Integration of the selective deconvolution algorithm into a multiresolution scheme

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Abstract

In 1987, Franke[2] introduced a new algorithm for the extrapolation of discrete signals. It allows the reconstruction and the extrapolation of texture segments on supports where the contour information is no longer included. This method is called the *selective deconvolution* and works as an iterative process using the Fourier transform and it inverse.

In this paper, we study the integration of the selective deconvolution method into a multiresolution scheme. In a first algorithm, we propose to apply the selective deconvolution introduced by Franke on each one of the subband signals obtained at the output of an analysis stage. The goal of this integration is both to gain computation time and to obtain a multiresolution representation of the texture segment.

Franke proposes the D.F.T. as spectral operator in his algorithm. Actually, more general spectral operators can also be considered. Therefore, in a second part of this paper, we study an iterative selective deconvolution method based on a subband decomposition rather on the D.F.T..

1.Introduction

In digital processing one often has to deal with the problem of extrapolating a function $f(n)$ observed over a finite interval only. As a consequence, the given signal samples usually contain perturbating effects due to the observation equipment. In fact, this corresponds to a so-called *windowing effect*, the window being the spatial domain where the signal is known. Figure 1 states the effect. The left-hand side signal represents a signal observed over a given spatial segment; the right-hand side signal is the signal extrapolated on the whole square segment. In this situation, the signal does not longer suffer from the influence of the triangular window.

Franke showed the influence of the window on the Fourier transform of an original signal which was a texture. The window introduces a subsequent spectrum dispersion. The extrapolated signal has a more concentrated spectrum. Concentration is better for coding but also for identification or pattern recognition.

The spectrum of $f(n)$ is obtained by means of an iterative algorithm. The idea of working with an iterative process directly on the Fourier transform is an old one: Gerchberg[3] and Papoulis[4] suggested an algorithm applicable for a band-limited function. Sabri et al.[6] gave an alternative non-iterative method where the total extrapolation method is reduced to a

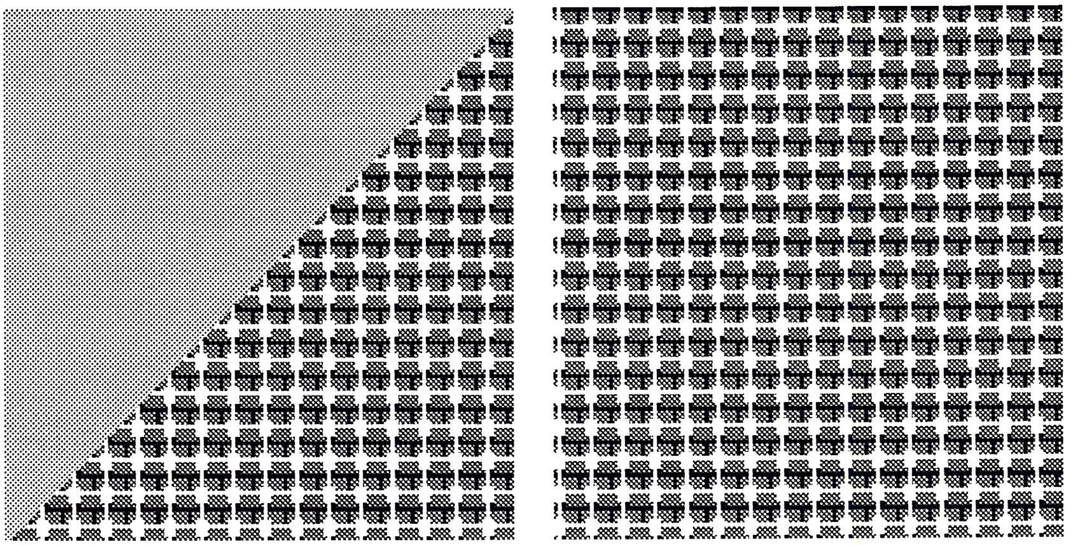

Figure 1: A texture and the corresponding extrapolated signal.

single matrix extrapolation. Unfortunately, it requires the inversion of a matrix that is ill conditioned in many cases. Another critical assumption is that signal is band-limited. Both of the precedent methods need to know exactly the cut-off frequency; with a higher or a lower cut-off frequency, problems like convergence, computation speed, estimation validity, ... are critical. This led Franke to suggest an algorithm (called *selective deconvolution*) for textures, signals containing dominant spectral lines. The chosen filter is adaptive and is intended to detect these dominant lines.

