Transport of Bose-Einstein Condensates through Aharonov-Bohm rings

R. Chrétien, J. Dujardin, C. Petitjean, and P. Schlagheck

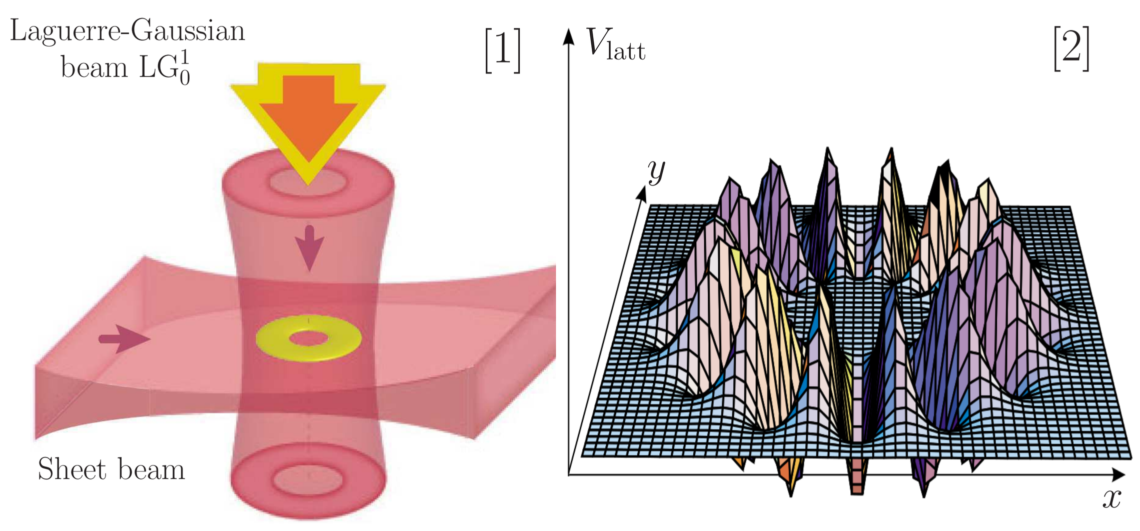
Département de Physique, University of Liege, 4000 Liège, Belgium

Abstract

We study the one-dimensional (1D) transport properties of an ultracold gas of Bose-Einstein condensed atoms through Aharonov-Bohm (AB) rings. Our system consists of a Bose-Einstein condensate (BEC) that is outcoupled from a magnetic trap into a 1D waveguide which is made of two semi-infinite leads that join a ring geometry exposed to a magnetic flux ϕ. We specifically investigate the effects of a small atom-atom contact interaction strength on the AB oscillations. The main numerical tools that we use for this purpose are a mean-field Gross-Pitaevskii (GP) description and the truncated Wigner (tW) method. The latter allows for the description of incoherent transport and corresponds to a classical sampling of the evolution of the quantum bosonic many-body state through effective GP trajectories. We find that resonant transmission peaks move with an increasing interaction strength and can be suppressed for sufficiently strong interaction. We also observe that the coherent transmission blockade due to destructive interference at the AB flux $\phi = \pi$ is very robust with respect to the interaction strength.

Aharonov-Bohm rings

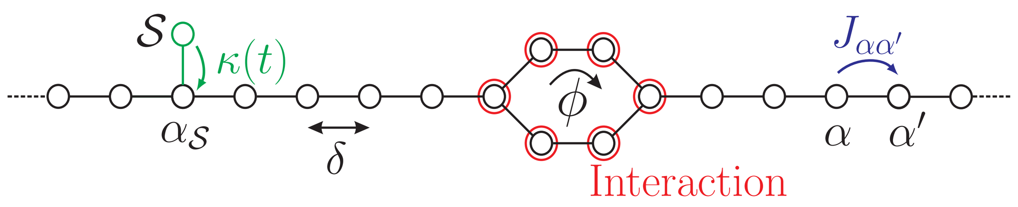
- Toroïdal optical dipole trap



- Intersection of two red-detuned beams
- Connection to two waveguides

Theoretical description

- Ring geometry connected to two semi-infinite homogeneous leads
- Perfect condensation of the reservoir ($T = 0\,\mathrm{K}$)
- Discretisation of 1D space
- Bose-Hubbard system [3]



