Extension of the Background/biResonant decomposition to the estimation of the kurtosis coefficient of structural response

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Abstract

Based on the observation of the existence of different timescales, this paper provides an approximate method to compute the kurtosis coefficient of the response of a linear-time-invariant system subjected to a low-frequency non Gaussian input. While the kurtosis coefficient is formally obtained by a multidimensional integration of the corresponding spectrum, the proposed method only requires the estimation of a single definite integral. The speedup performance is three to four orders of magnitude and the approximation is very accurate as it corresponds to the leading order expansion of the formal solution, with the ratio of the identified timescales considered as a small parameter.

1 Introduction

The stochastic analysis of a structure with deterministic properties, but subjected to a non Gaussian stationary random loading, may be advantageously performed with a Volterra approach [4]. Although this concept may be applicable to nonlinear structures [5, 14], the scope of this paper is limited to structures with a deterministic linear behavior. In this case, the Volterra kernels take a simple explicit expression and the statistical moments of the structural response are simply recovered by a multi-dimensional integration of the corresponding spectra[12].

In structural dynamics applications, Volterra models have already been used to model wind pressures resulting from the quadratic transformation of wind turbulence [9] or the response of offshore structures to random waves [2]. Although the concepts of Volterra models are appealing for single-degree-of-freedom systems, as they provide as simple physical interpretation to the filtering process of higher order statistical moments, they have also been applied in the context of multi-degree-of-freedom systems [3]. In realistic applications, the spectra describing the loading, be it wave or wind, take on heavy analytical expressions that discard any possible analytical estimation of the multi-dimensional definite integrals. This obviously starts with the expressions of the power spectral densities (psd) of wave heights [11] or wind velocity [13], and extends to bispectra, trispectra and higher spectra. For this reason, in a Volterra approach, statistical moments are computed with a numerical implementation of a multi-dimensional integration. The computational burden necessary for the estimation of these integrals turns out to be prohibitive as soon as high order moments are required (beyond and including 4th statistical moment). The numerical conditioning is in fact again worse when the structural damping is low as the Volterra kernels then exhibits very large gradients in the frequency space, which are difficult to capture properly in a numerical scheme. Although these issues related to this difficult numerical integration are evident and sometimes reported, there was barely not attempts at providing an efficient numerical integration scheme. An efficient meshing

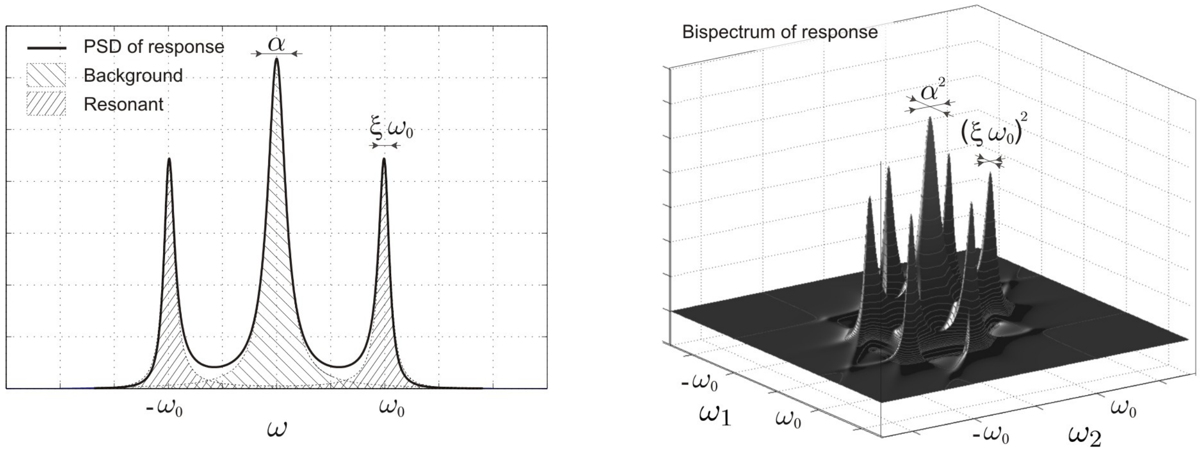

Figure 1: Typical psd (left) and bispectrum (right) of the response of a single degree-of-freedom system subjected to low frequency turbulence. The psd (left) is commonly decomposed as the sum of background and two resonant components. The bispectrum (right) is also decomposed as the sum of a background and six biresonant components.

technique was actually introduced in [7] for the integration of psds and bispectra, but it is hardly transposable to fourth and high orders.

