Duplicate Elimination

- During the phase of concatenation of forward and backward labels, the same path can be generated multiple times.

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- During the phase of concatenation of forward and backward labels, the same path can be generated multiple times.
- The path $P = s \rightarrow \cdots \rightarrow j \rightarrow i \rightarrow k \rightarrow \cdots \rightarrow t$ can be obtained by concatenating different pairs of labels, e.g. (l_i^{fw}, l_i^{bw}) or (l_j^{fw}, l_j^{bw}).

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- We accept it only if they are as close as possible to half of the overall consumption of the resource along the path, i.e. iff $\Phi_i := |R_{r,i}^{\text{fw}} - R_{r,i}^{\text{bw}}|$ is minimum.

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- The test is performed in constant time since we need only to check Φ_k if $R_{r,i}^{\text{fw}} < R_{r,i}^{\text{bw}}$ or Φ_j otherwise.

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DP improvements: Decremental State Space Relaxation

- In **State Space Relaxation**[5] we project the state-space $\mathcal{S}$ used in DP to a lower dimensional space $\mathcal{T}$, so that the new states retain the cost.

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- We maintain a set Θ of **critical** nodes on which the elementarity constraints are enforced at each iteration of DP.
- If at the end of DP the optimal path is not feasible, we update Θ with the nodes that are visited multiple times.

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DSSR strategies

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- These decisions involve trade-offs (e.g. cost of an iteration vs number of iterations).
- We can associate parameters to these decisions, which we can then tune. parameters.

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  - **Branch-and-Price based approaches**, classified in *restricted master heuristics*, *heuristic branching* approaches, and *relaxation based* approaches.

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Restricted Master Heuristics

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- The columns can either be generated heuristically or by solving exactly the pricing problem.
- However, the master problem defined over a subset of columns is often infeasible[8], so we have to adopt techniques to recover feasibility or devise ways to obtain a suitable set of columns.

[8] Joncour et al. 2010.

[9] Danna and Le Pape 2005.

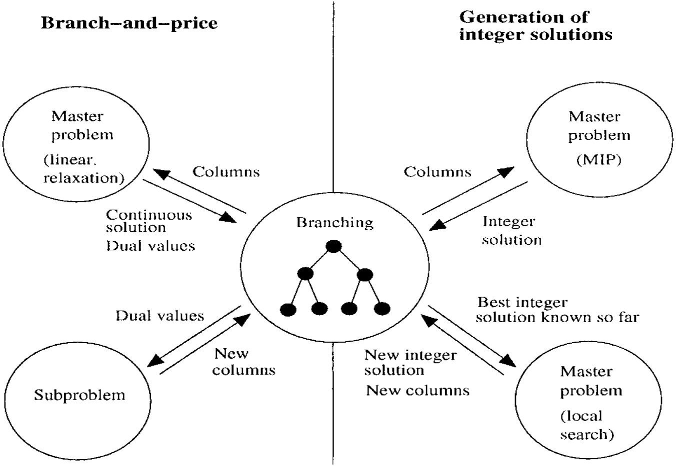
Restricted Master Heuristics

- The optimal solution of the master problem restricted to any subset of generated columns provides an heuristic solution.
- The columns can either be generated heuristically or by solving exactly the pricing problem.
- However, the master problem defined over a subset of columns is often infeasible[8], so we have to adopt techniques to recover feasibility or devise ways to obtain a suitable set of columns.
- Within the BP framework, we can use the RMH in a collaboration scheme with a metaheuristic[9], in order to obtain good solutions early in the procedure.

[8] Joncour et al. 2010.

[9] Danna and Le Pape 2005.

Collaboration scheme[10]



[10] Image from Danna and Le Pape 2005.

Thanks for your attention.

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