Results are quite satisfactory with this method. The drawback is the important computation time needed for the calculation of the Fourier transform and the inverse transform.

In the next section, we study the integration of the selective deconvolution method into a multiresolution scheme. The goal of this integration is to obtain a multiresolution representation of an extrapolated signal and to achieve a significant computation gain. The method proposed here consists in applying the selective deconvolution algorithm to each subband produced by an analysis stage.

The original method proposed by Franke is built around the D.F.T. (Discrete Fourier Transform). The D.F.T. is actually nothing but a particular choice of a spectral operator.

In a next section of this paper, we have built an iterative selective deconvolution method based on a subband operator rather on the D.F.T.

Results of two implementations illustrate the advantages and the drawbacks of the multiresolution representation. They are presented in section 4.

Formulation of the extrapolation problem

For commodity we use a one-dimensional notation; the formal extension to picture introduces no particular difficulty. Let $f(n)$ be the function to extrapolate and $y(n)$ the observed samples over the domain D_w. The functions $f(n)$ and $y(n)$ are related through the following expression:

$$y(n) = w(n)f(n) \qquad 0 \leq n \leq N-1 \tag{1}$$

where

$$w(n) = \begin{cases} 1 & \text{for} \quad n \in D_w \subseteq \{0, ..., N-1\} \\ 0 & \text{if} \quad n \notin D_w \end{cases} \tag{2}$$

is the window.

With the transform formalism, the equation is equivalent to the following convolution:

$$\mathcal{Y}(k) = \mathcal{W}(k) \otimes \mathcal{F}(k) \qquad 0 \leq k \leq N-1 \tag{3}$$

As clearly indicated by this last equation, the desired spectrum $\mathcal{F}(k)$ is affected by the window spectrum $\mathcal{W}(k)$; it is why a spectral extrapolation technique is also called a "deconvolution" technique.

2. Basic principles

2.1. Iterative extrapolation algorithm

The only way to solve the extrapolation problem with respect to the spectrum analysis is to proceed iteratively. With a consistent operator $\mathcal{O}$, we may form a list of successive values $..., f_i(n), f_{i+1}(n), ...$ obtained by

$$f_{i+1}(n) = \mathcal{O}[f_i(n)] \tag{4}$$

The classical solutions to the extrapolation problem consider different operators with the associated questions of convergence and unicity. It is not our concern here.

2.2. Subband representation

We develop a multiresolution version of the iterative operator. The signal $f(n)$ is replaced by its components $\phi_1(n), \phi_2(n), ..., \phi_l(n)$ forming a complete representation; the operator $\mathcal{O}$ is adapted in consequence for each function $\phi_j(n)$.

The multiresolution representation chosen here is the subband representation. Subband coding has been introduced by Crochiere et al.[1] for the coding of speech signals. The signal $f(n)$ is split in different bands by bandpass filters. The output of these filters are downsampled, generally so as to keep the initial amounts of information. Suppose we have 4 filters. The representation of $f(n)$ in a unique analysis stage decomposition is given by $\phi_1(n), \phi_2(n), \phi_3(n)$ and $\phi_4(n)$ with $n \in [0, \frac{N}{4} - 1]$. This is a 4-band parallel decomposition. The basic cell can serve again on the 4 signals or on a single one, leading to a *parallel* or to a *hierarchical* system respectively. The spectral analysis performed in this way is particularly interesting for an effective signal extrapolation problem where the signal is a texture and regroups parts of its energy in a few bands. Furthermore, the signals to be treated have a size of $\frac{N}{4}$. For deconvolution with the Fourier transform, where the computation complexity is in $N \log N$, the complexity greatly decreases.