- Hamiltonian

$$\hat{H} = \sum_{\alpha\in\mathbb{Z}} \left(E_\delta \hat{a}_\alpha^\dagger \hat{a}_\alpha + \sum_{\alpha'} J_{\alpha\alpha'} \hat{a}_{\alpha'}^\dagger \hat{a}_\alpha + g_\alpha \hat{a}_\alpha^\dagger \hat{a}_\alpha^\dagger \hat{a}_\alpha \hat{a}_\alpha \right) + \kappa(t) \hat{a}_{\alpha_{\mathcal{S}}}^\dagger \hat{b} + \kappa^*(t) \hat{a}_{\alpha_{\mathcal{S}}} \hat{b}^\dagger + \mu \hat{b}^\dagger \hat{b}$$

with :

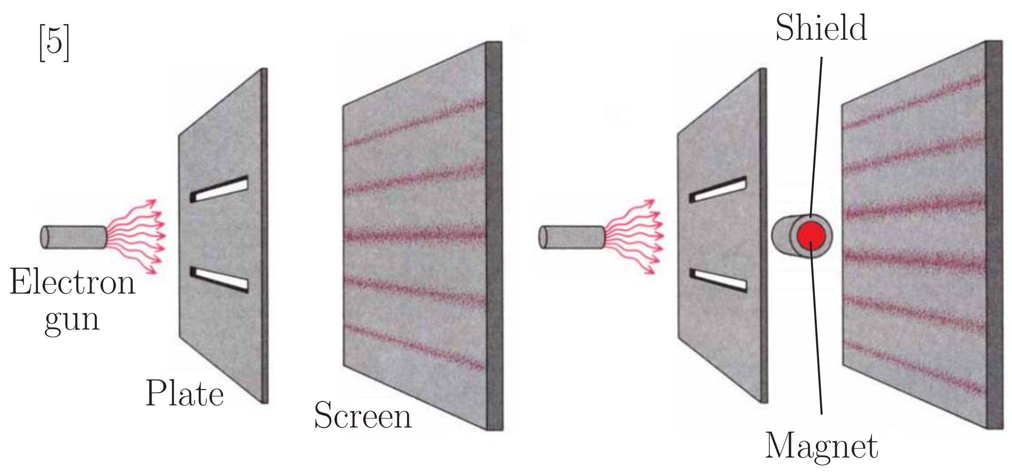
- $J_{\alpha\alpha'} = -\dfrac{E_\delta}{2} \begin{cases} 1 \text{ if site } \alpha \text{ next to site } \alpha' \\ 0 \text{ otherwise} \end{cases}$,
- $\hat{a}_\alpha$ and $\hat{a}_\alpha^\dagger$ the annihilation and creation operators at site α,
- $\hat{b}$ and $\hat{b}^\dagger$ the annihilation and creation operators of the source with chemical potential μ,
- $N \to \infty$ the number of Bose-Einstein condensed atoms within the source,
- $\kappa(t) \to 0$ the coupling strength, which tends to zero such that $N|\kappa(t)|^2$ remains finite,
- $g_\alpha = \begin{cases} g \text{ if site } \alpha \text{ within the ring} \\ 0 \text{ otherwise} \end{cases}$.

References

[1] A. Ramanathan *et al.* *Phys. Rev. Lett.* **106**, 130401 (2011).
[2] L. Amico *et al.* *Phys. Rev. Lett.* **95**, 063201 (2005).
[3] J. Dujardin *et al.* *Phys. Rev. A* **91**, 033614 (2015).
[4] Y. Aharonov and D. Bohm, *Phys. Rev.* **115**, 485-491 (1959).
[5] Y. Imry and R.A. Webb, *Scient. Am.* **260**, 56 (1989).
[6] R.A. Webb *et al.* *Phys. Rev. Lett.* **54**, 2696 (1985).

Aharonov-Bohm effect

- Potentials act on charged particles even if all fields vanish [4]

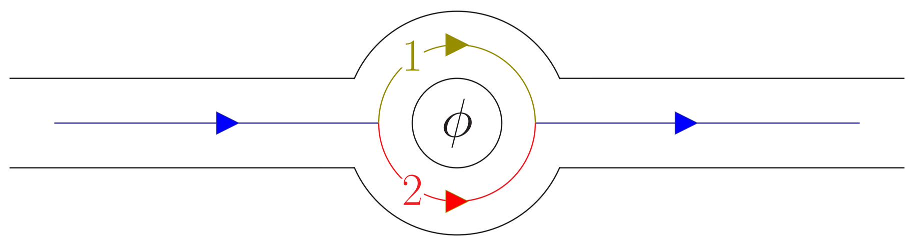


- Additional phase shift of the electron wavefunction due to the vector potential $\mathbf{A}$

$$\Delta\varphi = k\Delta l + \frac{e}{\hbar} \underbrace{\oint \mathbf{A} \cdot \mathrm{d}\mathbf{l}}_{\phi} = k\Delta l + 2\pi \frac{\phi}{\phi_0}$$

with $\phi_0 = h/e \approx 4.13566727 \cdot 10^{-15}$ Wb the magnetic flux quantum

