

## Non-Gaussian electrical fluctuations in a quasi-2d packing of metallic beads

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**Abstract.** – The electrical properties of a two-dimensional packing of metallic beads are studied. Small mechanical perturbations of the packing lead to giant voltage fluctuations. Fluctuations are found to be non-Gaussian and seem to belong to Lévy stable distributions. Anticorrelations have been also found for the sign of these fluctuations.

The granular state of matter exhibits fascinating physical properties such as memory effects [1], phase segregation [2], heterogeneous mechanical stresses [3], etc. Although many experiments have been performed for studying mechanical and geometrical aspects of the granular matter [3], only very few reports [4–6] can be found in the scientific literature for describing electrical properties of such systems. In 1890, Branly reported [4] that the electrical resistance of packed metallic grains is affected by an incident electromagnetic wave. Branly's coherer was the first reported antenna. He discovered also that a small vibration of the packing affects the electrical resistance. More recently, Ammi *et al.* [5] measured a non-Hertzian behavior of the electrical conductivity when a packing is submitted to high uniaxial pressures. In another experiment, Vandembrouck *et al.* [6] visualized the electrical paths in a packing of metallic beads for different injected current densities.

In the present letter, we report electrical measurements of a 2d packing of metallic beads submitted to small perturbations. Electrical fluctuations are statistically analyzed. A physical interpretation is given within the framework of Hertzian contacts. Comparison with recent experiments and models for stress fluctuations are also emphasized.

Our experimental set-up is illustrated in fig. 1. The main part of the system is a tilted plane of epoxy of  $25 \times 30 \text{ cm}^2$  on which  $\text{PbO}_2$  beads have been placed. A small tilt angle (typically  $\theta = 10^\circ$ ) can be adjusted by two screws. Contacts are placed on specific beads. A Plexiglas protection plane has been placed just above the epoxy plane to ensure the quasi-2d characteristic of the system. The mean diameter of the beads is 2.35 mm and a polydispersity of 2% has been measured. About 3000 beads have been placed on the epoxy plane. Above the protection plane, a wooden hammer driven by the parallel port of the computer can hit the epoxy plane at the bottom of the packing. The experiments have been performed in a

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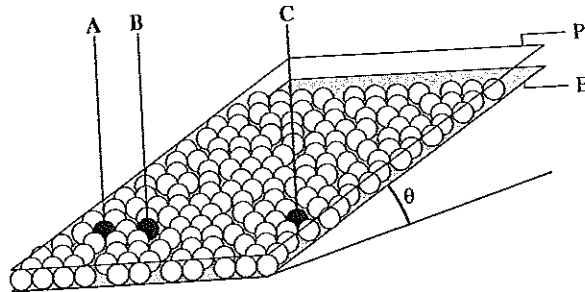


Fig. 1 - A schematic illustration of the experimental setup. Contacts A, B and C are on beads. Epoxy (E) and Plexiglas (P) are planes which are separated by the bead diameter and which are tilted by an angle  $\theta$ .

constant-current regime. A current source Keithley K2400 injects a current  $i$  through two distant beads (A and C in fig. 1) located at the bottom of the pile. The tension  $U$  between those beads is measured by a microvoltmeter and then sent to the computer. This method is the so-called "2-point measurement". A third contact (B in fig. 1) can be placed on a bead (a probe) anywhere in the pile. The tension  $U$  between a bead in the bottom and the probe is also measured ("3-point measurement" method). In each series of measurements, the following procedure has been repeated a large number  $n$  of times: i) the hammer hits the bottom of the packing every 10 seconds, and ii) the computer stores the tension at sampling rate of 0.2 seconds.

There are two disadvantages for using the 2pts measurements method. First, contact resistances are non-negligible and consequently the electrical fluctuations (studied herein) are quite small compared with the signal. Secondly, it is a relevant piece of information to probe the system in order to study the current paths. Also, 4-point measurements would be of interest, but an increase of the contact number increases also the probability to have bad contacts.

Two- and three-point measurement have been performed. Figure 2 presents a typical evolution of the voltage  $U$  as a function of the tap number  $n$  in the case of a 3pts measurement. The inset illustrates the tension at the scale of a few seconds: taps events are denoted by arrows at that scale. When a tap occurs,  $U$  is seen to be drastically affected and thereafter sticks at a new value. The rapid decrease of  $U$  (within a second) at each tap indicates the presence of capacitive effects. Those could be studied in AC current regimes but this is outside the scope of the present paper. One observes also that there is apparently no systematic behavior at each tap: i) the voltage fluctuation is either positive or negative, and ii) the jumps/drops of  $U$  seem to be characterized by a broad distribution of amplitudes. A compaction of the packing is also observed at the early stages of the experiment, and therefore one observes that the voltage  $U$  globally decreases since more and more beads are touching. After  $n = 2000$  taps, we have observed no macroscopic drift of the electrical voltage in our measurements! After  $t = 10000$  taps, the packing is still disordered. Indeed, numerous departures (defects) from an ordered structure are observed.