The final signal is reconstructed after applying a modified iterative operator. The subbands are interpolated by means of zero insertion. Without any iteration, the signal can be made of perfect reconstruction. We used 8 tap filters from Johnston[4]. They provide a quasi perfect reconstruction.

3. The iterative process

The extrapolation efficiency is completely conditioned by the choice of the iterative operator $\mathcal{O}$. Two questions are to be solved: first, which operator form, second, how to adapt an operator designed for $f(n)$ to the subbands signals $\phi_j(n)$. For the form we propose the selective deconvolution by means of the D.F.T, briefly described below, and next, the selective deconvolution by means of the subband transform. The adaptation will be discussed for each one of these two operator forms.

3.1. Selective deconvolution of the subbands by means of the D.F.T.

The iterative process acts as an adaptive filter $\mathcal{S}(k)$. At the beginning, the function is equal to zero for every k. The complete initialization step is then:

$$f_0(n) = y(n)$$
$$\mathcal{S}(k) = 0 \quad \forall k$$

The block diagram of the algorithm is shown in figure 2. At step $i+1$, the algorithm detects

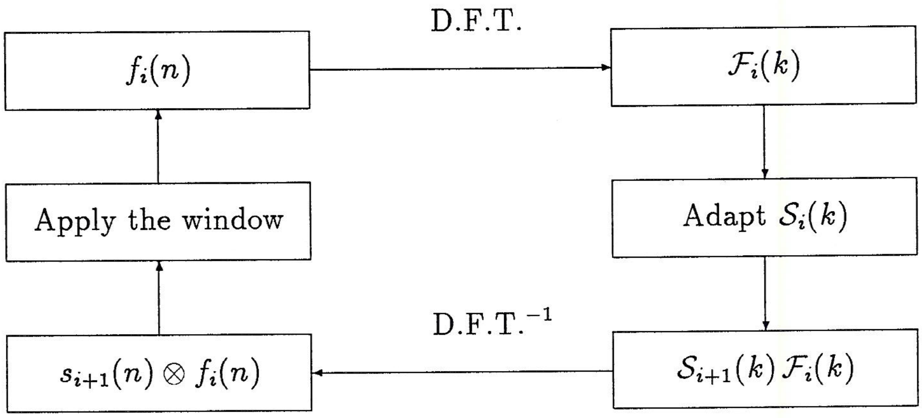

Figure 2: Block diagram of the D.F.T.-based selective deconvolution algorithm

the spectral line of $\mathcal{F}_i(k)$ having the largest magnitude and not considered in an earlier step. In accordance with Parseval's theorem, one then expects the error to be minimal. Let k_s

be the selected line (with the symmetrical position when working with real functions). The filter is modified by

$$\mathcal{S}_{i+1}(k) = \begin{cases} \delta(k - k_s) & \text{if } k = k_s \\ \mathcal{S}_i(k) & \text{elsewhere} \end{cases} \tag{5}$$

So, a supplementary spectral coefficient is added at each iteration. The corresponding estimation $f_{i+1}(n)$ results from the inverse transform of $\mathcal{S}(k)\mathcal{F}(k)$. If the spectral content of $f_{i+1}(n)$ is enriched, the values of $f_{i+1}(n)$ on the D_w segment differ form the observed samples. To achieve the iteration, it is necessary to apply the window by $f_{i+1}(n) \leftarrow [1 - w(n)]f_{i+1}(n) + w(n)y(n)$, after what the function is ready for a further step.

In this paper, we propose to extrapolate the subbands signals $\phi_j(n)$, not the original signal. Filtering and downsampling $y(n)$ or $f(n)$ is easy to do in accordance with the signal theory. It is not the case for the window function. The signal $w(n)$ is a binary function. For convenience, we conserve a binary signal by downsampling and tresholding the filtered window. In counterpart, it introduces an insignificant aliasing in the extrapolated signals.

The transform implemented by Franke is the Fourier transform. Other transforms as the discrete cosine transform suit as well.

