Implicit implementation of the Prevost model

(Cyclic loading of soils)

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Introduction

The **Prevost model** is dedicated to the modelling of the cyclic behaviour of cohesionless soils. It consists of conical nested surfaces, allowing plasticity effects in both loading and unloading. The volumetric hardening-rule is non associated and takes into account the phase transformation line, delineating contractive from dilative plastic behaviours.

The **implementation** of a model into the finite element code LAGAMINE is carried out. This crucial step transforms an analytical constitutive law into its discrete counterpart. The resulting algorithm must be accurate, efficient and robust. A closest-point projection algotihm is adopted to solve the set of non-linear equations. This method lies within the framework of return-mapping algorithms. It is **implicit**, i.e. the return direction is not known *a priori*. An iterative local procedure is then reuired. The Prager hardening rule is employed to describe the hardening of the yield surface.

Results presented ensure the implemented algorithm corresponds to the analytical model. Triaxial extensive and compressive tests are illustrated.

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Definitions

Cauchy effective stress tensor $\boldsymbol{\sigma}'$

Mean effective stress $\mathrm{p}' = (1/3)\cdot\boldsymbol{\sigma}':\boldsymbol{\delta}$

Deviatoric stress tensor $\mathbf{s} = \boldsymbol{\sigma}' - \mathrm{p}'\cdot\boldsymbol{\delta}$

Invariant of deviatoric stress $\mathrm{q} = \sqrt{(3/2)\cdot\mathbf{s}:\mathbf{s}}$

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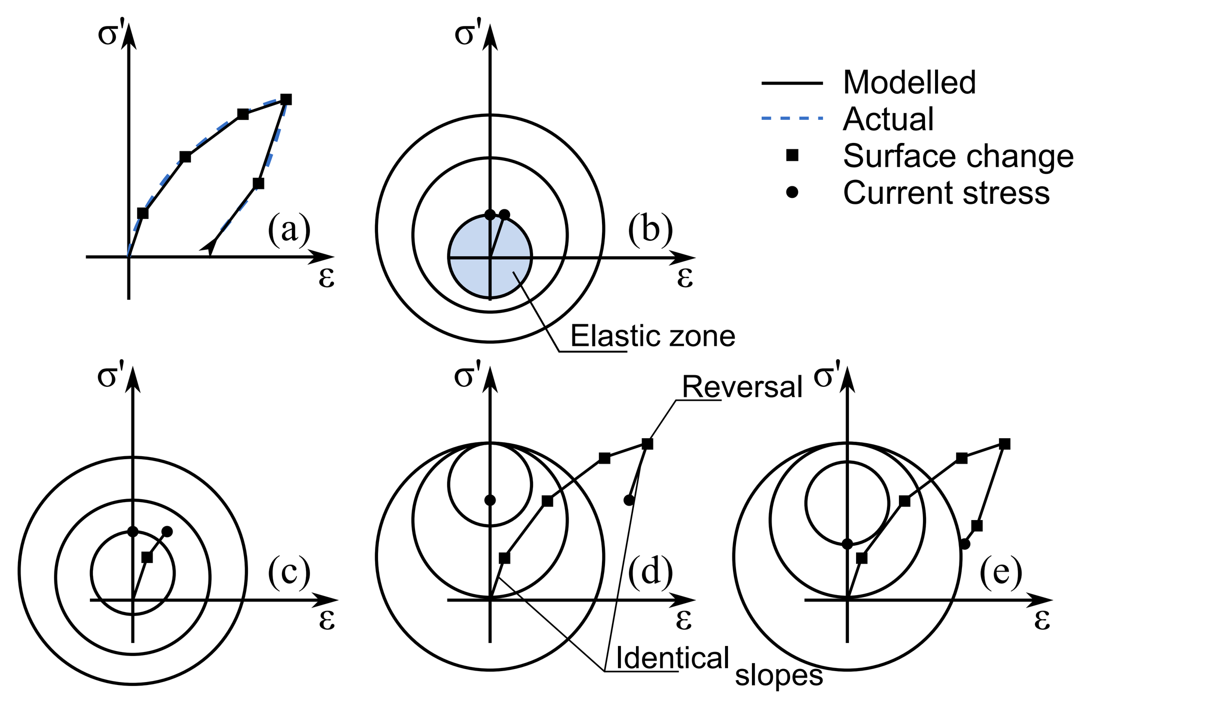
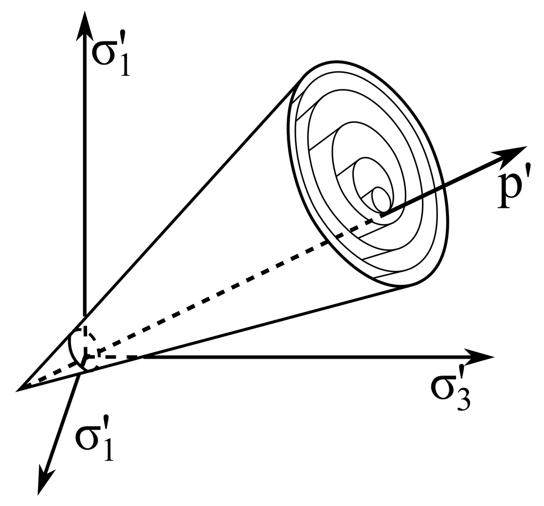
Prevost model