This short description of the current panorama indicates that the Volterra methods have sufficiently matured over the last 20 years regarding multi-degree-of-freedom linear systems, and over more than 10 years for nonlinear ones. However, very few applications out of the academic context have been presented so far. It is thus tempting to conclude that the methods are apparently hibernating, waiting for computational power required to make them applicable to large realistic structures, say modeled with a couple of thousands degrees-of-freedom.

One objective of this paper is to provide a method to simplify the estimation of the multi-dimensional integrals. More precisely, we focus on the estimation of the kurtosis coefficient, i.e. the integration of the trispectrum of the response in a 3-D frequency space. The paper is organized as follows. Section 2 summarizes existing assets for the second and third order analyses, while Section 3 provides the novel developments related to the estimation of the kurtosis, and Section 4 illustrates the concepts and benefits with a simple example.

2 Existing Assets

In the event that the timescale related to the dynamics of the structure (its natural periods) and the timescale of the random loading are well distinct —as it is typically the case in buffeting analysis—, a usual decomposition of the response into Background and Resonant contributions provides a very good estimation of the second statistical moment [6]. Recently the concept has been extended to the Background/biResonant (B/bR) decomposition for the estimation of the third statistical moment [8]. In this paper, we pursue this multiple timescale analysis, with the very same assumptions of slight damping and low-frequency loading as those that made Allan Davenport's Background/Resonant (B/R) decomposition fruitful.

The power spectral density of the structural response of a single degree-of-freedom linear system is obtained by

$$S_x(\omega) = S_f(\omega)\ K_2(\omega) = S_f(\omega)\ |H(\omega)|^2 \tag{1}$$

where S_x and S_f are respectively the psds of the response and of the loading, and $H = (-m\omega^2 + i\omega c + k)^{-1}$ is the linear frequency response function of a dynamical system, characterized by a mass

m, viscosity c and stiffness k. The kernel K_2 is equal to $|H|^2$, by definition. In this paper, we are concerned with psds of loading that are assumed to decrease very fast in a short frequency range, referred to as α in the following. Actually α is assumed to be small compared to the natural circular frequency $\omega_o = \sqrt{k/m}$ of the dynamical system. The ratios

$$\varepsilon = \frac{\alpha}{\omega_o} \qquad \text{and} \qquad \xi = \frac{c}{2m\omega_o} \tag{2}$$

are actually two small parameters of the problem at hand, which makes the topology of the psd of the response rather particular. Indeed, as seen in Fig. 1, the psd of the response features three sharp and distinct peaks. For this reason, the second statistical moment of the response, the variance, may be approximated as

$$m_{2,x} = \int_{-\infty}^{+\infty} S_x(\omega)\, d\omega \simeq m_{2,b} + m_{2,r} \tag{3}$$

with the background and resonant components respectively given by

$$m_{2,b} = \frac{m_{2,f}}{k^2} \qquad \text{and} \qquad m_{2,r} = \frac{\pi\omega_o}{2\xi}\frac{S(\omega_o)}{k^2} \tag{4}$$

where $m_{2,f}$ is the variance of the loading. This decomposition is known to be due to A. Davenport [6]. It has been further analyzed by Ashraf and Gould [1], and also later on generalized by Denoël [8]. In a very general context, this decomposition results from the estimation of an integral with small parameters by the addition of components that are successively and iteratively identified [10]. In short, it consists in (i) subtracting the background component, which is readily obtained (ii) providing a local and bounded approximation of the integrand in the vicinity of the resonance peak in the positive frequency range, (iii) provide an approximate analytical expression for the resonant component.

In a very similar manner, the description of the response of a dynamical system subjected to a non Gaussian load is complemented by the bispectrum of the response

$$\begin{aligned} B_x(\omega_1, \omega_2) &= B_f(\omega_1, \omega_2)\, K_3(\omega_1, \omega_2) \\ &= B_f(\omega_1, \omega_2)\, H(\omega_1)\, H(\omega_2)\, \bar{H}(\omega_1 + \omega_2)\,. \end{aligned} \tag{5}$$

Because of the smallness of the two parameters ε and ξ, this bispectrum features one distinctive quasi-static peak, six high biresonance peaks and, secondarily, six low biresonance peaks, see Fig. 1. Invoking the same general methodology for the estimation of the integral of B_x, the third statistical moment of the response may be expressed as

$$m_{3,x} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} B_x(\omega_1, \omega_2)\, d\omega_1 d\omega_2 \simeq m_{3,b} + m_{3,r} \tag{6}$$

with the background and biresonant components respectively given by

$$m_{3,b} = \frac{m_{3,f}}{k^3} \qquad \text{and} \qquad m_{3,r} = 6\pi\frac{\xi\omega_o^3}{k^3}\int_{-\infty}^{+\infty} \frac{B(\omega_o, \omega_2)}{(2\xi\omega_o)^2 + \omega_2^2} d\omega_2 \tag{7}$$

where $m_{3,f}$ is the third statistical moment of the loading. This approximation is valid no matter the expression of the bispectrum of the loading. It just requires the hypotheses of smallness to be fulfilled. Simplified expressions for particular cases may be found in [8], along with appropriate numerical schemes for the estimation of the remaining integral in (7).