3.2. Subband-based selective deconvolution

For an efficient extrapolation, the iterative algorithm must act as a *global* operator, which means that it affects the whole domain. The Fourier transform meets this requirement, but a subband transform does not. Indeed, the last one combines spatial and local localization. But, when the subband size is half the filter size, the synthesis filter operates as a global operator. This led us to try a subband-based iterative process. The way to proceed is:

- to analyse the signal into subbands,
- to select a coefficient in a subband; this coefficient may not have been considered in a previous step iteration,
- to pass the resulting signal through the synthesis stage, and
- to apply the window.

The corresponding block diagram is drawn in figure 3, where SB stands for subband transform, $\vec{k}$ is a vector containing the subband number and the position of the sample inside this subband.

Because the selection is made in the spatial domain, all coefficients must be selected, one by one.

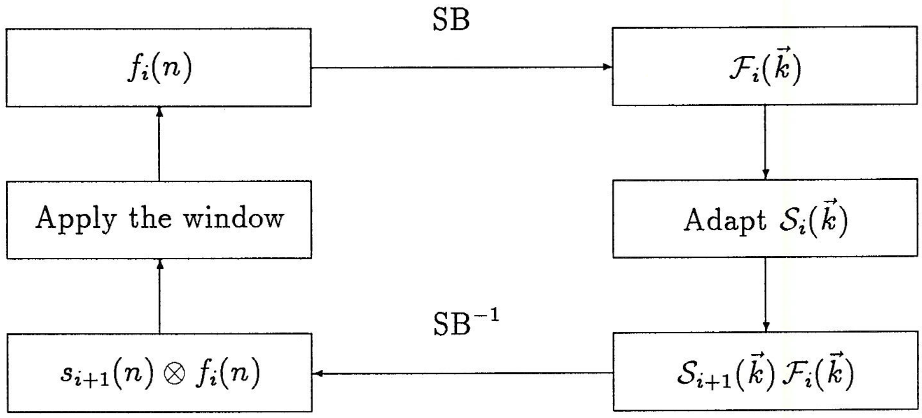

Figure 3: Block diagram of the subband-based selective deconvolution algorithm

4. Results

4.1. Selective deconvolution of the subbands by means of the D.F.T.

An implemented 4-stage hierarchical selective deconvolution (with a Fourier transform) algorithm is shown in figure 4; it is applied on an 8-bit picture of size 128x128. The upper left picture is the original signal. The extrapolated signal is drawned on its right. The corresponding multiresolution representations form the lower part of the figure. As it can be seen, each subband has been extrapolated correctly. The reconstruction method is detailed in figure 5. The lower resolution signals are first put together, and so on till the full resolution.

For texture signals, the amount of spectral coefficients to be selected is very low, about 10% is large enough for an excellent extrapolation. In a multiresolution representation, the coefficients are spread over all subbands according a semi-static, semi-dynamic criterion. Half the total amount of coefficients (the static one) is distributed in proportion to the subband sizes. The rest follows an energy criterion: the ratio of spectral lines to be selected in a band is the same as the ratio of the energy contained in each subband.

The performance of different combinations of the simple selection strategy and different multiresolution representation are compiled in table 1. The first row indicates the total number of coefficients selected during the iterative process. The second row represents the mean of the absolute errors on the window (E_w), on the complement of D_w (E_{wc}) and on the whole picture (E). Figure 5 was produced with 4 stages and 200 coefficients (only 2% of the spectrum size).

The time for the precedent experiments is given in table 2. There is a big difference between the real selective deconvolution (0 decomposition stage) and a multiresolution representation for results that are not discernible from the original.