Multi-surface concept consists in discretising the plastic modulus by a discrete number of hardening surfaces.





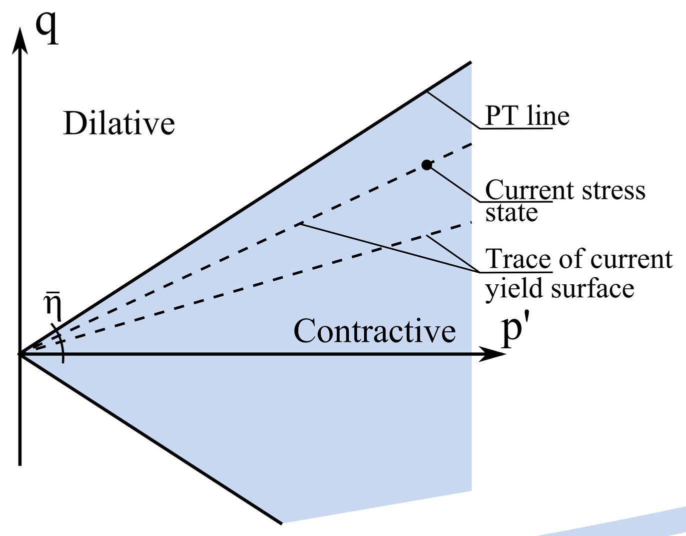
Each homothetic surface is characterised by the backstress tensor defining its centre $\boldsymbol{\alpha}$ and its opening M (Prevost 1985).

$$f \equiv (\mathbf{s} - \mathrm{p}'\cdot\boldsymbol{\alpha}):(\mathbf{s} - \mathrm{p}'\cdot\boldsymbol{\alpha}) - (\mathrm{p}'\cdot M)^2 = 0$$

Stress-states lying under the **P**hase **T**ransformation **line** ($\eta = \mathrm{q}/\mathrm{p}' < \bar{\eta}$) have a contractive volumetric plastic behaviour, i.e.

$$\dot{\epsilon}_v^p = \underbrace{\frac{1}{3}\cdot\frac{\eta^2 - \bar{\eta}^2}{\eta^2 + \bar{\eta}^2}}_{=P"}\cdot\dot{\lambda} < 0$$

where $\dot{\lambda}$ is the continuous plastic multiplier.



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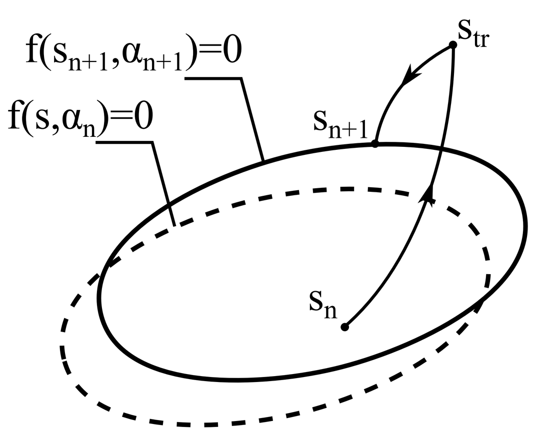
Implicit implementation