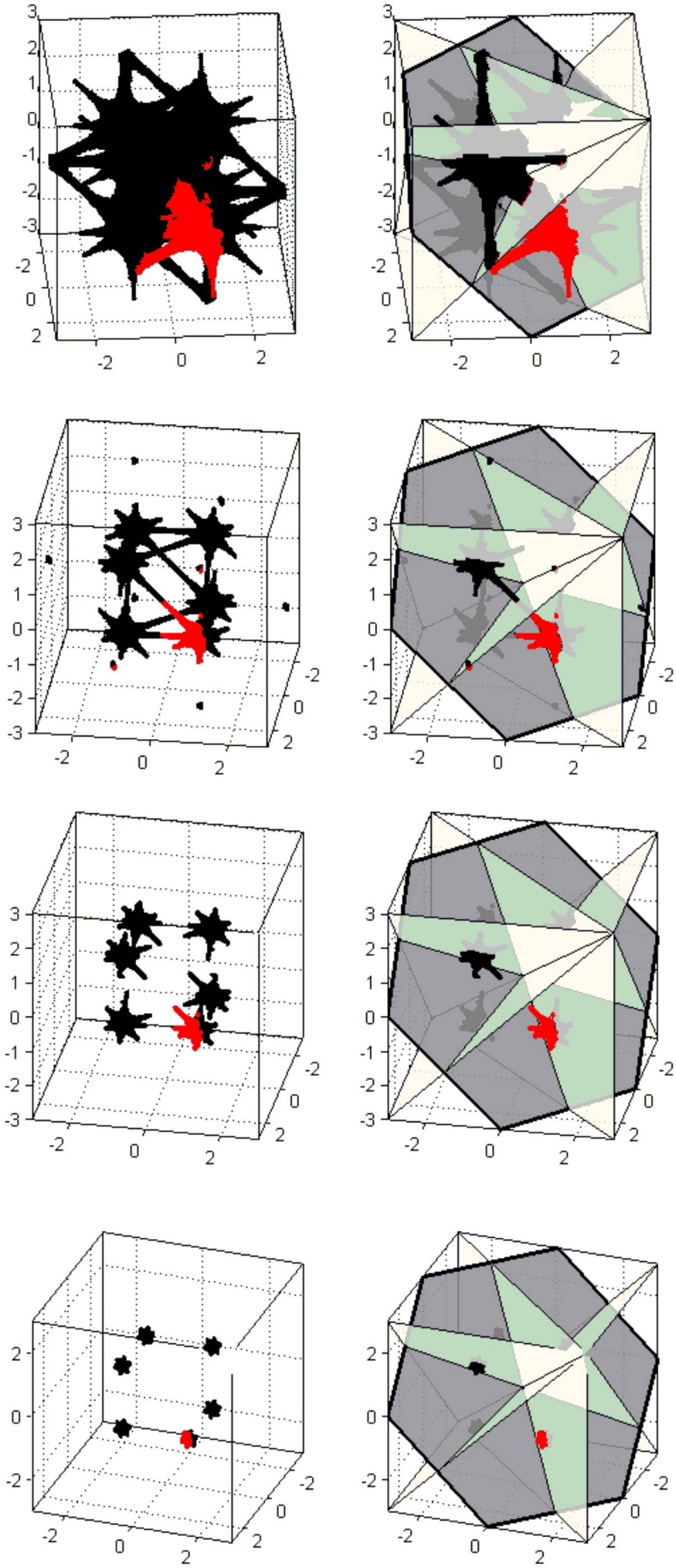

Figure 2: Level-set representation of the fourth order Volterra kernel ($\xi = 0.03$).