Coefficient amount	Error type	Decomposition stages				
		0	1	2	3	4
30	E_w	10.67	14.82	15.91	17.08	17.04
	E_{wc}	12.67	15.20	16.90	18.08	18.03
	E	11.89	15.01	16.40	17.57	17.53
60	E_w	2.02	8.33	9.04	9.85	9.87
	E_{wc}	2.65	8.64	9.87	12.31	12.29
	E	2.40	8.48	9.46	11.08	11.07
100	E_w	0.5	2.13	2.63	3.70	3.63
	E_{wc}	0.5	2.31	4.17	7.70	7.69
	E	0.5	2.22	3.40	5.69	5.65
200	E_w	0.5	1.06	1.50	2.52	2.40
	E_{wc}	0.5	1.97	3.83	7.06	7.02
	E	0.5	1.52	2.66	4.78	4.70
300	E_w	0.5	1.11	1.50	2.46	2.52
	E_{wc}	0.5	2.53	4.15	7.18	7.14
	E	0.5	1.82	2.82	4.80	4.68
600	E_w	0.5	1.13	1.55	2.44	2.28
	E_{wc}	0.5	3.27	4.70	7.62	7.59
	E	0.5	2.20	3.12	5.02	4.92
1000	E_w	0.5	1.13	1.57	2.44	2.32
	E_{wc}	0.5	3.62	5.00	7.88	7.84
	E	0.5	2.37	3.28	5.15	5.06

Table 1: Means of absolute errors.

4.2. Subband-based selective deconvolution

Figure 6 was computed with a parallel subband representation and a subband iterative scheme. Although the results show the interest of this extrapolation technique, there are still questions to be answered like the filter choice, the way of selecting coefficient, ... for better extrapolation results.

5. CONCLUSION

The concept of a multiresolution iterative operator is a helpful one for signal extrapolation. It decreases the computation time, gives a spectral analysis for coding or recognition purposes, and enhances the class of extrapolation operators. Results confirm the importance of the multiresolution representation for a D.F.T.-based selective deconvolution. We showed it is theoretically possible to achieve an extrapolation with a subband-based deconvolution algorithm.

Further works will concern the determination of an optimal strategy for spreading the amount of spectral lines to be conserved in every subband.

Coefficient amount	Decomposition stages				
	0	1	2	3	4
30	184	38	33	32	32
60	264	62	44	44	44
100	408	91	63	60	63
200	802	164	109	104	106
300	1194	241	149	148	146
600	2372	457	285	276	273
1000	3556	678	415	402	395

Table 2: Computation time in seconds for the D.F.T.-based selective deconvolution in a hierarchical subband representation.