- Interference pattern shifted when a shielded magnet is added
- Same effect within a two-arm ring



- Oscillations in transport properties due to interferences of partial waves crossing each arm of the ring
- Transmission periodic w.r.t. the AB flux ϕ

$T = |t_1 + t_2|^2 = |t_1|^2 + |t_2|^2 + 2|t_1| \cdot |t_2| \cos \Delta\varphi$ with period ϕ_0

Truncated Wigner Method

- Sampling of the initial quantum state by classical field $\{\psi_\alpha(t_0)\}_{\alpha\in\{\mathcal{S},0,\pm1,\pm2,\ldots\}}$.
- Time propagation according to classical (slightly modified) GP equation.
- Sampling the initial quantum state at t_0 [3]
  - Empty waveguide

$$\mathcal{W}_G(\{\psi_\alpha, \psi_\alpha^*\}, t_0) = \prod_\alpha \left(\frac{2}{\pi}\right) e^{-2|\psi_\alpha|^2}$$

  - $\psi_\alpha(t = t_0) = \frac{1}{2}(\mathcal{A}_\alpha + i\mathcal{B}_\alpha)$
  - $\mathcal{A}_\alpha$ and $\mathcal{B}_\alpha$: real gaussian random variables

$$\overline{\mathcal{A}_\alpha} = \overline{\mathcal{B}_\alpha} = \overline{\mathcal{A}_{\alpha'}\mathcal{B}_\alpha} = 0$$
$$\overline{\mathcal{A}_{\alpha'}\mathcal{A}_\alpha} = \overline{\mathcal{B}_{\alpha'}\mathcal{B}_\alpha} = \delta_{\alpha',\alpha}$$

  - Source populated with $|\psi_{\mathcal{S}}^0|^2 = N \gg 1$ atoms

$$\mathcal{W}_{\mathcal{S}}(\psi_{\mathcal{S}}, \psi_{\mathcal{S}}^*, t_0) = \left(\frac{2}{\pi}\right) e^{-2|\psi_{\mathcal{S}} - \psi_{\mathcal{S}}^0|^2}$$

  - Relative uncertainty negligible
  - $\psi_{\mathcal{S}}(t = t_0) = \sqrt{N} \approx \psi_{\mathcal{S}}(t)$ for $|\kappa| \to 0$
- Equation of propagation for the classical field

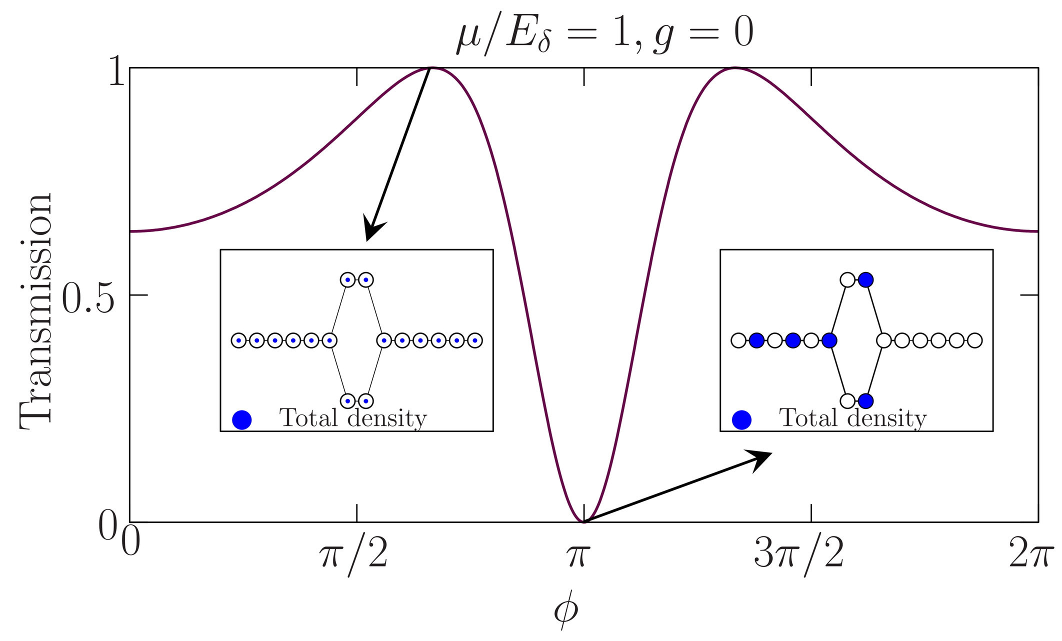
$$i\hbar \frac{\partial \psi_\alpha}{\partial t} = (E_\delta - \mu)\,\psi_\alpha + \sum_{\alpha'} J_{\alpha\alpha'} \psi_{\alpha'} + g_\alpha \left(|\psi_\alpha|^2 - 1\right) \psi_\alpha + \kappa(t)\sqrt{N}\delta_{\alpha,\alpha_{\mathcal{S}}}$$