	Nomenclature	Behavior around peak	Local integral	Total integral
2nd order:	Resonance	$K_2 = \mathcal{O}\left(\xi^{-2}\right)$	$\int K_2 d\omega \sim \mathcal{O}\left(\xi^{-1}\right)$	$\frac{1}{k^2}\frac{\pi\omega_o}{2\xi}$
3rd order:	Biresonance	$K_3 = \mathcal{O}\left(\xi^{-2}\right)$	$\int\int K_3 d\boldsymbol{\omega} \sim \mathcal{O}\left(1\right)$	$\frac{1}{k^3}\frac{8\pi^2\omega_o^2}{3(1+8\xi^2)}$
4th order:	Tetraresonance	$K_4 = \mathcal{O}\left(\xi^{-4}\right)$	$\int\int\int K_4 d\boldsymbol{\omega} \sim \mathcal{O}\left(\xi^{-1}\right)$	$\frac{1}{k^4}\frac{3\pi^3\omega_o^3}{4\xi(1+3\xi^2)}$

Table 1: Summary of the behavior of the kernel K_2, K_3 and K_4 around their resonance peaks.

3 Integration of the Trispectrum

From the decompositions existing at the second and third order, it is postulated that the fourth statistical moment may be decomposed as a sum of two contributions, namely the quasi-static or background one and the triresonant one, as shown next.

The trispectrum of the response of a single degree-of-freedom linear system is expressed as

$$\begin{aligned} T_x\left(\omega_1,\omega_2,\omega_3\right) &= T_f\left(\omega_1,\omega_2,\omega_3\right) K_4\left(\omega_1,\omega_2,\omega_3\right) \\ &= T_f\left(\omega_1,\omega_2,\omega_3\right) H\left(\omega_1\right) H\left(\omega_2\right) H\left(\omega_3\right) \bar{H}\left(\omega_1+\omega_2+\omega_3\right) \end{aligned} \tag{8}$$

and the corresponding moment is obtained as

$$m_{4,x} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} T_x\left(\omega_1,\omega_2,\omega_3\right) d\omega_1 d\omega_2 d\omega_3. \tag{9}$$

Before stepping into the development of an approximation for the computation of that integral, it is essential to understand the topology of the fourth order kernel K_4. It is represented by its level sets in Fig. 2, starting from low values set at the top to high values at the bottom. The six little stars in the bottom graph correspond to the high peaks of this Volterra kernel. The middle graphs show that these peaks are connected by bridges (that we have previously coined *ridge* in the third order context [8]). There exists also eight low peaks, identifiable by the little stars on the second graph. The rightmost plots also show the symmetry planes which owe the trispectrum of a real random process to be represented along only one twelfth of the complete frequency space.

Attention should of course be paid to the six high peaks, as they will contribute the most to the integral. They are located at $(\omega_0,\omega_0,-\omega_0)$, $(\omega_0,-\omega_0,\omega_0)$, $(-\omega_0,\omega_0,\omega_0)$, $(\omega_0,-\omega_0,-\omega_0)$, $(-\omega_0,\omega_0,-\omega_0)$, $(-\omega_0,-\omega_0,\omega_0)$. Interestingly, out of the four factors of the fourth order kernel K_4, all of them correspond to resonance at high peaks. As a result, the fourth order kernel is of order ξ^{-4} at high peaks. The high peaks in K_4 may therefore be termed *tetraresonance* peaks. This is a noticeable difference with respect to the third order where the kernel K_3 is of order ξ^{-2} at biresonance peaks. More interestingly again, as the peaks in K_4 spread along a volume of order $\xi^3\omega_o^3$, while they a limited to a base surface $\xi^2\omega_o^2$ at the third order, the contribution of the tetraresonance peaks is thus found to be singular in ξ, while that of the biresonance peaks was regular. These interesting observations are summarized in Table 1.

As a conclusion, the fourth order response is expected to resemble much more the usual second order response than the third order one.

The derivation of the approximation starts by recognizing the existence of a background component $m_{4,b}$ in the response, which is simply obtained by assuming that the structure responds statically to the random loading

$$m_{4,b} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{T_f\left(\omega_1,\omega_2,\omega_3\right)}{k^4} d\omega_1 d\omega_2 d\omega_3 = \frac{m_{4,f}}{k^4} \tag{10}$$

where $m_{4,f}$ is the fourth order statistical moment of the loading. This component is then trivially added and subtracted in (9), to obtain

$$m_{4,x} = m_{4,b} + \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} T_f\left(\omega_1,\omega_2,\omega_3\right)\left[K_4\left(\omega_1,\omega_2,\omega_3\right) - \frac{1}{k^4}\right] d\omega_1 d\omega_2 d\omega_3. \tag{11}$$

