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Generalized Iterative Algorithm for Spectral Signal Deconvolution

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Abstract

A generalized iterative algorithm for spectral signal deconvolution is presented in this paper. It works for signals characterized by a small number of spectral components. The algorithm combines the selective deconvolution which is an iterative spectral deconvolution technique and a subband decomposition. It offers a new way for finding optimal methods of spatial extrapolation and leads to important computation savings.

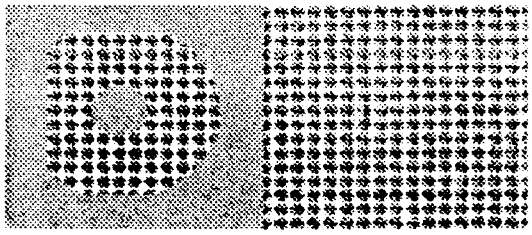

Figure 1: The right-hand side image results from the removal of the disturbating window effect.

1 Introduction

Situations occur where spectral information is corrupted by the observation equipment. To analyze the signal correctly it is necessary to remove this perturbating effect. Figure 1 shows an example: a textured signal is only known on a support, called *window* in this text, having the form of character D. This is the "windowing effect".

For spectral analysis the window has to be regular (rectangular, ...) and this is actually not the case. In order to remove the windowing effect, the basic idea is to extrapolate the spatial window content to a regular net. The corresponding extrapolated image is given in figure 1 on the right. No further window information is present.

In the same way the extrapolated signal spectrum reflects only the textured spectrum. We develop next a consistent extrapolation algorithm.

2 Model

For simplicity we adopt a one-dimensional formulation.

Let $y(n)$ be the observed signal samples. $y(n)$ results from "windowing" the true samples $f(n)$:

$$y(n) = w(n)f(n) \qquad 0 \leq n \leq N-1 \qquad (1)$$

with the window

$$w(n) = \begin{cases} 1 & \text{if} \quad n \in E_w \subseteq \{0, ..., N-1\} \\ 0 & \text{for} \quad n \not\subseteq E_w \end{cases} \qquad (2)$$

E_w is the window support. This means that the window is equal to 1 on this support.

In spectral terms the extrapolation problem is equivalent to a discrete deconvolution

$$\mathcal{Y}(k) = \mathcal{W}(k) \otimes \mathcal{F}(k) \qquad 0 \leq k \leq N-1 \qquad (3)$$

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because the desired spectrum $\mathcal{F}(k)$ is affected by $\mathcal{W}(k)$ due to the window.
In the digital case the extrapolation problem can be solved by iteration. With an operator $\mathcal{O}$, a list of function values $..., f_i(n), f_{i+1}(n), ...$ is constructed iteratively using

$$f_{i+1}(n) = \mathcal{O} f_i(n) \qquad (4)$$

The limit $f_\infty(n)$ tends to $f(n)$. Papoulis [1] proposed a now well-studied algorithm for band-limited signals, which is a strong restriction. Franke [3] added the idea of adaptive filtering but the computation complexity increased strongly. Moreover convergence and uniqueness of the solution are not guaranteed. The same goals can be achieved with a gradient based algorithm as established by Docampo-Ameodo [2]. But this algorithm introduces convergence troubles.
The proposed algorithm generalizes classical methods. It is composed of a succession of different operators we are going to discuss now.

2.1 Components

2.1.1 Direct and inverse transform

If the signal under analysis is known to be reconstructed with a small amount of transform coefficients, the principal operator to include is a transform operator and the inverse operation. The operator is for example the discrete Fourier transform (D.F.T.) or the discrete cosine transform (D.C.T.). The direct transform and its inverse will be noted respectively by M and M^{-1}.

2.1.2 Filtering

Applying the direct and the inverse transform leads to the original signal. For deconvolution purposes it is necessary to filter the signal in order to progressively separate noise and information signals. The filtering operation performs a modification of the transform data. Let Π be the filtering operator. Reading from right to left, the signal obtained after filtering is $M^{-1}\Pi M f(n)$.

2.1.3 Down and up-sampling

All the suggested deconvolution techniques work with the full resolution. Most signals concentrate the spectral energy in a limited bandwidth. It is true for image signals. If we could use this a priori information the computation time would certainly be drastically reduced. The algorithm described below contains a data reduction by down-sampling. The down-sampling operator keeps every second sample: $[\downarrow 2]f(j) = f(2j)$ for $j \in [0, (N/2) - 1]$. The corresponding dual operator is up-sampling. The up-sampling noted $[\uparrow 2]$ inserts zeros between samples.

2.1.4 Windowing operator

The last operator to be introduced is the windowing operation, noted D. It consists in placing the original samples on the window domain. In other terms,

$$D[f(n)] = \begin{cases} y(n) & \text{if} \quad n \in E_w \\ f(n) & \text{if} \quad n \notin E_w \end{cases} \qquad (5)$$

2.2 Generalized iterative deconvolution algorithm

The combination of the introduced operators conducts to a general iterative extrapolation algorithm:

$$f_{i+1}(n) = O f_i(n) \qquad (6)$$

where

$$O = DM^{-1}\Pi_2 M[\uparrow 2]M^{-1}SM[\downarrow 2]M^{-1}\Pi_1 M \qquad (7)$$

and

$$0 \le i \le \infty, \qquad f_0(n) = y(n) \qquad (8)$$

The role of $M^{-1}\Pi_1 M$ and $M^{-1}\Pi_2 M$ is to control aliasing difficulties during down and up-sampling operations.
The utilization of operator S is a idea of Franke. He chose the algorithm $O = DM^{-1}SM$ where S selects some coefficients in an *adaptive* way acting on the transform. At each step the operator conserves a supplementary coefficient which has not been selected before. This means that, if k_s is the new selected

coefficient,

$$S_{i+1}(k) = \begin{cases} 1 & \text{if} \quad k = k_s \\ S_i(k) & \text{if} \quad k \neq k_s \end{cases} \qquad (9)$$