Fluctuations  $\Delta U_n$  are defined as the differences of the stable electrical voltage before the  $(n + 1)$ -th and before the  $n$ -th taps. Since  $i$  is constant during the experiment, fluctuations  $\Delta U$  are mainly due to changes of the conductivity along electrical paths. From the Ohm law, we checked that the mean size of the fluctuations  $\langle \Delta U \rangle$  is proportional to the injected current  $i$ .

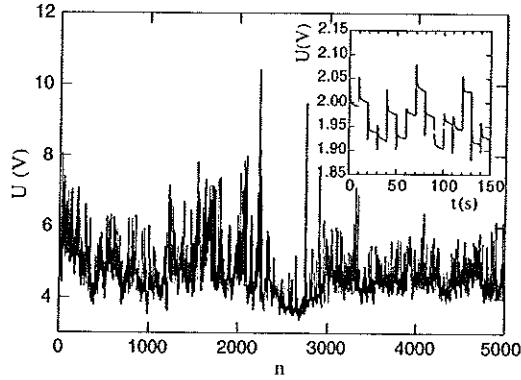


Fig. 2

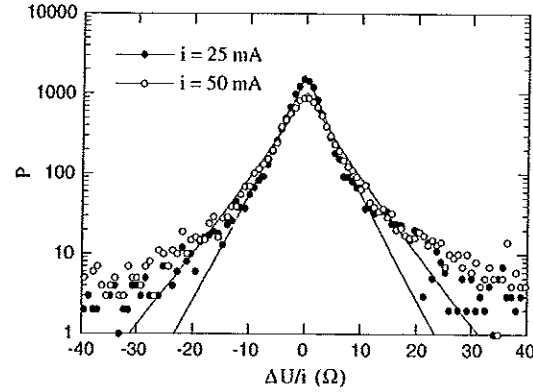


Fig. 3

Fig. 2 – The electrical fluctuations of  $U(n)$  in the quasi-2d packing of  $\text{PbO}_2$  beads. The inset presents the evolution of  $U$  at the scale of a few seconds.

Fig. 3 – Semi-log plot of fluctuation distributions  $P(\Delta U/i)$  for two different current intensities  $i$ . The continuous curves are fitted Lévy-like distributions using eq. (1).

Let us investigate the statistics of the voltage fluctuations. Figure 3 presents the distribution of fluctuations  $P(\Delta U)$  in a semi-log plot for two different values of the injected current  $i$ . Fluctuations are normalized by  $i$  for comparison. So-called “fat tails” for rare events are observed in fig. 3, the occurrence of large fluctuations is several orders of magnitude more frequent than those for Gaussian tails. The fluctuation histograms can be however described by a Lévy-like distribution [7]:

$$P(|\Delta U|) \sim \int_0^{+\infty} \cos(|\Delta U|q) \exp[-\gamma q^\alpha] dq, \quad (1)$$

which has fat tails

$$P(|\Delta U|) \sim |\Delta U|^{-(\alpha+1)} \quad (2)$$

for  $|\Delta U| \gg 1/\gamma$ . The exponent  $0 \leq \alpha \leq 2$  is the so-called Lévy index and  $\gamma > 0$  is a quantity related to the width of the distribution (for  $\alpha > 1$ ). When  $\alpha$  reaches 2, the distribution becomes a Gaussian distribution. When  $\alpha = 1$ , the Lévy distribution reduces to a Cauchy distribution. When  $\alpha < 1$ , the second moment of the distribution diverges. The continuous curves in fig. 3 are fits with eq. (1) giving a value  $\alpha = 1.1 \pm 0.1$  for both cases. Figure 4 presents in a log-log plot the fat tail of the distribution  $P(|\Delta U|)$ . Fluctuations are normalized by  $i$  for comparison. Power law behaviors (eq. (2)) are observed. We have found  $\alpha = 1.05 \pm 0.10$  in good agreement with the  $\alpha$  value of the first fits using eq. (1). Both fits in figs. 3 and 4 confirm the Lévy scenario for electrical fluctuations. This value is quite low and expresses that the voltage evolves erratically with frequent large drop and burst excursions. It is also worthwhile to point out that the Lévy index  $\alpha$  seems to be independent of the injected current  $i$  ranging from 3 mA up to 100 mA in our different series of measurements.