We will focus next on the estimation of that integral in the vicinity of the peak $(\omega_1, \omega_2, \omega_3) = (\omega_0, \omega_0, -\omega_0)$. To this aim, stretched coordinates (η_1, η_2, η_3) defined as a

$$\begin{aligned} \omega_1 &= \omega_o\left(1+\xi\eta_1\right) \\ \omega_2 &= \omega_o\left(1+\xi\eta_2\right) \\ \omega_3 &= -\omega_o\left(1+\xi\eta_3\right) \end{aligned} \tag{12}$$

are introduced in order to provide local approximation of the integrand. Considering that the damping ratio ξ is small, it is found that

$$K_4\left(\omega_1,\omega_2,\omega_3\right) - \frac{1}{k^4} \simeq \frac{1}{16k^4\xi^4\left(\eta_1-i\right)\left(\eta_2-i\right)\left(\eta_3-i\right)\left(\eta_1+\eta_2+\eta_3+i\right)}. \tag{13}$$

It is interesting to notice that this local approximation is bounded in the far field since

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\omega_o^3\xi^3\, d\eta_1 d\eta_2 d\eta_3}{16k^4\xi^4\left(\eta_1-i\right)\left(\eta_2-i\right)\left(\eta_3-i\right)\left(\eta_1+\eta_2+\eta_3+i\right)} = \frac{\pi^3\omega_o^3}{8k^4\xi}. \tag{14}$$

Furthermore, after consideration of the jacobian $\omega_o^3\xi^3$, the result (14) multiplied by 6 for the six high peaks indicates that the total integral of the kernel is well recovered (see Table 1), at least at the leading order term for small damping.

First option

By analogy with Davenport's developments at the second order, we may assume that the trispectrum of the loading T_f does not vary into the stretched frequency domain. It this case, the triple integral in (11) collapses to

$$m_{4,r1} = T_f\left(\omega_0,\omega_0,-\omega_0\right)\frac{\pi^3\omega_o^3}{8k^4\xi}. \tag{15}$$

The total tetraresonant contribution, obtained for symmetry reasons by multiplying this latter result by 6, is then simply given as

$$m_{4,r} = T_f\left(\omega_0,\omega_0,-\omega_0\right)\frac{3\pi^3\omega_o^3}{4k^4\xi}. \tag{16}$$

Second option

As a second option, the local behavior of the trispectrum of the loading could be approached more accurately. In this view, similarly to the developements presented in [8] for the third order analysis, the more or less rapid decrease of the trispectrum in the vicinity of the peak could be incorporated into the model.

Without too many details, because this second option is not illustrated next, this consists in determining the directions around the considered peak in which the gradient of the trispectrum of the loading is small. If it is found that the trispectrum mainly changes (according to the ratio of the timescales) in two orthogonal directions, then it may be assumed to be constant in a direction perpendicular to the plane defined by these two high gradient direction. In doing so, a local estimation of the trispectrum may be obtained, and this results in dropping by one the order of the integration, as for the third order analysis.

4 Illustration

As an illustration, we assume here that a single degree-of-freedom system characterized by the mass m, viscosity c and stiffness k is subjected to a quadratic loading

$$f = \gamma \left(1+u\right)^2 = \gamma + 2\gamma u + \gamma u^2 \tag{17}$$

where the low-frequency process u (regarded here as the *turbulence*) is modeled as a dimensionless Gaussian process with zero mean and standard deviation σ_u, and γ is a characteristic force. The intensity $I_u \equiv \sigma_u$ is assumed to be small so that

$$\begin{aligned} \mu_f &= \gamma \left(1 + \mathrm{ord}\left(I_u^2\right)\right) \\ m_{2,f} &= 4\gamma^2 \sigma_u^2 \left(1 + \mathrm{ord}\left(I_u^2\right)\right) \\ m_{4,f} &= 192\gamma^4 \sigma_u^6 \left(1 + \mathrm{ord}\left(I_u^2\right)\right) \end{aligned} \tag{18}$$

with $m_{4,f}$ the fourth order cumulant[1] of f, and so that the excess coefficient of the loading is, for small turbulence intensity,

$$\gamma_{e,f} = \frac{m_{4,f}}{m_{2,f}^2} = 12\sigma_u^2. \tag{19}$$

Furthermore the turbulence random process $u\left(t\right)$ is supposed to be an Ornstein-Uhlenbeck process with a psd expressed as

$$S_u\left(\omega\right) = \frac{\alpha}{\pi}\frac{\sigma_u^2}{\alpha^2+\omega^2}. \tag{20}$$

It is possible to show, see [7], that (at the leading order in I_u) the psd and the trispectrum of the force f are respectively expressed as

$$\begin{aligned} S_f\left(\omega\right) &= 4\gamma^2 S_u\left(\omega\right) \\ T_f\left(\omega_1,\omega_2,\omega_3\right) &= 16\gamma^4 \underset{1,2,3}{\mathcal{P}} \left[S_u\left(\omega_1+\omega_3\right)\left[S_u\left(\omega_2\right)+S_u\left(\omega_1+\omega_2+\omega_3\right)\right]\left[S_u\left(\omega_1\right)+S_u\left(\omega_3\right)\right]\right] \end{aligned} \tag{21}$$

where symbol $\underset{1,2,3}{\mathcal{P}}$ stands for the cyclic permutation over indices 1,2 and 3. This definition of the trispectrum allows to recover the fourth order cumulant, and not the raw moment. A simple check consists in observing that as the multiple integration of (21) renders $m_{4,f}$ as given in (18).