The algorithm is based on a 2-step **return mapping**

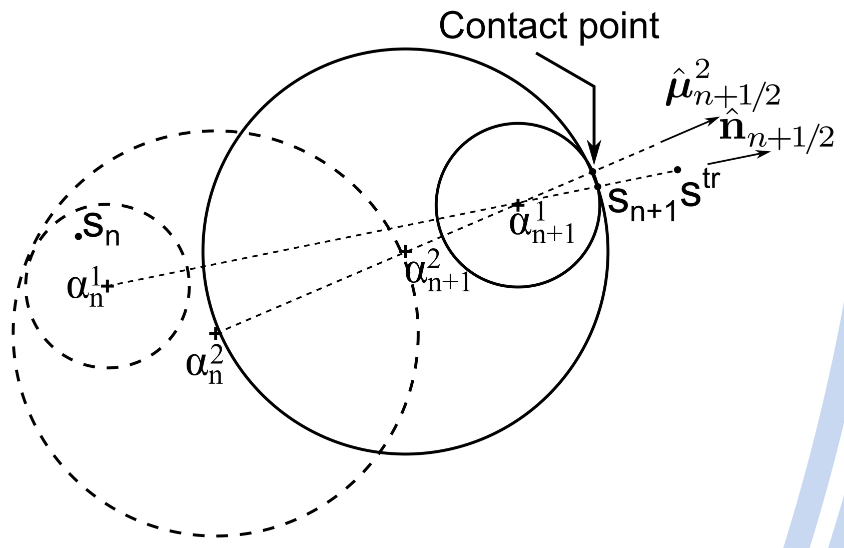
$$\Delta\boldsymbol{\sigma} = \underbrace{\mathbb{E}:\Delta\boldsymbol{\epsilon}}_{\text{Elastic predictor}} - \underbrace{\mathbb{E}:\mathbf{P}_{n+1/2}\cdot\Delta\lambda_{n+1}}_{\text{Plastic corrector}}$$

Elastic predictor $\mathbf{s}_n \rightarrow \mathbf{s}_{tr}$ Plastic corrector $\mathbf{s}_{tr} \rightarrow \mathbf{s}_{n+1}$

The implicit **Prager rule** describes the evolution of the yield surface, i.e. the evolution of its backstress (Montans 2001)

$$\Delta\boldsymbol{\alpha}^1 = \Delta\lambda^1_{n+1}\cdot \mathrm{H}^*_{1,n+1}\cdot\hat{\mathbf{n}}_{n+1/2}$$

and other surfaces are translated accordingly along $\hat{\boldsymbol{\mu}}^i_{n+1/2}$



The closest point projection algorithm consits in solving the set of equations corresponding to four unknowns (Mira 2009)

$$r_1 = \frac{H^*_{1,n+1}}{\bar{H}^1}\cdot\|\mathbf{Q}'_{n+1/2}\| - 1$$

$$r_2 = (\mathbf{s}^{tr}_{n+1} - \mathrm{p}'_{n+1}\cdot\boldsymbol{\alpha}^1_n):\hat{\boldsymbol{n}}_{n+1/2} - \sqrt{\frac{2}{3}}\cdot\mathrm{p}'_{n+1}\cdot M^1 - 2\cdot\mathrm{G}\cdot\Delta\lambda_{n+1}\cdot\|\mathbf{Q}'_{n+1/2}\| - \Delta\lambda^1_{n+1}\cdot H^*_{1,n+1}$$

$$r_3 = \mathrm{p}'_{n+1} - \mathrm{p}'_{tr} + 3\cdot B\cdot\Delta\lambda_{n+1}\cdot P"_{n+1/2}$$

$$r_4 = \|\mathbf{Q}'_{n+1/2}\| - 2\cdot\frac{\|\mathbf{s}_{n+1} - \mathrm{p}'_{n+1}\cdot\boldsymbol{\alpha}^1_{n+1}\|}{\left\|\frac{\partial f}{\partial \sigma}\right\|}$$