Let us also investigate the temporal correlations between two successive fluctuations. Figure 5 presents the phase space  $(\Delta U_n, \Delta U_{n+1})$  for a current  $i = 50$  mA. The data points are mainly dispersed in an ellipsoid shape which is tilted with respect to the horizontal and vertical axis. Moreover, branches are observed along the vertical, horizontal and diagonal

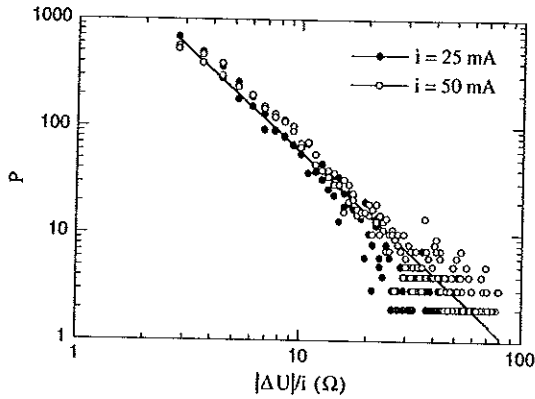


Fig. 4

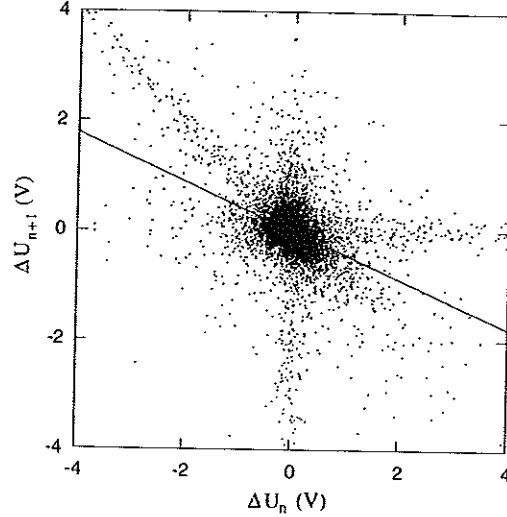


Fig. 5

Fig. 4 – Log-log plot for the tail of  $P(|\Delta U|/i)$ . The lines are power law fits using eq. (2).

Fig. 5 – Phase space  $(\Delta U_n, \Delta U_{n+1})$ . The continuous line is a linear regression  $\Delta U_{n+1} = a\Delta U_n + b$ .

directions. The latter branches indicate the existence of strong correlations for large fluctuations. The continuous line is a linear fit through the entire cloud of data points. The slope  $a = -0.45 \pm 0.01$  of the linear regression  $\Delta U_{n+1} = a\Delta U_n + b$  is significantly negative, and expresses that two successive fluctuations are highly anticorrelated. The slope  $a$  is found to be weakly dependent on the injected current  $i$ .

The series obtained with 2pts measurements lead similarly to Lévy statistics and anticorrelations. In 3-point measurements, the probe has been placed in various areas of the packing and the results are unchanged. Finally, it should be noted that nickel beads, which are more conducting than  $\text{PbO}_2$  beads, lead to similar results. We checked also that the results are not sensitive to the tilt angle  $\theta$ .

If the properties of the fluctuations characterizing the phenomenon are found to obey non-trivial laws, they should certainly arise from fundamental phenomena. A physical interpretation of these giant and non-Gaussian fluctuations should be found in the framework of the geometry of electrical paths. Indeed, the existence of multiple tortuous conducting paths has been reported in [6]. We conjecture that the fluctuations observed herein are most probably related to the fluctuations of stress chains along electrical paths. Also, the strong anticorrelations that we put into evidence should be related to memory effects in the packing.

As recently underlined by deGennes [8], both macroscopic electrical conductivity  $\sigma$  and macroscopic elastical modulus  $\mu$  of the packing are closely related if Hertzian contacts are assumed. This analogy between mechanical strain and electrical conductance implies that the distribution of the fluctuations measured herein could be related to the stress fluctuations along the electrical paths. Giant stress fluctuations have been recently reported in compression [9] and photoelastic experiments [10]. Cooperative rearrangements such as nucleation of cracks and the propagation of these cracks in the packing can lead to stress fluctuation of high amplitude. It has been put into evidence that these cracks have a transient life and heal after the displacement of blocks or single beads [9]. From the simulation point of view,

Coppersmith *et al.* [11] introduced the so-called  $q$ -model deducing a probability distribution of vertical forces varying as  $f \exp[-f/\langle f \rangle]$ . However, the  $q$ -model and more elaborated variants lead to exponential distributions of stress fluctuations with independent fluctuations. To our knowledge, the only model which is able to produce rearrangements of force paths at all scales is the so-called Scalar Arching Model (SAM) introduced by Claudin *et al.* [12]. The SAM gives rise to a broad distribution of the apparent weight variations  $\Delta W_a$  measured at the bottom of a silo. Claudin has found [13] a universal power law behavior  $P(|\Delta W_a|) \sim |\Delta W_a|^{-0.94}$  whatever the values of solid friction coefficients. This power law behavior is consistent with our results but the relationship between our exponent  $\alpha + 1$  and Claudin's exponent 0.94 is non-obvious.

In summary, the careful measurements of the electrical properties of a quasi-bidimensional packing of  $\text{PbO}_2$  beads have put into evidence the non-Gaussian character of conductivity fluctuations along electrical paths. Those non-Gaussian electrical fluctuations are related to stress fluctuations in the packing. Our measurements confirm some predictions of the Scalar Arching Model.

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