- Compute mean values of observables by average over GP trajectories
  - total density $n_\alpha = \overline{|\psi_\alpha|^2} - \frac{1}{2}$
  - coherent density $n_\alpha^{\rm coh} = |\overline{\psi_\alpha}|^2$
  - incoherent density $n_\alpha^{\rm incoh} = n_\alpha - n_\alpha^{\rm coh}$

Computational resources have been provided by the Consortium des Equipements de Calcul Intensif (CÉCI), funded by the Fonds de la Recherche Scientifique de Belgique (F.R.S.-FNRS) under Grant No. 2.5020.11

Aharonov-Bohm oscillations

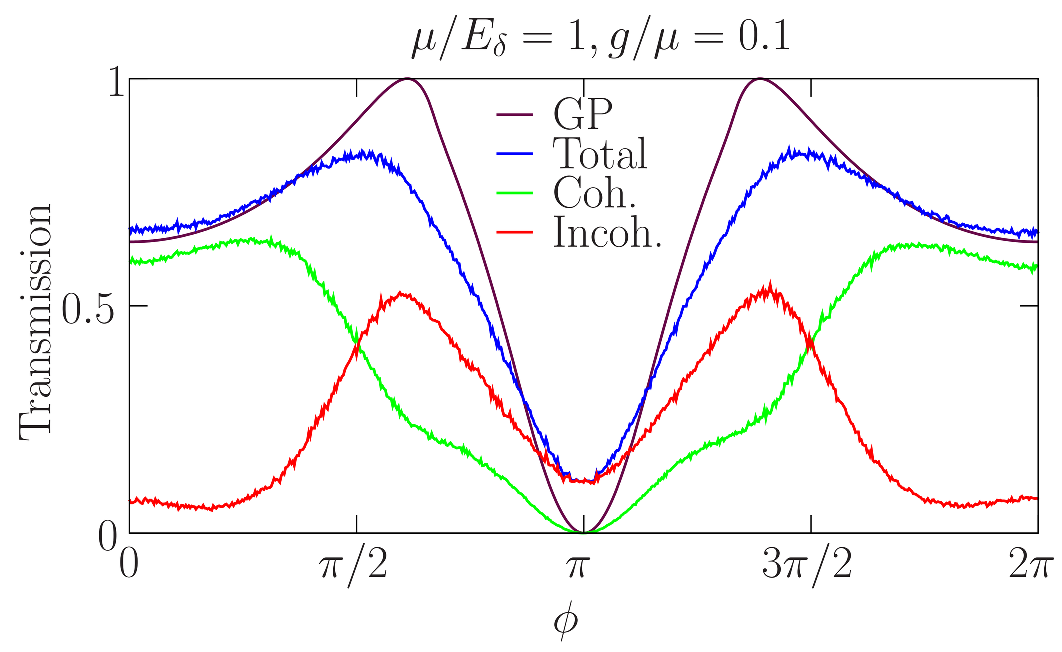
- Interferences induced by the AB flux ϕ [6].
- Periodic transmission oscillation with ϕ.



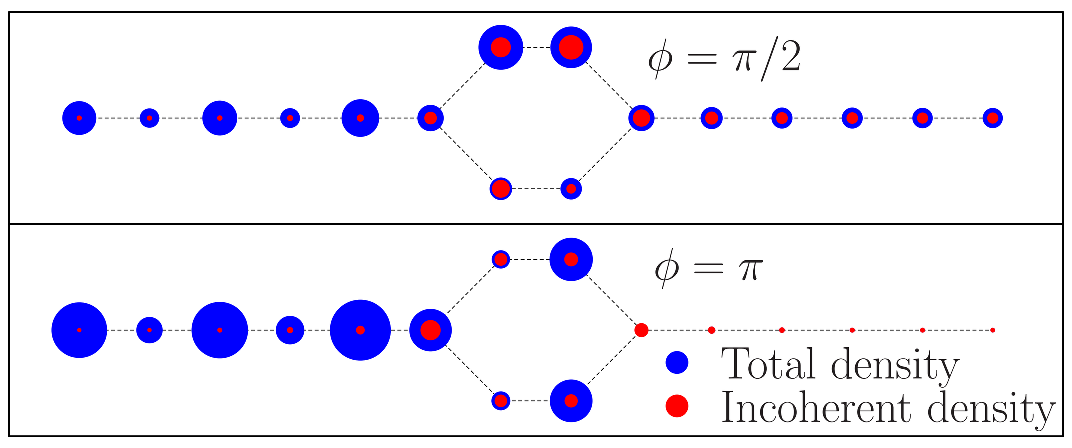
- Clear signature of the AB effect.
- No atom-atom interaction so far.