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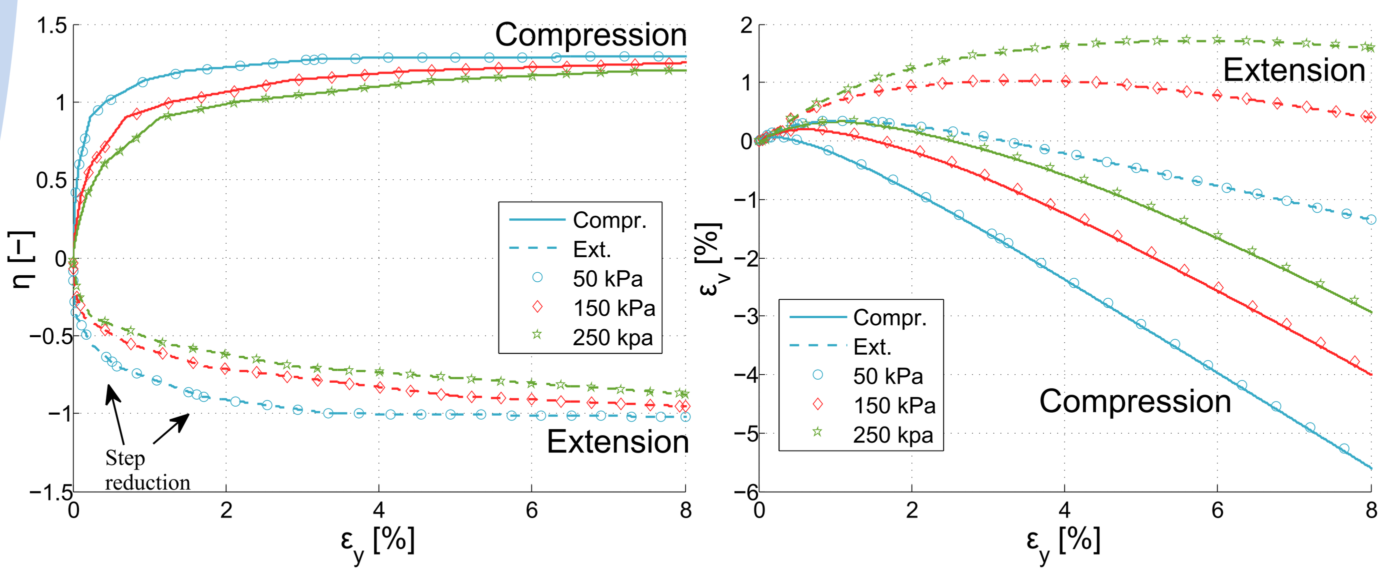
Results

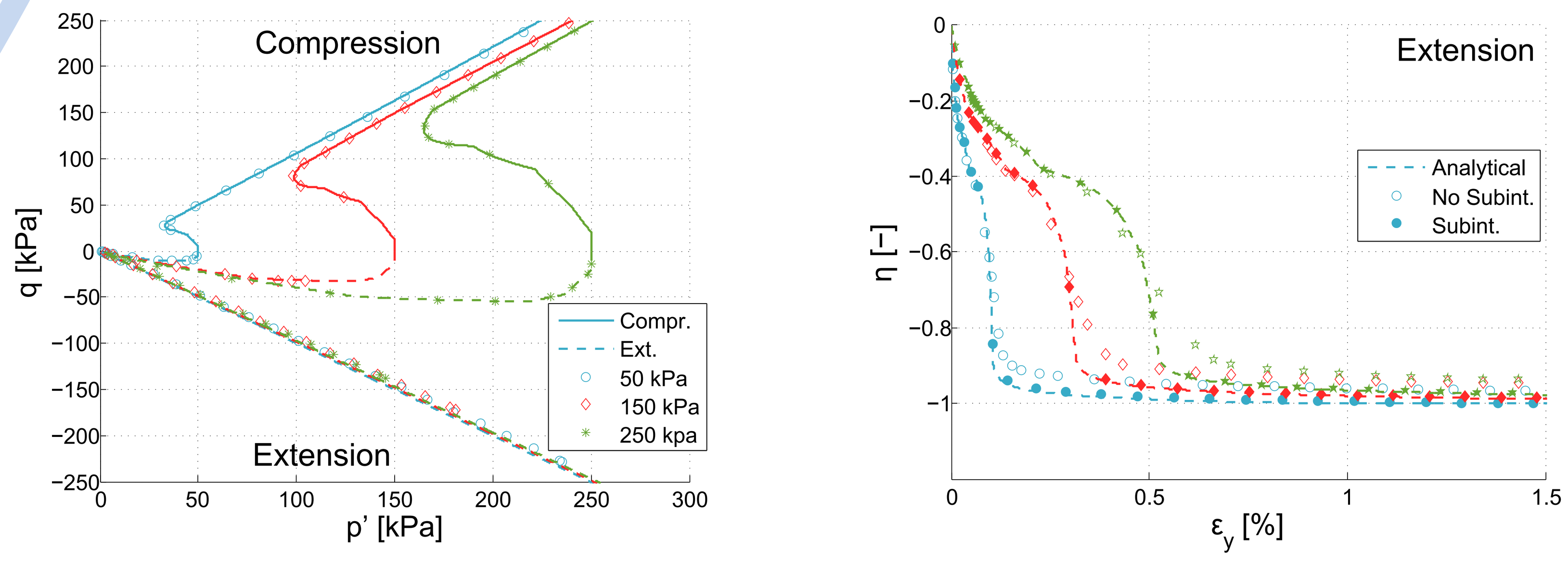
Analytical (solid or dashed lines) and implicit (markers) numerical simulations of triaxial tests are compared

