A comparison of grey-box and black-box approaches in nonlinear state-space modelling and identification

J.P. Noël, J. Schoukens
ELEC Department
Vrije Universiteit Brussel, Brussels, Belgium
jp.noel@ulg.ac.be, johan.schoukens@vub.ac.be

G. Kerschen
Space Structures and Systems Lab
Aerospace and Mechanical Engineering Department
University of Liège, Liège, Belgium
g.kerschen@ulg.ac.be

1 Abstract

Nonlinear system identification constantly faces the compromise between the flexibility of the fitted model and its parsimony. Flexibility refers to the ability of the model to capture complex nonlinearities, while parsimony is its quality to possess a low number of parameters. In this regard, a nonlinear state-space representation [1]

$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{A}\,\mathbf{x}(t)+\mathbf{B}\,\mathbf{u}(t)+\mathbf{E}\,\mathbf{g}(\mathbf{x},\mathbf{u}) \\ \mathbf{y}(t) &= \mathbf{C}\,\mathbf{x}(t)+\mathbf{D}\,\mathbf{u}(t)+\mathbf{F}\,\mathbf{h}(\mathbf{x},\mathbf{u}) \end{cases} \quad (1)$$

can be classified as very flexible but little parsimonious, two features typically shared by black-box models. In Eqs. (1), $\mathbf{A} \in \mathbb{R}^{n_s \times n_s}$, $\mathbf{B} \in \mathbb{R}^{n_s \times m}$, $\mathbf{C} \in \mathbb{R}^{l \times n_s}$ and $\mathbf{D} \in \mathbb{R}^{l \times m}$ are the linear state, input, output and direct feedthrough matrices, respectively; $\mathbf{x}(t) \in \mathbb{R}^{n_s}$ is the state vector; $\mathbf{y}(t) \in \mathbb{R}^{l}$ and $\mathbf{u}(t) \in \mathbb{R}^{m}$ are the output and input vectors, respectively. The linear-in-the-parameters expressions $\mathbf{E}\,\mathbf{g}(\mathbf{x},\mathbf{u}) \in \mathbb{R}^{n_s}$ and $\mathbf{F}\,\mathbf{h}(\mathbf{x},\mathbf{u}) \in \mathbb{R}^{l}$ are the nonlinear model terms coupling the state and input variables. The order of the model, *i.e.* the dimension of the state space, is noted n_s.

In the present contribution, it is shown that, in the case of mechanical systems where nonlinearities are physically localised, the model structure in Eqs. (1) can be drastically simplified. Assuming localised nonlinearities, the vibrations of a n_p-degree-of-freedom mechanical system obey Newton's second law written in the form

$$\mathbf{M}\,\ddot{\mathbf{q}}(t)+\mathbf{C}_v\,\dot{\mathbf{q}}(t)+\mathbf{K}\,\mathbf{q}(t)+\sum_{a=1}^{s} c_a\,\mathbf{g}_a(\mathbf{q}_{nl}(t),\dot{\mathbf{q}}_{nl}(t))=\mathbf{p}(t), \quad (2)$$

where $\mathbf{M}$, $\mathbf{C}_v$, $\mathbf{K} \in \mathbb{R}^{n_p \times n_p}$ are the mass, linear viscous damping and linear stiffness matrices, respectively; $\mathbf{q}(t)$ and $\mathbf{p}(t) \in \mathbb{R}^{n_p}$ are the generalised displacement and external force vectors, respectively; the nonlinear restoring force term is written using s basis function vectors $\mathbf{g}_a(t) \in \mathbb{R}^{n_p}$ associated with coefficients c_a. The subset of generalised displacements and velocities involved in the construction of the basis functions are denoted $\mathbf{q}_{nl}(t)$ and $\dot{\mathbf{q}}_{nl}(t)$, respectively.

The dynamics governed by Eq. (2) is conveniently interpreted by moving the nonlinear restoring force term to the right-hand side, *i.e.*

$$\mathbf{M}\,\ddot{\mathbf{q}}(t)+\mathbf{C}_v\,\dot{\mathbf{q}}(t)+\mathbf{K}\,\mathbf{q}(t)=\mathbf{p}(t)-\sum_{a=1}^{s} c_a\,\mathbf{g}_a(\mathbf{q}_{nl}(t),\dot{\mathbf{q}}_{nl}(t)). \quad (3)$$

The feedback structure of Eq. (3) suggests that localised nonlinearities in mechanical systems act as additional inputs applied to the underlying linear system. This, in turn, reveals that black-box nonlinear terms in a state-space model, such as $\mathbf{E}\,\mathbf{g}(\mathbf{x},\mathbf{u})$ and $\mathbf{F}\,\mathbf{h}(\mathbf{x},\mathbf{u})$ in Eqs. (1), are overly complex to address mechanical vibrations. A more parsimonious description of nonlinearities is achieved by translating Eq. (3) in state space, which provides the grey-box model

$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{A}\,\mathbf{x}(t)+\mathbf{B}\,\mathbf{u}(t)+\mathbf{E}\,\mathbf{g}(\mathbf{y}_{nl}(t),\dot{\mathbf{y}}_{nl}(t)) \\ \mathbf{y}(t) &= \mathbf{C}\,\mathbf{x}(t)+\mathbf{D}\,\mathbf{u}(t)+\mathbf{F}\,\mathbf{g}(\mathbf{y}_{nl}(t),\dot{\mathbf{y}}_{nl}(t)), \end{cases} \quad (4)$$

where $\mathbf{g}(t) \in \mathbb{R}^{s}$ is a vector concatenating the nonzero elements in the basis function vectors $\mathbf{g}_a(t)$, and $\mathbf{E} \in \mathbb{R}^{n_p \times s}$ and $\mathbf{F} \in \mathbb{R}^{l \times s}$ are the associated coefficient matrices; $\mathbf{y}_{nl}(t)$ and $\dot{\mathbf{y}}_{nl}(t)$ are the subsets of the measured displacements and velocities located close to nonlinearities, respectively.

For demonstration purposes, black-box and grey-box state-space models of the Silverbox benchmark, *i.e.* an electrical mimicry of a single-degree-of-freedom mechanical system with cubic nonlinearity, are identified using a maximum likelihood estimator. It is found that the grey-box approach allows to reduce markedly modelling errors with respect to a black-box model with a comparable number of parameters. It is also suggested that the greater accuracy of the grey-box model lends itself to the computation of reliable confidence bounds on the model parameters.

References

[1] J. Paduart *et al.*, Identification of nonlinear systems using Polynomial Nonlinear State-Space models, Automatica, 46:647-656, 2010.