A COMPARISON OF WITHIN-SEASON YIELD PREDICTION ALGORITHMS BASED ON CROP MODEL BEHAVIOUR ANALYSIS

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Abstract

The development of methodologies for predicting crop yield, in real-time and in response to different agro-climatic conditions, could help to improve the farm management decision process by providing an analysis of expected yields in relation to the costs of investment in particular practices. Based on the use of crop models, this paper compares the ability of two methodologies to predict wheat yield (*Triticum aestivum* L.), one based on stochastically generated climatic data and the other on mean climate data.

It was shown that the numerical-experimental yield distribution could be considered as a log-normal distribution. This function is representative of the overall model behaviour. The lack of statistical differences between the numerical realisations and the logistic curve showed in turn that the Generalised Central Limit Theorem (GCLT) was applicable to our case study.

In addition, the predictions obtained using both climatic inputs were found to be similar at the inter- and intra-annual time-steps, with the root mean square and normalised

deviation values below an acceptable level of 10% in 90% of the climatic situations. The predictive observed lead-times were also similar for both approaches. Given (*i*) the mathematical formulation of crop models, (*ii*) the applicability of the CLT and GLTC to the climatic inputs and model outputs, respectively, and (*iii*) the equivalence of the predictive abilities, it could be concluded that the two methodologies were equally valid in terms of yield prediction. These observations indicated that the Convergence in Law Theorem was applicable in this case study.

For purely predictive purposes, the findings favoured an algorithm based on a mean climate approach, which needed far less time (by 300-fold) to run and converge on same predictive lead-time than the stochastic approach.

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1. Introduction

Agricultural production is greatly affected by variability in weather (Semenov et al., 2009; Supit et al., 2012). Providing an opportunity to study the effects of variable inputs (such as weather events) on harvestable crop parts, crop models have been used successfully to support the decision-making process in agriculture (Basso et al., 2011; Ewert et al., 2011; Thorp et al., 2008). The development of methodologies for predicting grain yield, in real time and in response to different agro-climatic conditions (Dumont et al., 2014b; Lawless and Semenov, 2005), would further improve farm management decisions by providing an analysis of the trade-off between the value of expected crop yields and the cost of inputs.

Plant growth and development can be seen as systems linked to the environment in linear and non-linear ways (Campbell and Norman, 1989; Semenov and Porter, 1995). Many of the links between crop dynamics and atmospheric variables are non-linear and interdependent. Crop models were developed about 40 years ago as an effective substitute for ambiguous and cumbersome field experimentation (Sinclair and Seligman, 1996). The greater expectations from modelling rapidly led to increasingly detailed descriptions of the functioning of the biotic and abiotic components of cropping systems, leading to an increase in complexity and computer sophistication. Crop models provide the best-known approach for improving our understanding of complex plant processes as influenced by pedo-climatic and management conditions (Semenov et al., 2007), and they have proved to be more heuristic tools than simply a substitute for reality (Sinclair and Seligman, 1996). Most physically based soil-crop models operate on a daily time basis and simulate the evolution of variables of interest through daily dynamic accumulation.

In crop models, weather conditions need to be described as accurately as possible.

Weather data are the input data that drive the model and daily crop growth. It has been shown that weather data have a greater effect on yield than technical data and soil parameterisation (Nonhebel, 1994). In addition, crop model predictions (such as phenological development, biomass growth, or yield elaboration) are affected by temporal fluctuations in temperature and/or precipitation, even when the mean values remain similar (Semenov and Porter, 1995). It has been demonstrated that historical mean weather data might be inappropriate for predicting crop growth because of the non-linear response of crops to agro-environmental conditions (Porter and Semenov, 1999, 2005; Semenov and Porter, 1995). The sequencing of weather events greatly affects dynamic crop simulations; interactive stresses might have a greater impact on the final value of crop characteristics of interest (such as grain yield) than individual stresses (Riha et al., 1996).

Important research has been done on estimating the form of historical crop yield distributions. Day (1965) analysed crop yield distributions using the Pearson System and found that: (*i*) crop yield distribution is generally non-normal and non-log-normal, whereas (*ii*) the skewness and kurtosis of yield distribution (the mathematical third and fourth central moment, respectively) depend on the specific crop and the amount of available nutrients. His conclusions were corroborated by Du et al. (2012), who considered that the development of a complete theory on the effect of input constraints on yield skewness required empirical studies on diverse crops grown in different production environments. Several authors (Just and Weninger, 1999; Ramirez et al., 2001) have tried to assess the normality of crop yield distribution, but have not been able to do so. Just and Weninger (1999) identified three specific reasons for this: (*i*) the misspecification of the non-random components of yield distributions, (*ii*) the misreporting of statistical significance, and (*iii*) the use of aggregate time-series data to represent farm-level yield distributions. Numerous works have referred to the ‘usual left-tail problem’, which deals with the low probability of occurrence of some very low yields, characterised by particularly poor climate conditions (Hennessy, 2009a). More recently, Hennessy (2009a, b, 2011) analysed crop yield expectations with reference to the Law of the Minimum Technology and the Law of Large Number.

Within the context of yield prediction, there is a distinction between statistical models and process-based models. In the early 1960s the National Agricultural Statistics Service (NASS) of the United States Department of Agriculture (USDA) developed a method for assessing crop yield based on several sources of information, including various types of surveys and field-level measurements. These yield forecasting models are based on analysing relationships of samples at the same stage of maturity in comparable months over the preceding 4 years (Allen et al., 1994; Keller and Wigton, 2003). More recently, the statistical models have been coupled with remote data and recorded climatic measurements covering a preliminary period of a few months (Doraiswamy et al., 2007). As the yield prediction model is empirical and not physically based, this approach has serious limitations: (*i*) the future impact of past stress effects is not integrated into the physiological plant growth and (*ii*) the compensation mechanisms of crop management are not fully considered.

Process-based crop model approaches appear to be better alternatives for yield prediction, but crop models should rely on data that reflect hypothetical future scenarios. An appropriate and sophisticated approach for predicting grain yield with incomplete weather data was described by Lawless and Semenov (2005). It is based on the use of the Sirius crop simulation model (Jamieson et al., 1998; Semenov et al., 2007; Semenov et al., 2009) and the LARS-WG stochastic weather generator (WG) (Racsko et al., 1991; Semenov and Barrow, 1997). The methodology for predicting grain yield with incomplete weather data was related to the crop's life cycle: based on observed weather for the first part of the growing season, the authors used a stochastic WG to produce a probabilistic ensemble of synthetic weather time-series for the remainder of the season. WGs can be used to generate multiple stochastic realizations of extended sequences of real historical weather data (Lawless and Semenov, 2005; Mavromatis and Hansen, 2001; Mavromatis and Jones, 1998; Singh and Thornton, 1992), allowing risk assessment studies to be performed. The weather time-series built in this way were then used as an input in a crop simulation model to generate distributions of crop characteristics (such as phenological stages, end-season grain yields). As the season progressed, the uncertainty of the crop simulations decreased. This approach is interesting, but time-consuming and machine intensive.

Another method would involve replacing future data by forecasted weather. The initial problems here, though, are that forecasting has a time limit and that forecast accuracy diminishes with the long-time predictions. An added problem is the need to downscale data from a Global or Regional Climate Model (GCM/RCM) to local conditions at a resolution suitable for crop simulation models. The EU-funded DEMETER and ENSEMBLES projects are probably the two most representative examples of this application in Europe (Cantelaube and Terres, 2005; Challinor et al., 2005; Hewitt, 2004; Palmer et al., 2005). It is worth mentioning that GCM/RCM downscaling can be achieved by linking a seasonal forecast with a WG (Semenov and Doblas-Reyes, 2007), which allows yield prediction to be performed. It has been shown, however, that this approach is not any better at yield prediction than the approach based on historical climatology (Semenov and Doblas-Reyes, 2007).

Dumont et al. (2014b) have developed a similar approach. They assessed the potential of overcoming the lack of future weather data by using seasonal averages. For each of the climatic variables necessary to run the crop model (temperature, precipitation, solar radiation, vapour pressure, wind speed), they computed the seasonal averages as the daily mean values calculated from a 30-year historical weather database. Being based on only one future projection, it was very light in terms of computational requirement.