[1]Notice that we use symbol $m_{4,\cdot}$ for quantities related to cumulants, whereas the classical notation is $k_{4,\cdot}$.

Application of Davenport's Background/Resonant decomposition yields

$$\begin{aligned} m_{2,b} &= \frac{m_{2,f}}{k^2} = \frac{4\gamma^2\sigma_u^2}{k^2} \\ m_{2,r} &= \frac{\pi\omega_0}{2\xi}\frac{S_f\left(\omega_0\right)}{k^2} = \frac{m_{2,f}}{k^2}\frac{1}{2\xi}\frac{\alpha\omega_0}{\alpha^2+\omega_0^2}, \end{aligned} \tag{22}$$

while application of the proposed Background/TetraResonant (B/tR) decomposition yields

$$\begin{aligned} m_{4,b} &= \frac{m_{4,f}}{k^4} = \frac{192\gamma^4\sigma_u^6}{k^4} \\ m_{4,r} &= \frac{m_{4,f}}{k^4}\frac{1}{4\xi}\frac{\alpha\left(8\omega_0^2+3\alpha^2\right)\omega_o^3}{\left(\omega_0^2+\alpha^2\right)^2\left(4\omega_0^2+\alpha^2\right)}. \end{aligned} \tag{23}$$

An illustrative way to represent the statistical moments of the response it to normalize them by the corresponding background contribution. In this perspective, the dynamic amplification coefficients at the second and fourth orders respectively are defined as

$$\mathcal{A}_2 = \frac{m_{2,x}}{m_{2,b}} = 1 + \frac{m_{2,r}}{m_{2,b}} \qquad \text{and} \qquad \mathcal{A}_4 = \frac{m_{4,x}}{m_{4,b}} = 1 + \frac{m_{4,r}}{m_{4,b}}. \tag{24}$$

They are represented in Fig. 3 along with the exact results obtained through an analytical integration of the corresponding spectra. (Notice parenthetically that this analytical integration is possible only because the expression of the psd of the turbulence is simple. In fact, these reference results were obtained in this illustration by application of Cauchy's residue theorem.)

At the second order, the perfect match between the analytical solution (labeled "Exact") and Davenport's B/R decomposition is just another illustration of the good performance of the approximation. The globally decreasing profile of this curve conveys the well-known idea that at high damping ratios, the dynamic amplification factor is unitary. This statement is actually valid for any moment

$$\lim_{\xi\to+\infty} \mathcal{A}_j = 1 \qquad \forall j \tag{25}$$

since, as the response is quasi-static, any statistical moment of the response is directly obtained from the same statistical moment of the loading.

As explained before, the global profile of the fourth order dynamic amplification factor $\mathcal{A}_4$ is much closer to the second order one than the third order one. Despite the drastic decrease in the estimation of the fourth statistical moment, the quality of the B/tR decomposition is respectable. It captures quite precisely the profile of the exact solution. This is all the more impressive as the first option described above has been retained, i.e. the heavy numerical integration in the 3-D space is simply replaced by a single point estimate. On account that one hundred integration points at least are required in each dimension of the frequency space, the B/tR decomposition may thus be seen as a speedup by six orders of magnitude. We should admit however that the discrepancy may reach up to 25% for moderate values of the damping coefficient.

The profile of the third order dynamic amplification factor $\mathcal{A}_3$ is discussed in detail in [8]. It is just provided here so as to highlight the significant difference between odd and even statistical orders.