6. Acknowledgements

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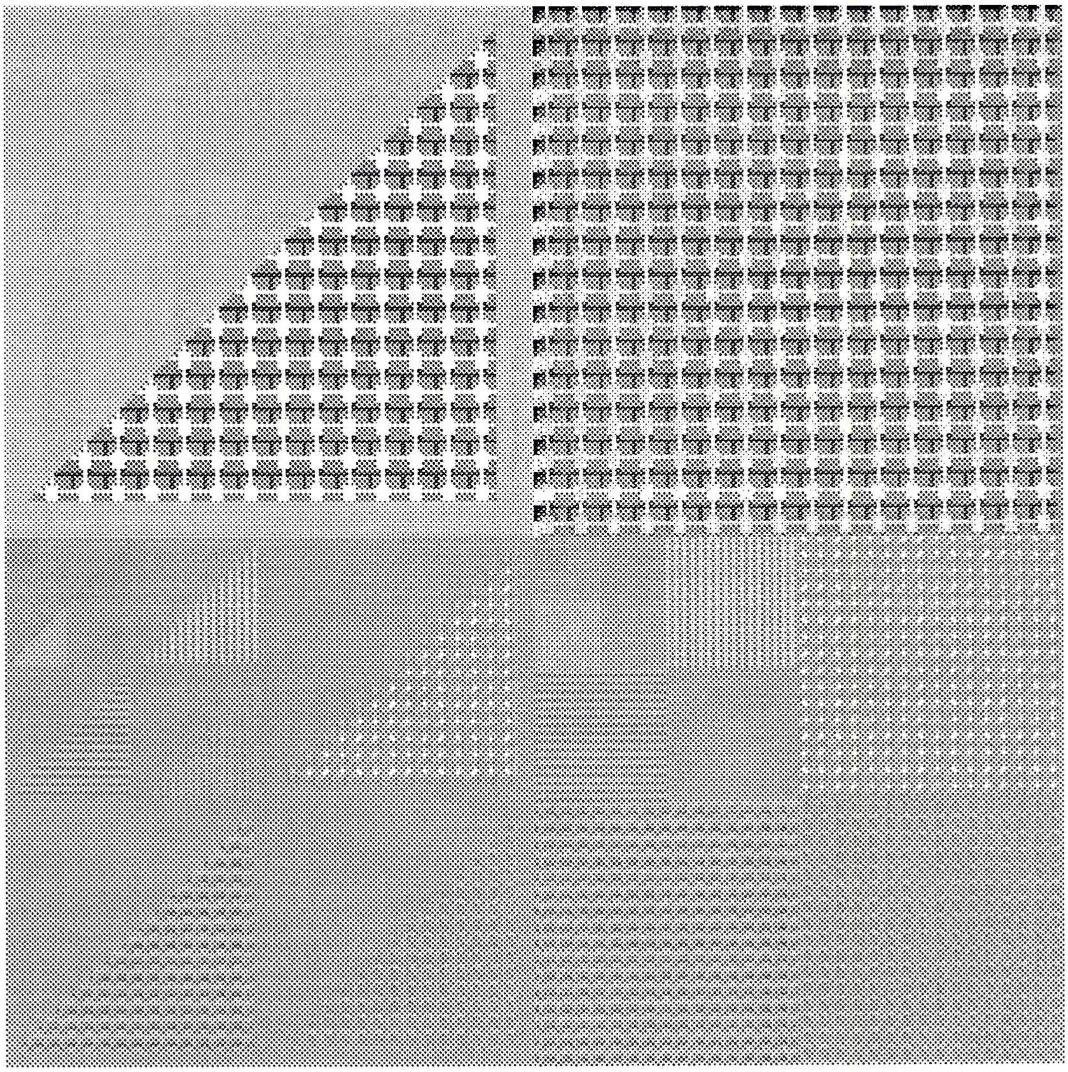

Figure 4: Illustration of the hierarchical selective deconvolution algorithm. The signals are in order: $y(n)$, $f(n)$, and the subband decompositions of these signals.