The aim of our study was to compare the efficiency of two crop yield prediction methodologies that are based only on historical records. To make the yield predictions, the Lawless and Semenov (2005) approach, based on using a high number of stochastically generated climate data, and the Dumont et al. (2014b) methodology, based on using seasonal averages, were selected. Both approaches benefit from the same amount of realized information. In each of the studies, relevant yield predictions could be made only at a late stage, but no research had ever compared the methodologies in an identical case study or using the same crop model. Comparing the efficiency of the two methodologies relied on an in-depth analysis of crop model behaviour based on a sound statistical foundation. The research findings reported by Day (1965) and Hennessy (2009a, 2009b, 2011) were applied to our study of crop model behaviour and the mathematical nature of the computed weather time-series is discussed in relation to the Convergence in Law Theorem and Central Limit Theorem (CLT).

2. Material and methods

2.1 Overview of the procedure

To answer the question of whether the predictive approaches have equal potential in terms of their ability to predict yield with the same accuracy and lead-time, we developed a four-step procedure (see Figure 1). The first step focused on the applicability of the CLT to the weather input generation. In other words, it has to be verified that the stochastically generated climates used by Lawless and Semenov (2005), denoted X_n, converged on the mean climate computed by Dumont et al. (2014b), denoted as X. This was ensured by the properties of the LARS-WG, and was thus only reminded in the material and method section.

The second step sought to determine if the crop model answers (i.e., in this case, the simulated end-season grain yields) could be approximated by a general function 'f' being representative of the whole model and linking the climatic inputs and the simulated variable output. The numerical-experimental crop yield distributions obtained with stochastically generated climate data were analysed. In compliance with the Generalised Central Limit Theorem (GCLT), the approximation of the simulated yield distribution by a log-normal distribution was assessed.

In the third step, which was divided into two successive phases, the simulations obtained using both sets of climatic data were compared. In the first phase, the within-season yield predictions were compared on an annual basis. In the second phase, the corresponding predictive lead-times were compared. If the two approaches were found to be equivalent (i.e., if the mathematical expectation of the Lawless and Semenov [2005] approach, denoted as $E[f(X_n)]$, did not differ significantly from the other approach, where the mathematical expectation of the outcomes was denoted $E[f(X)]$) this would validate the applicability of the Convergence in Law Theorem.

2.2 Case study

The data used in this paper are derived from an experiment conducted to study the growth response of wheat (*Triticum aestivum* L., cultivar Julius) in the agro-environmental conditions of the Hesbaye region in Belgium. The soil at the experimental site was a classic loam type.

Biomass growth was monitored over 3 years (crop seasons 2008-09, 2009-10 and 2010-11). In 2008-09, the yields were fairly high under adequate nitrogen fertiliser rates, due mainly to good weather conditions. In the 2009-10 and 2010-11 seasons, there was severe water stress, resulting in yield losses. In 2009-10 the water stress occurred in early spring and early June; in 2010-11 it occurred from February to the beginning of June. In the summer rainfall returned, ensuring a normal growth rate for the last part of the season. Reasonable grain yield levels were achieved, but the straw yield remained low, giving a high harvest index.

The current practice in Belgium is to apply a total of 180 kgN.ha^{-1} in three equal fractions (60 kgN.ha^{-1}) at the tiller, stem extension and flag-leaf stages, which is known to be close to the optimum nitrogen rate for crop growth under the climatic conditions prevalent in the country (Dumont et al., 2014a). Over the 3-year experiment, at this fertilisation level, the grain yields reached 12.6, 7.8 and 7.1 ton.ha^{-1} of dry matter, respectively Among the replicates, the highest yield was 14.0 ton.ha^{-1} in 2009 and the lowest was 5.8 ton.ha^{-1} in 2011.

2.3 Modelling crop growth

2.3.1 The STICS crop model

The STICS crop growth model (Brisson et al., 2003; Brisson et al., 2009; Brisson et al., 1998) was used to simulate the end-season grain yields (expressed in tons of dry matter per hectare [ton.ha^{-1}]) that were the focus of the study. In this model, dry matter is related to absorbed radiation according to the radiation-use efficiency (RUE) concept (Monteith and Moss, 1977). STICS allows the effect of water and nutrient stress on development rate (Palosuo et al., 2011) to be taken into account The actual and potential evapotranspiration were computed using the Penman formalism (Penman, 1948). The STICS model requires daily weather inputs (i.e., minimum and maximum temperatures, total radiation and total rainfall, vapour pressure and wind speed).

The STICS model parameterisation, calibration and validation were performed on the 3-year database used for the case study. For the calibration process, the DREAM(-ZS) algorithm (Dumont et al., 2014c; Vrugt et al., 2009) was used. The highly contrasting climatic data in the 3-year database were used to parameterise crop water, thermal and nitrogen stress dependence. Times-series of leaf area index (LAI) measurements (once a month), biomass and grain yield estimates (once a fortnight and at the time of final grain yield), soil N-NO_3^- and N-NH_4^+ (once a fortnight) and plant N uptake (once a month) were used to parameterize the various aspects of plant development (i.e., grain yield components, plant growth rate, soil water and nitrogen uptake). There is more detail on the model calibration process and the accuracy of the model in Dumont et al. (2014c).

2.3.2 The simulation process

It was assumed that cultivar, soil and management remained the same for all simulations, and therefore that the simulations differed only in terms of weather inputs. In order to ensure that the simulated plant growth would be limited only by climatic factors, simulations were conducted with adequate nitrogen fertilisation levels. The simulated fertiliser rate used for the study was a total of 180 kgN.ha^{-1} applied in three equivalent fractions (60 kgN.ha^{-1}) at the tiller, stem extension and flag-leaf stages.

In order to simplify the simulation process, the same management techniques were applied to each simulation, following the 2008-09 itinerary. The sowing date was in late October, on 10/25.. Each simulation was run with the sowing date as the starting point. The same soil description was used for all simulations. The soil-water content was initialized at field capacity, and the soil initial inorganic N content corresponded to real measurements taken in the first year of the experiments. The three 60 kgN.ha^{-1} nitrogen fertilizer doses were applied at fixed dates (i.e., at the tillering, stem extension and flag-leaf stages in 2008-09) on on the 03/23, 04/16 and 05/25, respectively.

2.4 Weather database generation

2.4.1 Historical climatic database

The complete 30-year (1980-2009) Ernage weather database (WDB) was used in this study to generate the crop model inputs. Part of Belgium's Royal Meteorological Institute (RMI), the Ernage weather station is 2 km from the experimental field. The measurements carried out by the station involved all the climatic variables required to run a crop model.

2.4.2 Generating a probabilistic ensemble of synthetic weather data

The first approach used for within-season yield predictions was based on the work of Lawless and Semenov (2005). In essence, the 30-year Ernage WDB was analysed using the

LARS-WG, which computed a set of parameters representing the experimental site (daily mean values, daily standard deviations, daily maxima and minima, successive wet and dry series and frequency of rainfall events). They the LARS-WG can be used to generate a set of stochastic synthetic weather time-series representative of the climatic conditions in the area. According to Lawless and Semenov (2005), and for reasons detailed at section 2.6.1, 300 time-series were generated and then input into the model.

Using a WG is an appropriate way of simulating yields under new combinations of probable weather scenarios. If the crop model is correctly calibrated and validated, this would lead to a simulation of stress conditions not observed during the limited time of a field experiment.

2.4.3 Generating the mean climate data

The second approach, based on the work of Dumont et al. (2014b), used a daily mean climate dataset. The dataset was drawn from the Ernage WDB, and the daily mean data for each climate variable was computed. In other words, for each variable and day, each element of the mean climate matrix was computed as the mean of the corresponding 30 values of the same day over the 30 years.

This approach relies on the strong assumption that climate conditions are very close to the seasonal norms. This is particularly the case with precipitation, for which a minimum value is thus available each day, ensuring reduced water stress. As discussed by Dumont et al. (2014b), such an assumption leads to simulations that, at any time of the year, show the remaining yield potential. Other assumptions and limitations of this approach are described by Dumont et al. (2014b).

