Regularized focusing inversion of time-lapse electrical resistivity data: an approach to parametrize the minimum gradient support functional
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## 1. Introduction

Inversion of time-lapse resistivity data allows obtaining 'snapshots' of changes occurring in monitored systems for applications such as tracer tests or geothermal heat exchange. Based on these snapshots, one can infer qualitative information on the location and morphology of changes, but also quantitative estimates on the degree of changes in temperature or TDS content [1]

Analysis of these changes can provide direct insight into flow and transport and associated processes. However, the reliability of the analysis is dependent on survey design data error, and regularization. Regularization may be chosen depending on available information collected during the monitoring. Common approaches consider smoothing model changes both in space and time but it is often needed to obtain a sharp temporal anomaly, for example in fractured aquifers.

## 2. Formulation of the problem

We have implemented a time-lapse inversion scheme using the minimum gradient support functional as regularization operator in a difference inversion scheme [2]. This approach limits the occurrences of changes in the model [3]. The model functional is expressed as

$$
\psi_{\mathrm{m}, \mathrm{MGS}}=\int \frac{\nabla(\Delta \mathbf{m}) \cdot \nabla(\mathbf{\Delta} \mathbf{m})}{\nabla(\Delta \mathbf{m}) \cdot \nabla(\Delta \mathbf{m})+\beta^{2}} \mathrm{dv}
$$

$-\Delta \mathrm{m}$ is the parameter change (resistivity)
$-\beta$ is an additional parameter to stabilize the functional for small values of $\Delta \mathrm{m}$

We propose here to optimize $\beta$ by considering a univariate line search at the first iteration to find the value of $\beta$ that minimizes the data misfit. The parameter is then kept constant during the Gauss-Newton iterative scheme.
3. Numerical model

Fig 1. (A) Background model (B) Time-lapse model (C) Background inversion (D) Smoothness constraint time-lapse inversion (E) Optimization of $\beta$ ( F to K ) MGS time-lapse inversion with various $\beta$


The numerical model represents a sharp time-lapse contrast inside fractured limestones (A and B). We tested various value of $\beta$ ( F to K ) in the MGS inversion.

The value giving the minimum data misfit after 1 iteration (E) yields the time-lapse changes that are closer to the reference (H).

The optimum value of $\beta$ also leads to the image with the smaller amount of artifacts of inversion.

The magnitude of the recovered anomaly is underestimated but is one order of magnitude above the one recovered with the smoothness constraint inversion.

The position and shape of the anomaly are much better recovered.

## 4. Field Application

Fig 2. (A) Background inversion (B) Smoothness constraint time-lapse inversion (C) Optimization of $\beta$ ( D to H) MGS time-lapse inversion with various $\beta$ (I) Field survey (J) Geological conditions

We applied the methodology on a salt tracer experiment carried out in fractured limestones (J) [4].

The 72 electrodes ( 3 m spacing) ERT profile was located 15 m downgradient of the injection well (I) and the error was estimated using reciprocal measurements.

The inversion with smoothness constraint inversion (B) displays a large negative anomaly (-6\%). The diffuse nature of the anomaly is likely the result of the regularization.

We tested several values of $\beta$ for the MGS inversion ( D to H ). For large $\beta$, the anomaly is similar to the smoothness constraint solution. For very small $\beta$, the inversion fails to converge. The value of $\beta$ yielding the minimum of the data misfit after the first iteration (C) provides the results (G) with a sharp and focused anomaly as can be expected from the geology (J).


## 5. Conclusion

- We propose an optimization procedure to select the value of $\beta$ in MGS inversion
- Through a line search, we select the $\beta$ which minimizes the data misfit of the first iteration of the inversion procedure - This optimization is robust, it does not require prior information and relies on the data only
- The numerical benchmark validates the methodology for a challenging target
- The similitude of the synthetic and field cases provides increased confidence in the results obteained with the MGS inversion compared to the smoothness constraint inversion


## References

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