Drained triaxial tests



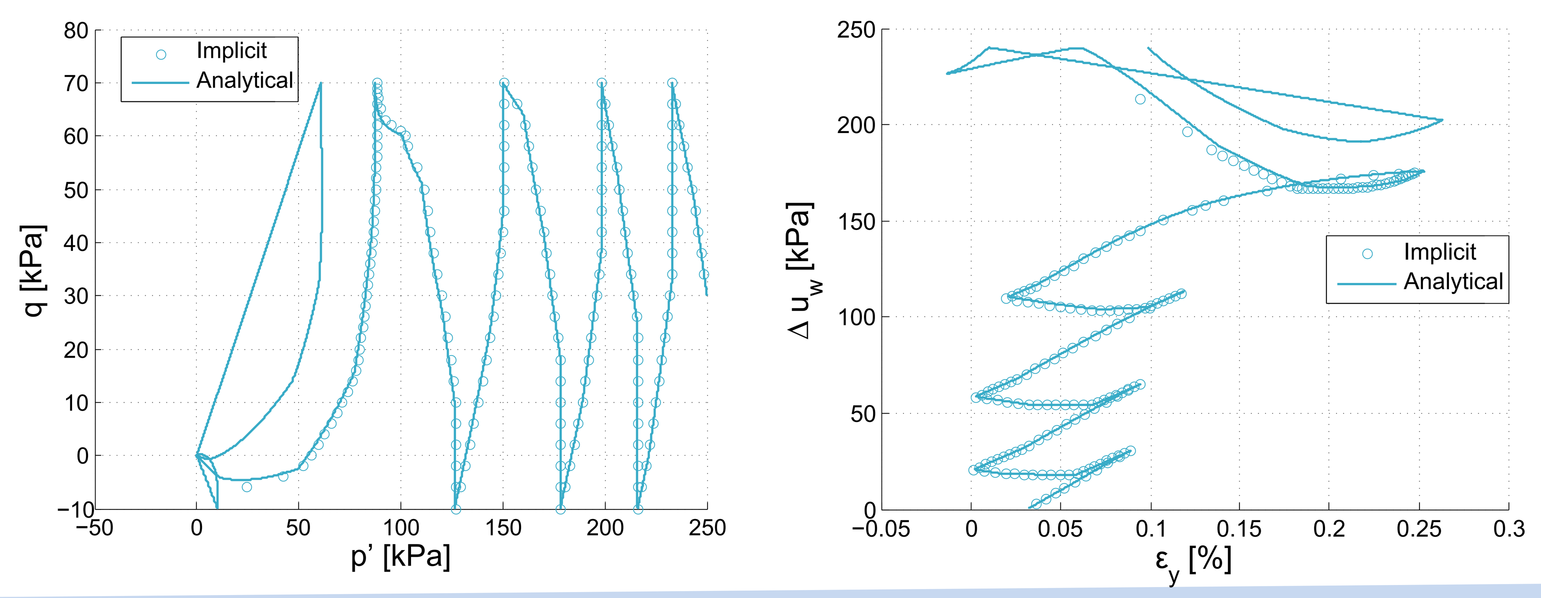
Undrained triaxial tests

Extensive simulations suffer a large variation of η over a small range of ε_y. Sub-stepping is required to ensure an accurate integration



Cyclic undrained tests

Implicit scheme correctly captures the progressive accumulation or pore water pressure along the test.



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Montans, F.J. (2001). Implicit multilayer J2 plasticity using Prager's translation rule. *International Journal for Numerical Methods in Engineering*, 50, 347-375

Mira, P., Tonni, L., Pastor, M., Fernandez-Merodo,J.A. (2009). A generalized mid-point algorithm for the integration of a generalized plasticity model for sands. *International Journal for Numerical Methods in Engineering*, 77, 1201-1223

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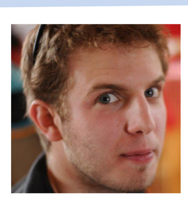

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