2.4.4 Within-season prediction

These two types of synthetic weather data were used to perform within-season yield prediction. Climate series were generated from recorded historical climatic data. At a predetermined rate (e.g., every 10 days), the observed weather sequences were replaced by either the probabilistic ensemble of synthetic climatic time-series or the mean climatic data. The climatic matrix ensembles of data thus generated could then be used as inputs for the crop growth model. The effect of such probable climatic conditions could be studied for the various yield components. With this methodology, the proportion of the hypothetical future data diminished as the growing season progressed, as did the uncertainty about the corresponding simulated yield.

2.5 Statistical considerations

2.5.1 The Convergence in Law Theorem

The convergence in law ($\rightarrow_L$) or in distribution ($\rightarrow_d$) is considered to be one of the weaker laws of convergence, but underpins the demonstration of many theorems and is key to our analysis of crop model behaviour. It can be enunciated as follows: Let $\{X_n\}$ be a sequence of n random variables x and let X be a random variable. Denote by $F_n(x)$ the distribution function of X_n for all real x. The convergence in law theorem then states that $\{X_n\}$ converges in distribution to X ($X_n \rightarrow_d X$) as $n \rightarrow \infty$, if there is a function f, which extends over the real space ($R \rightarrow R$), continuous and bounded such that:

$$E[f(X_n)] \rightarrow E[f(X)] \qquad \textbf{(Eq. 1)}$$

2.5.2 The Central Limit Theorem and the log-normal distribution

The Central Limit Theorem (CLT) (de Moivre, 1976) can be enunciated as follows: Let $\{Y_n\}$ be independent random variables, of the same law (i.e., identically distributed), of integrable square. We denote μ its expectation and σ^2 its finite variance; here we assume that $\sigma^2 > 0$. Then:

$$\frac{\sqrt{n}}{\sigma}\left(\frac{S_n}{n} - \mu\right) \rightarrow_L Y, \text{ as } n \rightarrow \infty \qquad \textbf{(Eq. 2)}$$

where S_n is the sum of the Y_n values. Y follows a Gaussian distribution, centred in zero, with variance one: $Y \sim \mathcal{N}(0,1)$. In practical terms, the CLT implies that for 'large' n, the distribution of Y_n may be approximated by a Normal distribution with mean μ and variance σ^2/n.

The CLT allows for different generalisations in order to ensure the convergence of a sum of random variables under a weaker hypothesis (particularly with regard to the distribution from which they originated), but relies on conditions that ensure that no variable has significantly greater influence than any other variable. In particular, the CLT has been extended to the product of functions, the logarithm of a product being the sum of the logarithms of each factor. This extension is known as the Generalised Central Limit Theorem (GCLT).

Day (1965) suggested assessing the following generalised log-normal transformation of data in order to determine if crop yields Y_n responded to a log-normal distribution:

$$Y_{n-\log} = \ln\left(Y_{\max} - Y_n\right), \qquad Y_n < Y_{\max} \qquad \textbf{(Eq. 3)}$$

where Y_{max} corresponded to a theoretical maximal threshold and $Y_{i,\ i \in \{1,\ldots,n\}}$ corresponded to the observed yield under given climate X_i, in other words $Y_i = f(X_i)$.

An easy way to assess the log-normal behaviour of a yield sampling Y_n is to evaluate the normality of the corresponding normalised and zero-centred log-transform vector Y_{Norm} (computed according to Eq. 3). Such an evaluation relies on the use of the Kolmogorov-Smirnov test (Dagnelie, 2011; Feller, 1948). The vector of observations Y_n could therefore be transformed according to Eqs. 2 and 3, leading to Eq. 4 where the corresponding distribution (Eq. 5) is assumed to follow the log-normal distribution.

$$Y_{Norm} = \frac{\ln\left(Y_{\max} - Y_n\right) - \mu_{\ln\left(Y_{\max} - Y_n\right)}}{\sigma_{\ln\left(Y_{\max} - Y_n\right)}} \qquad \textbf{(Eq. 4)}$$

$$p(y) = \left[\frac{1}{\sqrt{2\pi}.\ln\left(Y_{\max} - y\right).\sigma_{\ln\left(Y_{\max} - y\right)}}\right].\exp\left\{-\frac{1}{2}.\left[\frac{\ln\left(Y_{\max} - y\right) - \mu_{\ln\left(Y_{\max} - y\right)}}{\sigma_{\ln\left(Y_{\max} - y\right)}}\right]^2\right\} \qquad \textbf{(Eq. 5)}$$

2.6 Practical implementation of the statistical basis of general model behaviour assessment

2.6.1 LARS-WG and mean climate data

The LARS-WG was specifically designed "to generate synthetic data which have the same statistical characteristics as the observed weather data" (Semenov and Barrow, 2002). It is therefore clear that the CLT applies to the inputs, ensuring that the stochastically generated climatic time-series (X_n) used in the Lawless and Semenov (2005) methodology converge in law with the mean climatic data (X) proposed by Dumont et al. (2014b). The statement $X_n \rightarrow_L X$, however, does not say how large n must be for the approximation to be practically useful. Lawless and Semenov (2005) demonstrated that a set of 60 synthetic weather time-series was enough to achieve a stationary prediction of mean grain yield. As the stochastic component of LARS-WG is driven by a random seed number, however, Lawless and Semenov (2005) recommended using at least 300 stochastically generated weather time-series, which latter was therefore the number of time-series used to conduct this research.

2.6.2 Hypothesis underlying the GCLT

Crop models are known to have a non-linear response to weather conditions. They also have limitation factors affecting yield components, attributable mainly to genetic specification, such as a maximum number of grains in place or a maximal weight of individual grains. A third feature of crop models is that, within them, growth is simulated as a differential daily increment (Eq. 6) and that most of the increment ($f(Y(t), X(t), \theta)$) is determined by functions that are themselves either multiplicative (e.g., growth function x stress function) or hierarchical (e.g,. biomass growth being exponentially connected to LAI value).

$$Y(t + \Delta t) = Y(t) + f\left(Y(t), X(t), \theta\right) \qquad \textbf{(Eq. 6)}$$

where $Y(t)$ and $Y(t+\Delta t)$ are the outputs simulated at the daily Δt time step, $X(t)$ is the vector of input variables, θ is the vector of model parameters and f accounts for the simulated model processes.

We can reasonably assume that each simulated end-season yield (i.e., Y_n) is the result of a unique combination of climatic variables X_n: different combinations of variables (e.g., temperature, vapour pressure); different dynamics over the seasons for each individual variable (stochastic generation of values such as $X(t)$, $X(t+1)$, $X(t+2)$ and so on); and different dynamics of interacting variables (successive dry and wet series). To some extent, this ensures that the simulated yields are independent random variables, which is a necessary condition for assessing CLT applicability.

The second assumption is that the output variables have the same law. The objective of the second step of the procedure is to find this general law and validate the CLT applicability to the model outputs. Some discussions, however, have to be made at this stage. Each input variable X_n (known to comply with the CLT) is used to pilot the simulations through the same complex model summarized as Eq. 6. The sum term in Eq. 6, which constitutes the daily increment, is therefore also consistent with the CLT. On the other hand, due to the structure of a crop model, it is known that under the $f(Y(t), X(t), \theta)$ term there are hidden hierarchical ($Y = f(X) \equiv g(h(X))$) and multiplicative ($Y = f(X) \equiv g(X) \times h(X)$) functions. The model $f(Y(t), X(t), \theta)$ remains the same for all assessed input variables. Provided that none of the climatic variables has a significantly greater influence than others, the main objective is therefore to determine if the generated outputs respond to a unique distribution law compliant with the CLT.

2.6.3 The log-transformation of simulated outputs to assess the GCLT

Among the generalisations of log-transformation proposed by Day (1965), the one proposed at Eq. 3 appeared suitable for the observed yield distributions and the ‘left-tail’ problem. Day (1965) stated, however, that it would be difficult to find the threshold Y_{max} (Eq. 3) that would correspond to the potential maximal yield of the crop, for which the probability of occurrence should be zero.

An easy, yet relevant, way to find the potential yield Y_{max} in Eqs. 3 to 5 would be to consider that the maximal yield obtained under *n* climatic scenarios generated with LARS-WG was the upper limit of the distribution. The probability that such an optimal climatic scenario had occurred would be quite low (close to zero) and due exclusively to a particular combination of climatic variables resulting from the stochastic generation performed using LARS-WG.