Interaction effects

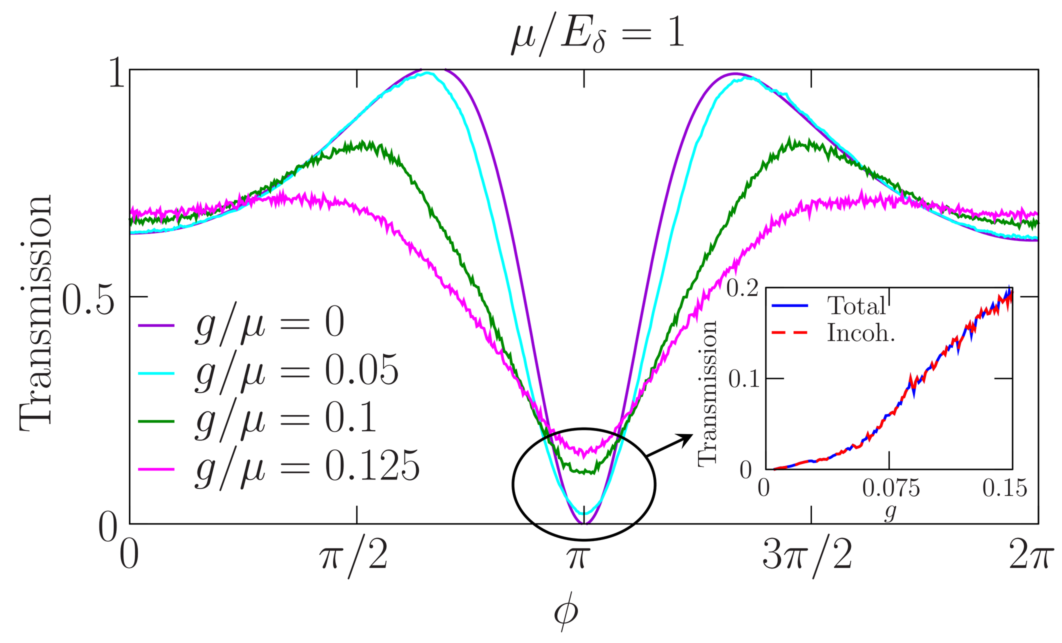
- Incoherent transmission even at $\phi = \pi$



- Inhibition of perfect transmission (see [3])
- Suspension of transmission blockade at $\phi = \pi$ due to incoherent atoms created within the ring



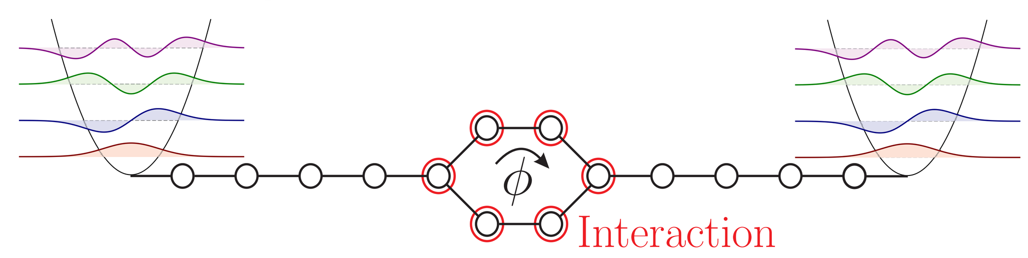
- Resonant transmission peaks move with g and disappear if g is strong enough



- Destructive interferences at $\phi = \pi$, which is robust on the variations of g.
- More incoherent particles created within the ring as g increases.
- Transmission totally incoherent at $\phi = \pi$, for all $g > 0$.

Perspectives

- Disorder within the ring
- Continuous limit $\delta \to 0$
- Finite temperature for the reservoirs



- Boson sampling

http://www.pqs.ulg.ac.be

rchretien@ulg.ac.be