The unbounded increase of the fourth order dynamic amplification factor $\mathcal{A}_4$ for low damping ratios should be interpreted with care. This increase is indeed not related to a non-Gaussian characteristic of the response. To assess the non-Gaussianity of the response, the excess coefficient of the response is established. Based on the B/R and B/tB approximation, it is given by

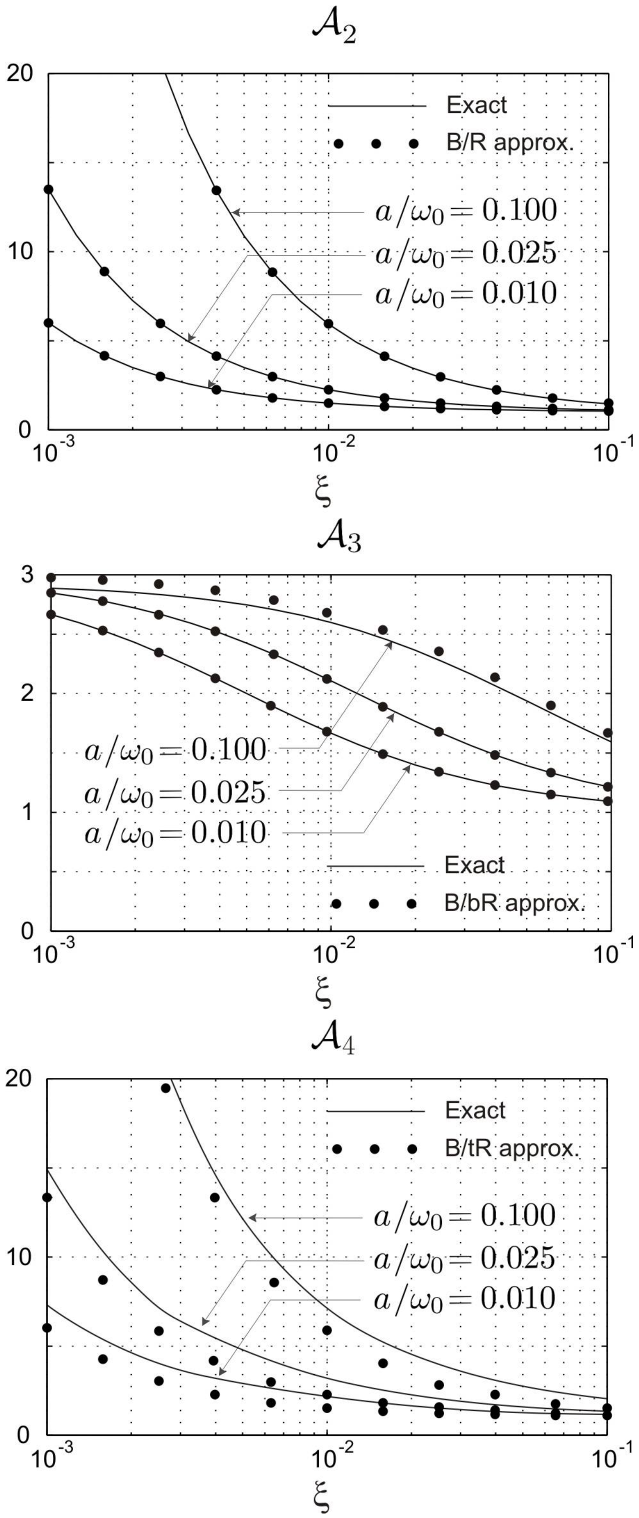

Figure 3: Dynamic amplification factors: ratio of the statistical moment of the response to the statistical moment that would be obtained if the response was quasi-static.

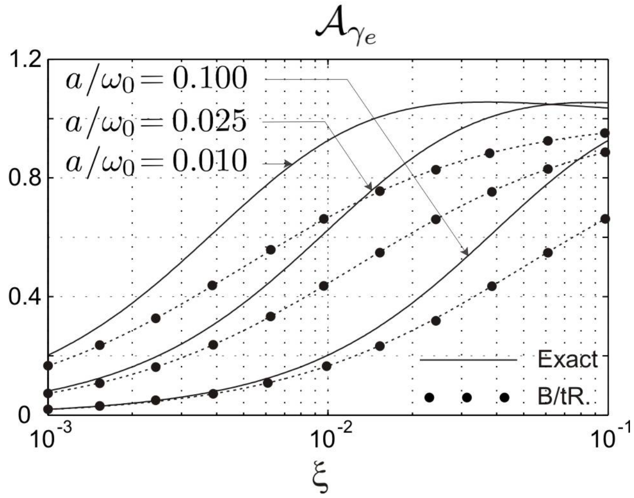

Figure 4: Level-set representation of the fourth order Volterra kernel ($\xi = 0.03$).

$$\gamma_{e,x} = \frac{m_{4,x}}{m_{2,x}^2} = \frac{\mathcal{A}_4 m_{4,b}}{\mathcal{A}_2^2 m_{2,b}^2} = \frac{\mathcal{A}_4}{\mathcal{A}_2^2}\gamma_{e,f}. \tag{26}$$