2.7 Comparisons of model output distributions and yield prediction abilities

The third and fourth steps of the procedure focus on comparing the distribution of the simulated grain yields obtained using the Lawless and Semenov (2005) methodology with the results obtained using the Dumont et al. (2014b) approach. As a high number of synthetic climate data was used, and provided that a general law *f* can be highlighted, the mathematical expectation of the end-season yields (*i.e.,* $E[f(X_n)]$) could be computed as its empirical mean. It could then be compared with the unique yield value simulated, using mean climate as the climatic projection (i.e., $E[f(X)]$).

There were three levels of comparison. First, the model was run on inputs consisting only of stochastic climate data on the one hand and only of daily mean data on the other. The end-season yield value obtained from the second dataset was positioned within the yield distribution obtained from the first dataset. As the main aim of the study was to compare the two within-season yield prediction algorithms, the equivalence of the yields simulated using the two approaches would then be evaluated throughout the season (2.7.1). Finally, the predictive lead-time for both approaches would then be compared (2.7.2).

2.7.1 Single year analysis and model output distributions

In order to see if the two methodologies led to same output simulations, two statistical criteria were used: relative root mean square error (RRMSE) and normalised deviation (ND) (Eqs. 7 and 8). The two approaches would be considered as equivalent if the value of both criteria was less than 10%. The 10% threshold was seen as appropriate for two reasons. First, an ND value less than 10% is usually thought to validate model simulations (Beaudoin et al., 2008; Brisson et al., 2002). Second, the within-season predictive ability would be assessed considering a plus or less 10% error around the final simulated grain yield (cfr 2.6.4 - Analysed data).

$$RRMSE = \frac{\sqrt{\frac{1}{k}\sum_{i=1}^{k}\left(Y_i - \hat{Y}_i\right)^2}}{\frac{1}{k}\sum_{i=1}^{k}\left(Y_i\right)} \text{, with expected } RRMSE < 0.1 \qquad \textbf{(Eq. 7)}$$

$$ND = \frac{\sum_{i=1}^{k}\left(Y_i\right) - \sum_{i=1}^{k}\left(\hat{Y}_i\right)}{\frac{1}{k}\sum_{i=1}^{k}\left(Y_i\right)} \text{, with expected } ND < 0.1 \qquad \textbf{(Eq. 8)}$$

where Y and $\hat{Y}$ refer to the end-season yields simulated using the two approaches and i refers to the i^{th} simulation of end-season yields performed during the season.

2.7.2 Inter-year analysis and prediction ability of the approaches

The ability of both approaches to predict yield was assessed finally by comparing the predictive lead-time curves observed for the original 30-years Ernage weather database. The computation of the curves followed the process proposed by Lawless and Semenov (2005) and consisted of plotting the cumulative probability distribution of the first day for which the yield could have been predicted. There is more detail on how this distribution is computed in Lawless and Semenov (2005) and Dumont et al (2014b).

With regard to the predictive ability of the model, the within-season predictive simulations were compared to the simulated final grain yield, with an error of plus or minus 10% considered as an acceptable predictive value. There is more detail on this in the work reported by Lawless and Semenov (2005) or Dumont et al. (2014b),

3. Results

3.1 Assessing the crop model behaviour

3.1.1 Analysis of the experimental probability density function for purely synthetic climate data

Figure 2 shows the probability density function and cumulative distribution function of grain yield simulations conducted on purely synthetic climate data generated using the LARS-WG. The simulated outputs were subjected to the log-normal distribution. The log-normal distribution was not fitted to the data, but the theoretical distribution was computed on the basis of the characteristic values of the simulated output that were the mean and standard deviation of the log-transformed values (Eq. 5). The computed theoretical function (solid black lines) matched the numerical-experimental distribution (solid grey line or grey histogram) fairly well. The log-normal distribution therefore seemed particularly suitable for representing the crop model answer.

Using this approach, it was possible to compute the mean (vertical black line in Fig. 2B) or median of the experimental distribution, intercepted at the 50th percentile (horizontal black line in Fig. 2B), which was 11.25 ton.ha^{-1} and 11.82 ton.ha^{-1}, respectively. From a probabilistic point of view, at sowing there was a 50% chance of achieving at least 11.82 ton.ha^{-1}, without any prior knowledge of the forthcoming weather. In comparison, the mean of the distribution occurred at a probability level of 40%. The simulated yields accorded with the observations performed during the original 3-year experiments, the values of which were presented at section 2.2.

The yield simulated using the pure mean dataset was 12.14 ton.ha^{-1}. In the previous distribution this would have occurred at a probability level of 56%, implying that, if mean climate data were used instead of stochastic data, there was a 16% chance of overestimating the yields by about 7.5%. This latter value was computed as the relative difference between the yield prediction obtained via the mean climatic projections (i.e., $E[f(X)]$) and that obtained via the stochastic simulations (i.e., $E[f(X_n)]$).

With regard to the theoretical computed log-distribution, the cumulative distribution function curve showed a left-tail, with a theoretical minimum value fixed at -∞, whereas the minimum simulated grain yield was 3.4 ton.ha^{-1}. The maximum simulated Y_{max} value was 14.9 ton.ha^{-1}.

Finally, the Y_{Norm} vector was computed according to Eq. 4 and its normality was evaluated using the Kolmogorov-Smirnov test. The *p-value* was 0.837, far higher than the expected value of 0.025 (= α/2). This led to the conclusion that the experimental distribution could not be considered as differing from a log-normal function, and confirmed the validation of the GCLT and its applicability to the crop model. In other words, the STICS crop model could be considered as a global *f*-function that links the $X(t)$ random climatic inputs and the $Y(t)$ simulated grain yield outputs.

3.1.2 Climate data combination and the log-normal behaviour

When performing within-season yield prediction using the Lawless and Semenov (2005) approach, the stochastic projections were coupled with observed time-series. The issue then was to determine to what extent (i.e., till which amount of observed weather data) the crop model could exhibit a log-normal behaviour ? An example of the simulated grain yields based on combined synthetic and observed data, and drawn from 300-year weather simulations, was computed for the 1981-1982 crop season (see Fig. 3). Progressing through the crop lifecycle, the uncertainty about the weather data lessened as the amount of observed time-series increased. The surrounding bounds on corresponding yield predictions therefore gradually tightened until a final value (11.6 t.ha^{-1}) was reached with purely observed time-series.

For each section of data that could be extracted from this figure, an analysis conducted as described in the previous section was performed. Table 1 shows the *p-value* resulting from the Kolmogorov-Smirnov test, applied on the normalised vector of data (Eq. 4). The 30 years of the database were studied individually, as year 1981-82 (Fig. 3), using a 10-day replacement rate of the observed time-series. The *p-value* under the acceptable 0.025 ($\alpha/2$, α = 5%) expected criteria are underlined in grey. Until the day of the year (DOY) 06/15, our analyses showed that in almost 95% of cases the model could be considered as having log-normal behaviour. The test generally failed later in the season (between 06/15 and 08/24), whatever the year. For example, the 1981-82 crop season (Fig. 3) failed the Kolmogorov-Smirnov test for DOY 06/15, when the p-*value* was 0.01, below the acceptable value of 0.025.

Figure 4 presents same results as Figure 2, but for 1981-82 and taking account of real time-series observed until 06/15. The corresponding simulations (Fig. 3) showed that the period between DOY 05/16 and 06/15 corresponded to a transient period where simulation distribution evolved from widely spread to closely tightened around the final simulation obtained only for real climate. At DOY 06/25 (Fig. 4), a *p*-value of 0.02 was obtained. The distribution seemed closer than a normal/symmetric distribution, as confirmed by the proximity of the mean and median of the distribution (Fig. 4B)

In conclusion, for most of the season (from sowing until DOY 06/15), the log-normal distribution seemed able to account for crop yield distribution. This confirmed the applicability of the GCLT. Later in the season, as the part represented by the observed time-series became dominant within the model inputs (at DOY 06/15, 230 days of real weather had been observed), the log-normal behaviour disappeared. At that point, on one hand there was no longer any independence of the climate series, and on the other hand the number of grains was fixed.

3.2 Assessing the potential of yield prediction

3.2.1 Single-year analysis of model outputs

The follow-up to the research focused on determining if the Converge in Law theorem could be applied to STICS model simulations. Thus, the mathematical expectation of the simulation conducted on 300 stochastic climate data ($E(f[X_n])$) was compared with the simulation conducted using the mean climate data $E(f[X])$.