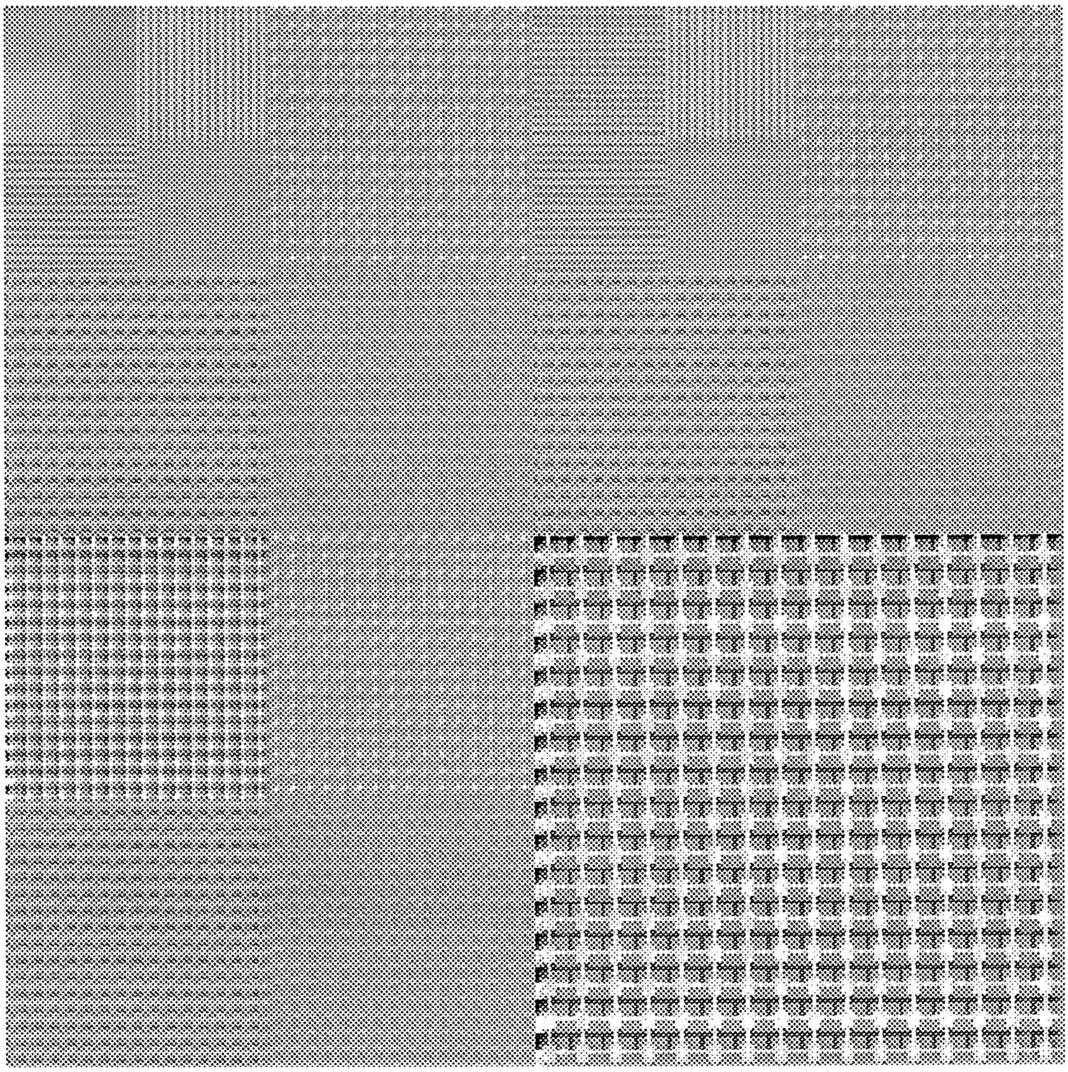

Figure 5: Progressive reconstruction of the subband representation. The last picture contains the extrapolated signal.

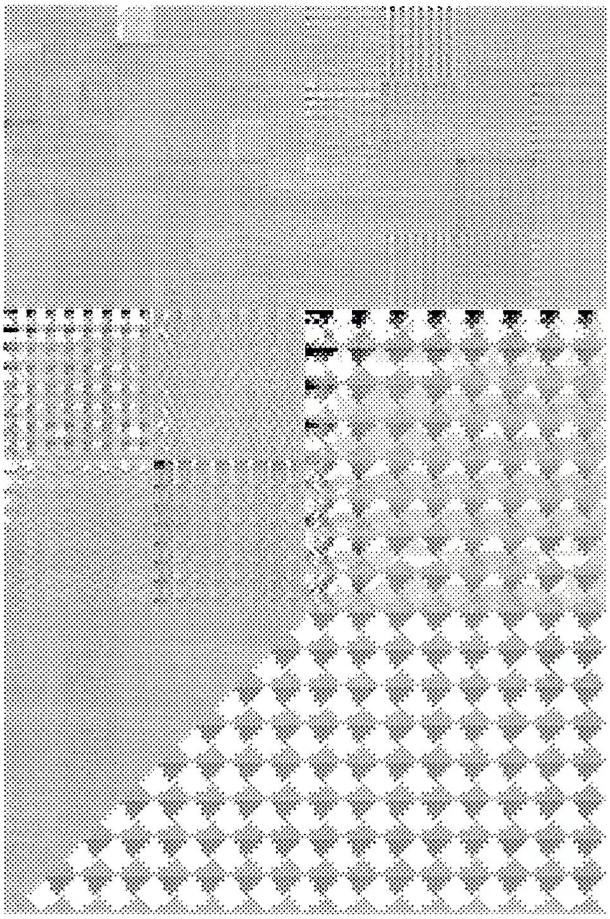

Figure 6: Illustration of the reconstruction steps of a parallel subband representation with an iterative subband scheme. The last row shows $y(n)$ and $f(n)$.