The added coefficient is for example the one having the largest magnitude and not selected before. As usual for iterative algorithms, the convergence must be established analytically. In his demonstration, Franke does not take into account that the selection function $S(k)$ is modified at each step.
The convergence of our general algorithm is not always assured. But in the practical case of real signals with dominant spectral lines it can be proven [4] that it converges and the solution is unique.
The algorithm of Papoulis corresponds in fact to $D = M^{-1}\Pi M$ where Π is a cut-off frequency filter.
So most of the existing algorithms correspond to particular forms of equation 7. But this equation also leads to new strategies like the following example.

3 Illustration – A new extrapolation scheme: one-step subband decomposition with separate band deconvolution

The complete generalized algorithm makes no use of computation time reductions. But if the iterative work is done on the central part $M^{-1}SM$ only completed with a windowing operator $\tilde{D}$ similar to D, the calculation time is reduced by a factor 4 for an image. We have then the following algorithm, where $u(n)$ and $v(n)$ are intermediary functions and where the second equation only is iterative:

$$\begin{aligned} u_0(n) &= [\downarrow 2] M^{-1}\Pi_1 M y(n) \\ u_{i+1}(n) &= \tilde{D} M^{-1} S M u_i(n) \\ v(n) &= D M^{-1}\Pi_2 M [\uparrow 2] u(n)_{i\to\infty} \end{aligned}$$

$v(n)$ is not the desired function $f(n)$. The frequency content of $v(n)$ is fixed by the two filters Π_1 and Π_2.
The new concept introduced is that of hierarchical decomposition. With an adequate choice of subband filters there is perfect reconstruction and the kernel equation works at reduced resolution. Using the linearity property of spectral decompositions we extrapolate independently the four bands $v(n)$ calculated with four distinct filter pairs Π_1, Π_2 and group them again at the end. Figure 2 illustrates the global strategy. 'A supplementary step is taken in this example. The number of iterations and in fact the number of spectral lines retained in each band is directly proportional to the energy contained in the band. Other selecting strategies may also be imagined.

4 Conclusion

A general spectral deconvolution algorithm is introduced. It groups together most of the existing algorithms and allows the synthesis of new strategies based on the utilization of a priori information of any kind and even on physical notion like energy. In addition, the introduction of multi-resolution operations permits important computation savings.

Acknowledgement
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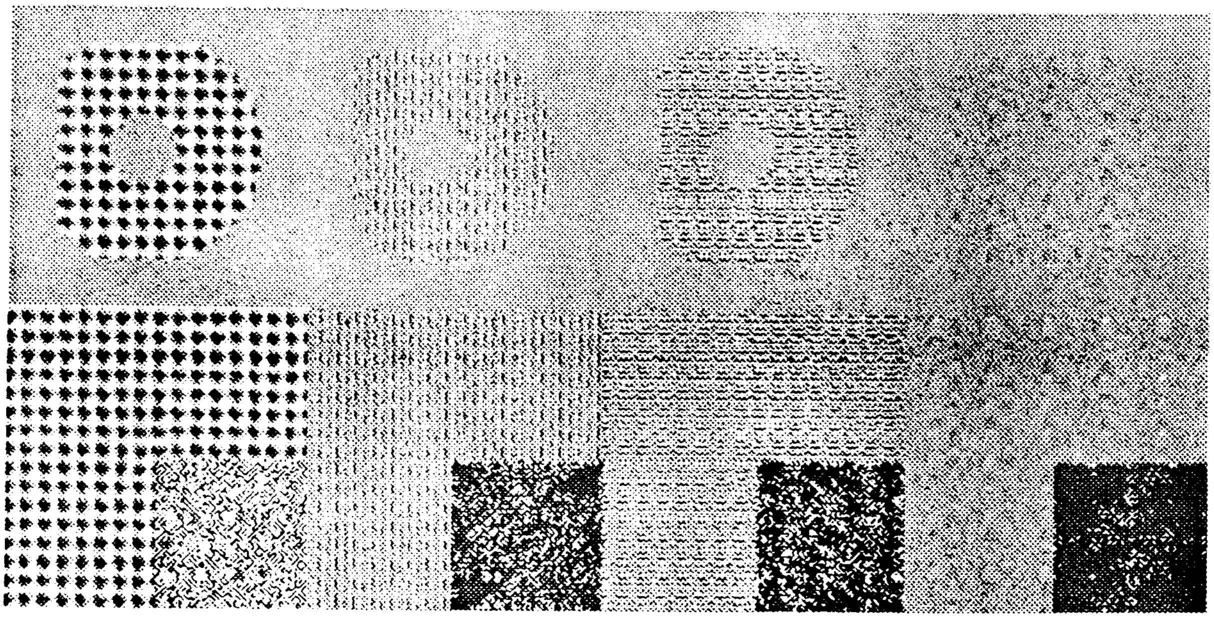

Figure 2: One-step subband decomposition with separate band deconvolutions.
The signal shown in figure 1 has been processed. The first row gives the 2-D subbands at the output of the synthesis stage.
The second row presents the corresponding extrapolated subbands (at the output of the synthesis stage). The lower right part of the images show the amount and positions of selected spectral lines (white pixels). The number of spectral lines which were chosen is proportional to the energy present in the given band. The total number of conserved spectral lines is equal to the quarter of the original pixel number. When summing the four subband signals, we obtain the right-hand side image of figure 1 which is quite satisfactory.