Figure 5 presents the variation in predicted model output during within-season simulation, using the both Lawless and Semenov (2005) and Dumont et al. (2014b) approaches. In terms of the outputs of the methodologies, there were contrasting results in the 1991-92 (Fig. 5A) and 2007-08 (Fig. 5B) seasons. Figure 5 is based on Figure 3, which summarised the information using three characteristic values: the average and the percentile 2.5 and 97.5 of the 300 simulations.

For the 1991-92 season, the mean values of the 300 simulations (solid grey line) were very close to the results generated using the Dumont et al. (2014b) approach (solid black line). The RRMSE and ND values were 0.026 and -0.015, respectively.

This was not the case for the 2007-08 season. The main differences between the two seasons could be explained by the first 10 days of the observed time-series (drastic autumn conditions) for the crop seasons from 2005 to 2008. For these years, there was a significant reduction in the predicted final grain yield values because the sowing for the simulations was based on stochastic climate assumptions. It is likely that the first 10 days of the observed time-series had such an impact on the simulations that only very good climatic conditions, such as the mean climate assumption, could have compensated for this. This effect had repercussions for each simulation out of 300 climate ensembles and over the main part of the season. After DOY 07/15, the simulations based on both projective assumptions (mean and stochastic climate) were very close, which indicates the importance of the observed time-series in the crop model inputs.

When comparing the two crop seasons, the projected mean climate assumptions (solid black line) also led to more constant yield simulations over the years (about $12 t.ha^{-1}$), at least for the first part of the season.

The final aim of this section is to determine if the mean yield of the 300 stochastic climate inputs is equivalent to the yield predictive curve obtained using the Dumont et al (2014b) methodology. In other words, the equivalence between the expectations $E[f(X_n)]$ and $E[f(X)]$ needs to be assessed.

Table 2 summarizes the criteria (RRMSE and ND) computed on the basis of the outputs from the two methodologies where data were replaced every 10 days for each individual year (lines 1981 to 2009 in Table 2) and when the data originating from all the simulations were aggregated (line 'Overall' in Table 2). In 90% of cases, ND values were below the expected 10%, whereas RRMSE values were above the threshold in only 5 years out of 29. In general, both approaches gave very close results. To a lower extend, the two approaches were also equivalent for the 1984-85 and 1996-97 crop seasons, with the RRMSE very close to the imposed thresholds (0.102 and 0.101, respectively). As illustrated by Figure 5, the 2007-08 crop season exhibited bad RRMSE and ND criteria when comparing the two approaches, which was also the case for the 2005-06 and 2006-07 seasons.

Figure 6 presents the graphical comparison of the two approaches resulting from the concatenated data. The RRMSE and ND values were also computed with these data (corresponding to the last 'overall' row in Table 2). The overall ND value revealed a slight overestimation (-5.8%) using the Dumont et al. (2014b) methodology compared with the Lawless and Semenov (2005) methodology. The overall RRMSE was close to the acceptable value (0.112). This was due mainly to the crop seasons from 2005 to 2008; which simulations are shown by the cloud of small dots in the upper left of the graph (Fig. 6)

The close simulations seemed qualitative enough to be able to conclude that there was equivalence between the two approaches, supporting the validity of applying the Convergence in Law theorem to the use of crop model.

3.2.2 Multiple-year analysis and prediction ability

Finally, the statistical predictive ability of both predictive methods was compared (Fig. 7) using the Lawless and Semenov (2005) approach. This approach is based on determining the cumulative probability function associated with the first days for which the predictions would have been possible, given an error level around the final simulated value (10% in this case, represented by the horizontal light dotted grey lines in Fig. 5).

The 2-sample Kolmogorov-Smirnov test was applied to these distributions, enabling the equivalence of both distributions (*p-value* = 0.31) to be validated. The RMSE between the two approaches was evaluated at 9 days, which is less than the rate of data replacement (10 days). Both approaches produced yield predictions with an equivalent lead-time.

4. Discussion

When developing decision-support systems, crop modellers are faced with antagonist decisions. On one hand, it is very important to build models and systems that can compute a reasonable and reliable answer as fast as possible. At critical moments, when important management decisions have to be made, farmers, who are the users of the information produced, are not concerned about the time a model needs to run – they just want clear, rapid answers to their questions. On the other hand, with regard to statistics, a modeller needs to characterise the quality and certainty of a simulation, which makes it essential to perform multi-simulations from which statistical values can be computed, to give a mean accompanied by a confidence interval (e.g., 95% uncertainty limit). In addition, both practical approaches need to be implemented in the spirit of the philosophy of the methodologies developed by Dumont et al. (2014b) and Lawless and Semenov (2005).

It is worth mentioning that, although the two methodologies are generic, the results presented here are site-specific. The model was parameterised and calibrated on a specific soil type and for a specific crop culture. The 30-year WDB was also representative of the climatic conditions of a specific area. Although generic, however, the procedure could be applied to other models or model outputs.

4.1 Crop model behaviour analysis

Crop yields have finite lower and upper ranges, even under favourable climatic conditions (Day, 1965), and this is especially true for crops that have a determinate growth, such as wheat. Day (1965) observed, however, that determinate-growth crops skewed the probability function under random weather effects, particularly when nitrogen was fertilised. Our analysis confirms the observation by Day (1965) of a left-tail dissymmetry under different climates.

It was therefore necessary to find a distribution that could account for these behaviourial traits of dissymmetry and upper limitations. Our study showed that the behaviour of the model could usually be correctly approximated by a log-normal distribution. This was so for the stochastic climate approach and at the early stages of the within-season yield prediction, *i.e.*, provided *(i)* that the observed time-series were not predominant in the climatic combinations or (*ii*) that, in the early season, observed time-series did not have a significant effect on the end-season simulated yield (as illustrated in the years from 2005 to 2008).

With a few exceptions, the properties of the GCLT could be used to account for the whole model behaviour. By extension, in this case, it is reasonable to assume that the STICS model could be considered to operate as a product of functions that are themselves dependent on random climatic variables.

4.2 Grain yield results

The results analysis showed a systematic and important tightening of the 95% confidence curves between DOY 05/16 and 07/05. At this level, the crop had been sown about 200-250 days earlier. This transient period corresponds to the stages between flag-leaf emergence and anthesis, the exact date being determined by the climatic conditions of the relevant year. In real life, over its whole life cycle, wheat is able to compensate in order to optimise its reproduction abilities. Once the number of grains is established, however, the yield result depends entirely on grain filling, no matter it is driven by climatic condition (linked to future data) or biomass reallocation (linked to past growing conditions).

Therefore, according to the simulation processes and the within-season prediction methodology, as the season progresses and the hypothetical projective climatic conditions are replaced by observed time-series, the number of grains is progressively fixed for each simulation at a time and according to the different scenarios. Once the real weather has been monitored up to the day when the number of grains has been fixed for all simulations, however, the confidence boundaries become very close. From that time, as in real life, the simulated yield depends entirely on grain filling and exhibits normal behaviour. During this period, an observed normal distribution of grain yields would argue in favour of the applicability of the CLT, instead of GCLT. Further research is needed to validate this statement.

4.3 Predictive ability of the two approaches

As Dumont et al. (2014b) discussed in their work, the mean climate hypothesis is a strong assumption. Seeing the climatic conditions as the mean data over the studied period is equivalent to make crop growth predictions in almost non-limiting growing conditions. Under such conditions, the plant will grow with little or no stress because a minimum amount of water, solar radiation energy and sum of temperature are provided each day to the crop. These assumptions imply that the simulated yield will correspond to the remaining yield potential of the crop. This answers the question: “At a given point in the season, what could I still expect at harvest if the climate tends to come back closer to the seasonal norms ?” This also implies that the simulated yield could often be slightly overestimated, as confirmed by the observed overall ND value (+5.8 %).

The conclusion that emerges from our analysis, however, is that from a strictly predictive point of view the Dumont et al. (2014b) approach is equivalent to the Lawless and Semenov (2005) approach (2005). In addition, during the single-year analysis the RRMSE and ND criteria were close to or lower than the 10% threshold in 90% of the cases. Finally, when no climatic data replacements were performed (i.e., when the yields were simulated based only on pure projective stochastic climatic data or pure mean data), the difference was about 7.5%. This clearly shows that the Convergence in Law theorem is applicable.