The ratio of the excess coefficients of the response to that of the loading

$$\mathcal{A}_{\gamma_e} = \frac{\gamma_{e,x}}{\gamma_{e,f}} = \frac{\mathcal{A}_4}{\mathcal{A}_2^2} \tag{27}$$

indicates by how much the mechanical system reduces the non-Gaussianity of the loading. This ratio is represented in Fig. 4 as a function of the damping ratio and for different values of α/ω_o. As a result of the rescaling perhaps, the discrepancy between the exact result and the proposed approximation is now more evident. However, as the profile of $\mathcal{A}_4$ is properly captured by the approximation, the global trend of this new dynamic amplification (one should rather say reduction as the excess of the response is in general lower than the excess of the loading) is acceptably sketched.

Regarding the non-Gaussianity of the response, it is now also clear that the response tends to be Gaussian as the damping ratio decreases, i.e. as the response tends to be resonant. This is a consequence of the central limit theorem, on account that the memory time of the linear system increases as ξ^{-1}.

Finally, it is interesting to observe that, for the particular case of the Ornstein-Uhlenbeck turbulence model the excess coefficient of the response may in some instances be larger than the excess coefficient of the loading ($\mathcal{A}_{\gamma_e}$). This specificity of a linear system subjected to the square of such a random process is however not captured by the B/tR decomposition.

5 Conclusions

Based on the assumption of small structural damping and the consideration of separated timescales for the loading and the structure, we have extended the B/R and B/bR decompositions to the fourth order. These assumptions allow a drastic reduction of the computational costs related to the estimation of a three dimensional integral.

The surprising correspondence between the fourth and second order analyses (rather than the third one) drove us to the development and assessment of a first simple option in this paper which consists in avoiding the multiple integration and replacing it by a simple estimation of the integrand. It was thus estimated that this option reduces the computational costs by six orders of magnitude, but it provides a solution with moderate accuracy however. The development of the second option, addressing several refinement levels, should provide additional solutions with different trade-offs between accuracy and computational costs.

References

[1] Ashraf Ali, M. and P.L. Gould, On the resonant component of the response of single degree-of-freedom systems under random loading. Engineering Structures, 1985. 7(4): p. 280-282.

[2] Benfratello, S. and G. Falsone, Non-Gaussian Approach For Stochastic-Analysis Of Offshore Structures. Journal Of Engineering Mechanics-Asce, 1995. 121(11): p. 1173-1180.

[3] Benfratello, S., M. Di Paola, and P.D. Spanos, Stochastic response of MDOF wind-excited structures by means of Volterra series approach. Journal Of Wind Engineering And Industrial Aerodynamics, 1998. 74-6: p. 1135-1145.

[4] Carassale, L. and A. Kareem, Dynamic analysis of complex systems by Volterra approach. Computational Stochastic Mechanics, 2003: p. 107-117.

[5] Carassale, L. and A. Kareem, Modeling nonlinear systems by Volterra Series. Journal of Engineering Mechanics ASCE, 2010. 136: p. 801-818.

[6] Davenport , A.G., The application of statistical concepts to the wind loading of structures. Proceedings of the Institute of Civil Engineers, 1961. 19: p. 449-472.

[7] Denoël, V., Application of stochastic analysis methods to the study of the effects of wind on civil engineering structures, PhD thesis, Department of Mechanics and Structures 2005, University of Liège: Liège.

[8] Denoël, V., On the background and biresonant components of the random response of single degree-of-freedom systems under non-Gaussian random loading. Engineering Structures, 2011. 33(8): p. 2271-2283.

[9] Gusella, V. and A.L. Materazzi, Non-Gaussian response of MDOF wind-exposed structures: Analysis by bicorrelation function and bispectrum. Meccanica, 1998. 33(3): p. 299-307.

[10] Hinch, E.J., Perturbation Methods. Vol. 1. 1991, Cambridge: Cambridge University Press. 160.

[11] Kareem, A., J. Zhao, and M.A. Tognarelli, Surge response statistics of tension leg platforms under wind and wave loads: A statistical quadratization approach. Probabilistic Engineering Mechanics, 1995. 10(4): p. 225-240.

[12] Schetzen M., The Volterra and Wiener Theories of Nonlinear Systems1980.

[13] Solari, G. and G. Piccardo, Probabilistic 3-D turbulence modeling for gust buffeting of structures. Probabilistic Engineering Mechanics, 2001. 16(1): p. 73-86.

[14] Worden, K. and G. Manson, Random vibrations of a Duffing oscillator using the Volterra series. Journal Of Sound And Vibration, 1998. 217(4): p. 781-789.