This fact is very important because the Dumont et al. (2014b) approach needs less time (by 300-fold) to run and reach the same conclusions as the Lawless and Semenov (2005) approach. The Lawless and Semenov (2005) approach is very important, however, because it allows prediction uncertainty to be characterised, which is not possible with the Dumont et al. (2014b) approach. When analysing climate variability or climate changes, this issue of uncertainty associated with the simulations is significant. When predicting yield, however, running time is a crucial factor in terms of building decision-support systems.

4.4 Further discussion on climatic assumption and yield distribution analysis

There is clear evidence that yield simulated using mean climatic data is close to the yield mean obtained under stochastically generated climatic data. An overestimation has been observed, though. Ongoing research (Dumont et al., 2014a ; Dumont et al., 2013) has suggested that under the specific agro-pedo-climatic conditions of this case study, greater skewness occurred under a fertilisation level corresponding to three applications of 60 $kgN.ha^{-1}$ at the tillering, stem extension and flag-leaf stages, which is the fertilisation regime simulated in this study. A higher degree of asymmetry leads to greater differences between the mean, the median and the mode of the yield distribution.

This raises other discussions. First, the applicability of the Convergence in Law Theorem is attractive and is compatible with the mathematical nature of crop models. As the level of asymmetry is likely to decrease with other practices, the legitimacy of applying the Convergence in Law Theorem should be easier to demonstrate.

Second, Day (1965) suggested that mode or median estimates of yield might be preferred to the mean estimates, both for forecasting and prescription purposes. Our study seemed to confirm this statement. The median value of yield distribution obtained using only stochastic climate data (11.82 ton.ha^{-1}) was much closer to that for yield simulated with mean climate data (12.14 ton.ha^{-1}). The analysis described in this paper should be performed using the median value instead of the mean value.

Third, mean climate data was used as a model input. It is fairly evident that some weather variables, such as temperature and solar radiation, show normal daily distributions, suggesting an equivalence of the mean and median of these distributions. For some other climatic data, however, daily distribution is itself asymmetric. In Belgium, rain records exhibit a right-tail dissymmetry, with a high frequency of low rainfall, and low return times of substantial rain. It would be interesting to assess the impact of median climatic data on the corresponding simulated yield, and compare it with the yield distribution obtained stochastically.

Finally, it is worth commenting on the generic nature of the results presented in this paper. With regard to the statistical references, it could be concluded that using a model that relies on similar formalisms as those of STICS models should not contradict our conclusions and the GCLT would still be applicable. With regard to the crop, wheat has a determinate growth and therefore it is likely that the conclusions we reached could be extended to any other crop with determinate growth. Further research needs to be conducted on tuberous crops, by example, such as potatoes and sugar beet, because the factors involved in tuberous yield elaboration differ greatly from those in grain yield elaboration. Finally, the main question to address was whether or not the Convergence in Law theorem could apply in other contexts, particularly in other climatic conditions (e.g., southern Europe Mediterranean weather, as in Italy or Spain) or under climatic changes. Our research suggested that if climatic-induced stress remains limited in intensity or length, the GCLT would be applicable to crop modelling. More work needs to be done, however, to determine the extent to which this would apply given greater climatic-induced stress levels.

5. Conclusion

In this paper, two validated methodologies for within-season wheat yield prediction, one proposed by Dumont et al. (2014b) and the other by Lawless and Semenov (2005), were compared. Both approaches offer the main advantage of being able to use historical data, the first based on the computed mean climate and the second on using stochastically derived time-series. The comparison was made using sound statistical procedures to study crop model behaviour. Based on the Convergence in Law Theorem and the CLT (as well as GCLT), we developed a procedure that shows how the two approaches, relying on the same weather input database, could be used to make yield predictions and how close the predictions thus obtained could be.

The generalised log-normal distribution was seen as a good way of assessing model behaviour, especially when the model was run on a high number of stochastic climate inputs. This is attractive because it means the model can be seen as a product of variables, which is consistent with the mathematical nature of the model. It also validated the applicability of the GCLT, which was a requirement in assessing the applicability of the Convergence in Law Theorem.

Once the model behaviour had been characterised, the comparison of the yield prediction ability of the two methodologies was investigated. On a year-to-year basis, the analysis showed that some climatic combinations of variables could induce a bias from the beginning of the season, leading to a divergence at an early stage of the predictive curves. In

90% of the cases, however, the differences between the two methodologies were close enough to consider them as equivalent (RRMSE and ND < 10%). The inter-year analysis, which related to the statistical ability of yield prediction, led to the conclusion that the two methodologies had equivalent lead-time. These observations suggest that the Convergence in Law theorem was validated by our case study.

It is important to note, however, that our work was carried out under temperate Belgian weather conditions, simulating the development of a determinate wheat crop and using the STICS model and the formalisms inherent in it. The procedure we designed, however, is generic and should be tested on other models, under other climatic conditions and with other crops before any generalisations can be made. Some generalised model behaviour was highlighted, though. Crop models have been built to match reality, but contrary to real-life, they operate entirely according to their mathematical construction. Under fixed agro-pedological conditions, it should thus be possible to summarize the crop model behaviour under a wide variety of climate conditions and put it in relation to a specific but relevant distribution. The methodology described in this paper constituted an attempt to achieve this.

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List of figures :

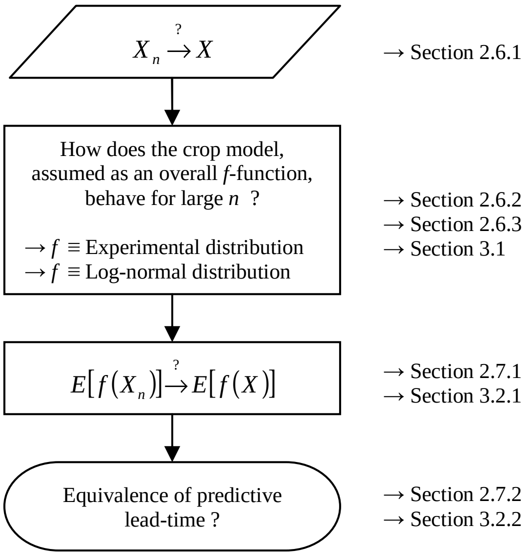

Figure 1: Schematic representation of the procedure used to compare the predictive ability of the Dumont et al. (2014b) and Lawless and Semenov (2005) methodologies. X_n represents the *n* stochastic weather realisations, *X* represents the mean climate data, *f* represents a general function and *E* is the mathematical expectation.

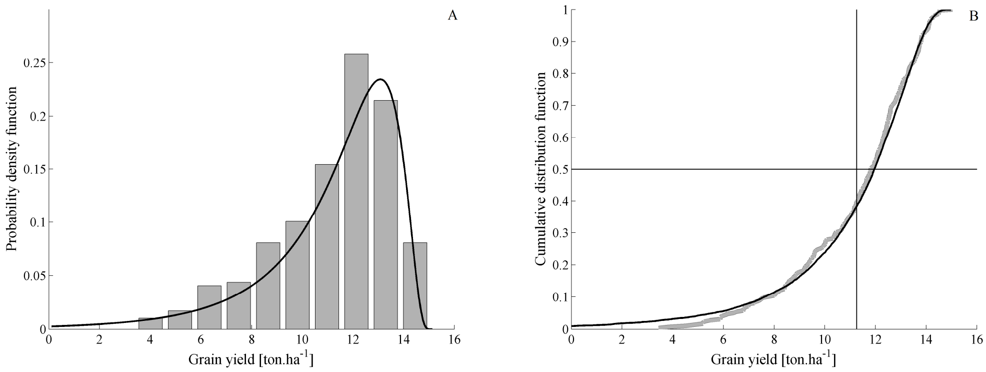

Figure 2: Probability density function (A) and cumulative distribution function (B) of the simulation conducted on pure synthetic-stochastic climate data. Simulated data are represented by a grey bar (A) or a bold solid grey line (B) and the computed log-normal distribution is represented by a solid black line. In graph B, the mean value is represented by a vertical thick black line and the 50th percentile by a horizontal thick black line).

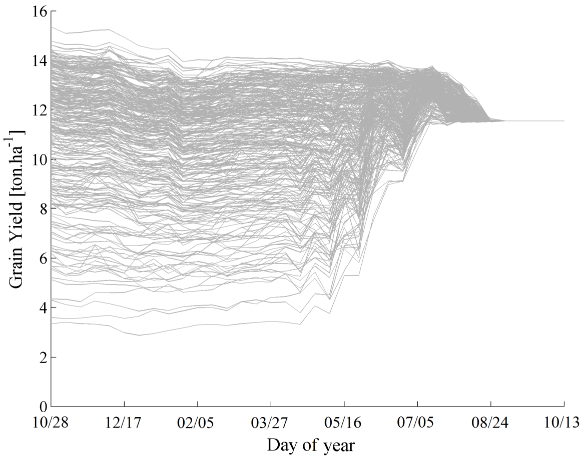

Figure 3: Variation in predicted model outputs (grey line) from 300 years of weather ensemble simulations based on a combination of synthetic and observed data for the 1981-1982 crop season

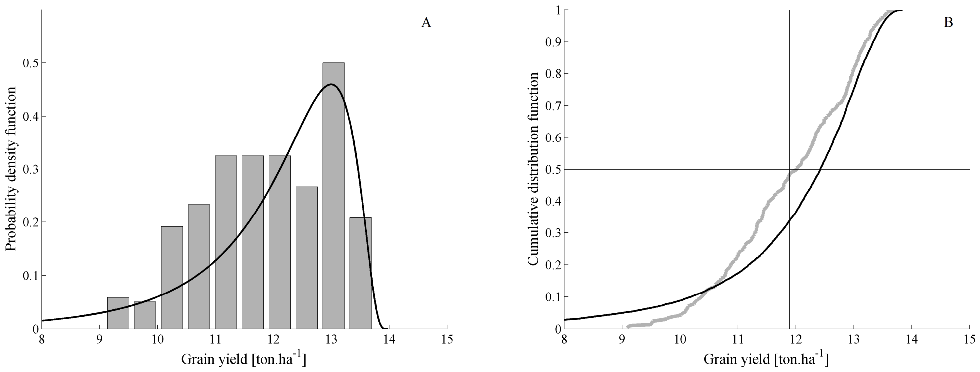

Figure 4: Probability density function (A) and cumulative distribution function (B) of the predicted yield for which observations were made up to DOY 06/25 for 1981-82. Simulated data are represented by a grey bar (A) or bold solid grey line (B) and the computed log-normal distribution is represented by a solid black line. In graph B, mean value is represented by a vertical thick black line) and the 50th percentile by a horizontal thick black line.

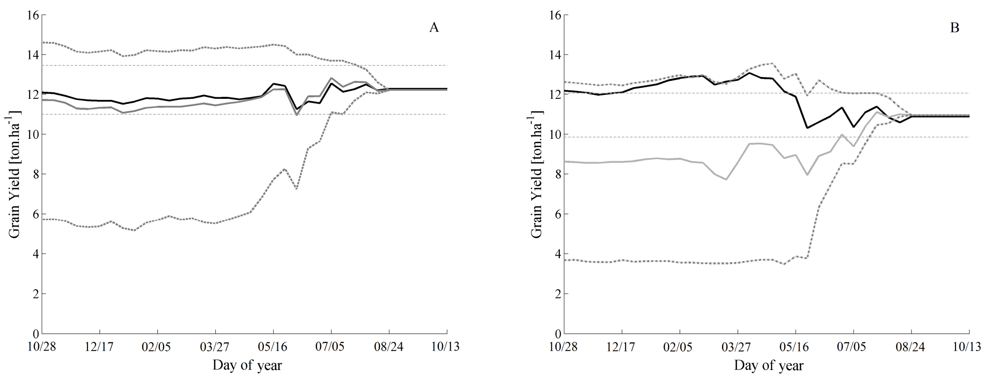

Figure 5: Variation in the predicted grain yield simulations based on a combination of synthetic and observed data using the methodology proposed by Lawless and Semenov (2005) for the 1991-92 (A) and 2007-08 (B) seasons. The solid grey line represents the mean value and the dashed grey lines represents the 2.5 and 97.5 percentiles (confidence interval at 95%). The solid black line represents the simulations obtained with the mean climate assumptions of Dumont et al. (2014b). The 10% error prediction level around the final yield simulation obtained with pure real climate is represented by a horizontal dotted light-grey line.

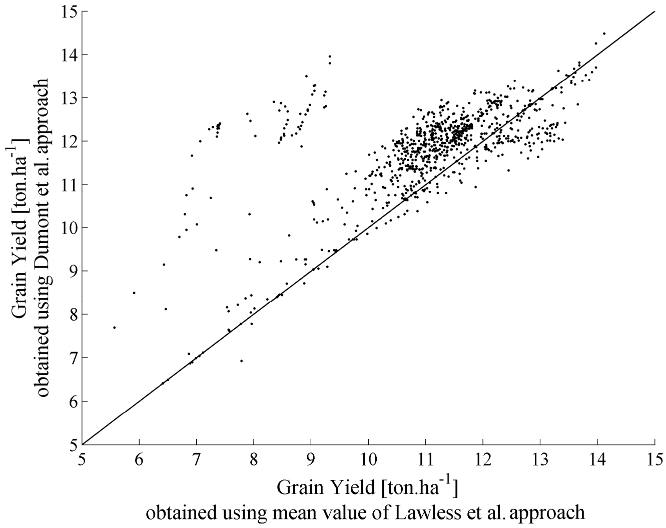

Figure 6: Graphical representation of predictive simulation output for the two assessed method.

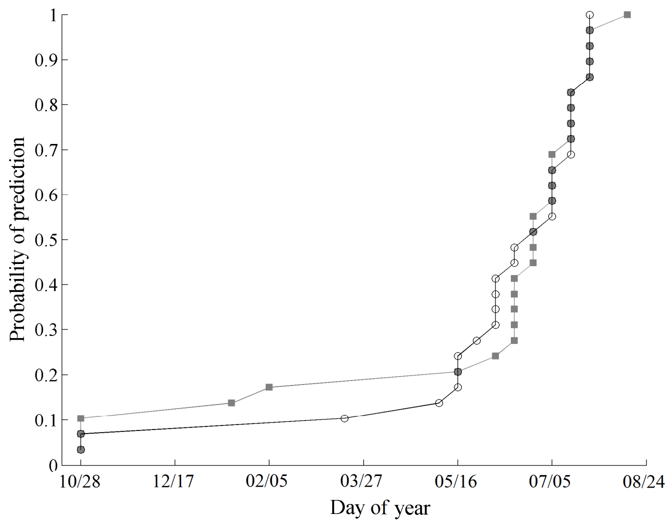

Figure 7: Graphical representation of the predictive ability, using the method of determining the first day of possible prediction, of the mean climate approach (black line with empty circles) and the mean value of 300 simulations (grey line with filled squares).

List of tables :

Table 1: Results of the Kolmogorov-Smirnov test (*p-value* of the statistical test) on the simulated end-season grain yields distributions, according to climatic year of harvest and the day of the year when observed time-series were replaced by synthetic time series.

Year DOY	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
10/28	0,75	0,74	0,64	0,74	0,80	0,67	0,64	0,56	0,72	0,65	0,82	0,76	0,77	0,81	0,82	0,64	0,88	0,84	0,39	0,65	0,71	0,68	0,67	0,46	0,67	0,27	0,95	0,89	0,54
11/07	0,79	0,77	0,56	0,79	0,86	0,82	0,78	0,68	0,92	0,73	0,85	0,91	0,87	0,78	0,63	0,72	0,93	0,78	0,56	0,72	0,79	0,62	0,57	0,52	0,59	0,12	0,98	0,86	0,49
11/17	0,80	0,90	0,80	0,83	0,82	0,75	0,71	0,75	0,85	0,70	0,80	0,91	0,84	0,72	0,72	0,64	0,86	0,70	0,78	0,82	0,65	0,61	0,63	0,50	0,68	0,21	0,98	0,74	0,48
11/27	0,80	0,91	0,64	0,74	0,78	0,74	0,78	0,62	0,81	0,71	0,88	0,96	0,90	0,75	0,95	0,76	0,83	0,62	0,71	0,79	0,67	0,80	0,44	0,59	0,45	0,21	1,00	0,62	0,47
12/07	0,80	0,93	0,76	0,83	0,93	0,64	0,84	0,77	0,80	0,81	0,85	0,88	0,97	0,73	0,87	0,76	0,58	0,62	0,88	0,74	0,82	0,89	0,51	0,81	0,72	0,27	1,00	0,81	0,44
12/17	0,66	0,95	0,88	0,77	0,93	0,55	0,86	0,89	0,93	0,67	0,53	0,84	0,98	0,77	0,88	0,86	0,53	0,67	0,66	0,62	0,85	0,90	0,50	0,42	0,63	0,33	0,99	0,94	0,45
12/27	0,65	0,92	0,82	0,80	0,89	0,69	0,83	0,84	0,91	0,86	0,57	0,91	0,93	0,84	0,88	0,67	0,68	0,80	0,78	0,57	0,66	0,91	0,13	0,66	0,41	0,30	0,98	0,78	0,38
01/06	0,58	0,89	0,81	0,78	0,89	0,60	0,82	0,95	0,77	0,78	0,72	0,90	0,85	0,59	0,83	0,49	0,80	0,76	0,79	0,59	0,48	0,62	0,21	0,71	0,64	0,24	0,99	0,72	0,60
01/16	0,64	0,86	0,81	0,77	0,88	0,44	0,78	0,79	0,73	0,83	0,79	0,93	0,67	0,56	0,84	0,64	0,76	0,84	0,37	0,55	0,75	0,82	0,71	0,88	0,51	0,35	0,99	0,59	0,71
01/26	0,66	0,66	0,56	0,77	0,74	0,51	0,58	0,90	0,93	0,81	0,69	0,88	0,77	0,60	0,76	0,78	0,25	0,71	0,51	0,76	0,71	0,59	0,73	0,79	0,44	0,38	0,98	0,58	0,58
02/05	0,85	0,71	0,78	0,78	0,69	0,63	0,52	0,81	0,91	0,84	0,69	0,71	0,84	0,58	0,72	0,78	0,34	0,84	0,42	0,74	0,48	0,83	0,77	0,65	0,23	0,38	0,96	0,51	0,45
02/15	0,80	0,72	0,46	0,74	0,70	0,51	0,66	0,71	0,94	0,89	0,70	0,90	0,82	0,51	0,77	0,70	0,40	0,88	0,38	0,48	0,86	0,87	0,39	0,80	0,24	0,39	0,94	0,60	0,62
02/25	0,71	0,66	0,35	0,66	0,77	0,29	0,52	0,59	0,94	0,74	0,76	0,84	0,64	0,46	0,80	0,59	0,48	0,75	0,22	0,39	0,57	0,80	0,07	0,95	0,17	0,33	0,83	0,48	0,45
03/07	0,91	0,73	0,50	0,61	0,73	0,22	0,65	0,66	0,79	0,55	0,61	0,84	0,59	0,51	0,88	0,62	0,36	0,62	0,37	0,70	0,34	0,76	0,13	0,78	0,16	0,31	0,58	0,45	0,45
03/17	0,81	0,58	0,26	0,76	0,63	0,34	0,47	0,83	0,67	0,63	0,24	0,79	0,82	0,55	0,83	0,49	0,70	0,88	0,60	0,73	0,89	0,77	0,21	0,38	0,16	0,35	0,91	0,44	0,39
03/27	0,84	0,48	0,68	0,69	0,66	0,18	0,69	0,87	0,86	0,73	0,42	0,97	0,87	0,33	0,82	0,35	0,51	0,76	0,45	0,67	0,88	0,60	0,03	0,17	0,04	0,28	0,79	0,71	0,42
04/06	0,76	0,38	0,37	0,68	0,55	0,25	0,33	0,89	0,81	0,63	0,59	0,60	0,82	0,36	0,55	0,47	0,66	0,83	0,34	0,79	0,99	0,69	0,05	0,40	0,16	0,58	0,68	0,91	0,39
04/16	0,65	0,48	0,35	0,65	0,62	0,22	0,22	0,78	0,91	0,50	0,32	0,54	0,63	0,33	0,85	0,62	0,22	0,93	0,33	0,58	0,93	0,56	0,04	0,25	0,36	0,43	0,46	0,49	0,44
04/26	0,50	0,30	0,12	0,42	0,41	0,31	0,34	0,67	0,70	0,77	0,49	0,50	0,83	0,40	0,75	0,66	0,77	0,71	0,41	0,66	0,89	0,27	0,03	0,40	0,57	0,47	0,56	0,95	0,38
05/06	0,64	0,31	0,43	0,33	0,57	0,15	0,52	0,60	0,40	0,78	0,64	0,34	0,66	0,33	0,49	0,43	0,69	0,45	0,28	0,43	0,39	0,71	0,30	0,89	0,12	0,13	0,24	0,66	0,44
05/16	0,18	0,13	0,11	0,55	0,18	0,06	0,20	0,57	0,34	0,49	0,21	0,13	0,70	0,56	0,40	0,23	0,59	0,45	0,48	0,34	0,52	0,58	0,02	0,43	0,08	0,30	0,00	0,66	0,38
05/26	0,09	0,11	0,02	0,17	0,14	0,07	0,30	0,31	0,32	0,42	0,14	0,07	0,34	0,21	0,12	0,37	0,15	0,17	0,07	0,38	0,12	0,28	0,01	0,02	0,04	0,07	0,10	0,73	0,09
06/05	0,27	0,07	0,01	0,09	0,31	0,09	0,40	0,69	0,63	0,09	0,49	0,41	0,50	0,50	0,23	0,60	0,19	0,33	0,56	0,06	0,69	0,32	0,00	0,54	0,42	0,02	0,06	0,21	0,06
06/15	0,01	0,01	0,00	0,02	0,08	0,00	0,05	0,32	0,13	0,03	0,45	0,58	0,19	0,27	0,01	0,25	0,08	0,38	0,00	0,07	0,08	0,87	0,00	0,11	0,07	0,07	0,00	0,05	0,11
06/25	0,02	0,02	0,00	0,01	0,08	0,00	0,00	0,02	0,18	0,00	0,06	0,10	0,01	0,55	0,30	0,00	0,16	0,00	0,00	0,10	0,00	0,01	0,00	0,29	0,00	0,00	0,00	0,01	0,00
07/05	0,33	0,00	0,00	0,64	0,00	0,14	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,37	0,13	0,24	0,44	0,00	0,02	0,07	0,00	0,00	0,00	0,00	0,00	0,00
07/15	0,01	0,01	0,00	0,50	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,54	0,00	0,00	0,00	0,00	0,44	0,00	0,00	0,00	0,00	0,00	0,00	0,21	0,00	0,00	0,00	0,00	0,00
07/25	0,02	0,39	0,20	0,00	0,00	0,00	0,02	0,00	0,00	0,00	0,00	0,70	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,33	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
08/04	0,28	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,71	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
08/14	0,79	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,21	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
08/24	0,13	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

Table 2: RRMSE and ND values computed for each crop and as an aggregated dataset in order to evaluate the equivalence of the yield prediction simulation approaches: comparison of the mean climate assumptions with the mean value of 300 simulations

Year	**RRMSE**	**ND**
1980-81	0.044	-0.034
1981-82	0.038	-0.022
1982-83	0.047	-0.032
1983-84	0.057	-0.037
1984-85	**0.102**	-0.087
1985-86	0.037	-0.022
1986-87	0.072	-0.055
1987-88	0.048	-0.032
1988-89	0.051	-0.041
1989-90	0.085	-0.068
1990-91	0.091	-0.075
1991-92	0.026	-0.015
1992-93	0.077	-0.064
1993-94	0.082	-0.062
1994-95	0.078	-0.058
1995-96	0.079	-0.061
1996-97	**0.101**	-0.082
1997-98	0.041	-0.033
1998-99	0.079	-0.063
1999-00	0.040	-0.032
2000-01	0.061	-0.056
2001-02	0.080	-0.058
2002-03	0.058	+0.049
2003-04	0.041	+0.033
2004-05	0.062	+0.051
2005-06	**0.492**	**-0.393**
2006-07	**0.354**	**-0.281**
2007-08	**0.326**	**-0.264**
2008-09	0.038	-0.029
Overall	**0.112**	-0.058