1986A&A...170..167S

Astron. Astrophys. 170, 167–173 (1986)

ASTRONOMY
AND
ASTROPHYSICS

Photoelectric photometry of 22 Kalliope during the 1985 opposition and determination of its pole orientation: the "magnitude-aspect" relations revisited*

J. Surdej[1,**], A. Pospieszalska-Surdej[1], T. Michalowski[2], and H.J. Schober[3]

[1] Institut d'Astrophysique, Université de Liège, Avenue de Cointe 5, B-4200 Cointe-Ougrée, Belgium
[2] Obserwatorium Astronomiczne UAM, ul. Stoneczna, 36, PL-60-286 Poznań, Poland
[3] Institut für Astronomie, Universitätsplatz 5, A-8010 Graz, Austria

Received April 25, accepted July 22, 1986

Summary. Using photoelectric photometry of 22 Kalliope from nine previous oppositions as well as a photometric lightcurve recorded with the ESO 50 cm telescope during the 1985 opposition, we have derived a unique solution for the pole orientation of this minor planet: $\lambda_0 = 203° \pm 2°$, $\beta_0 = 29° \pm 5°$, $a/b = 1.33 \pm 0.01$, and $b/c = 1.24 \pm 0.08$, on the basis of the revisited "amplitude-aspect" relation. Furthermore, we show that whenever a minimum of three accurately calibrated photometric lightcurves of an asteroid are available at suitable oppositions, it is possible to make use of "magnitude-aspect" relations in order to determine the ecliptic coordinates (λ_0, β_0) of the rotation axis, the absolute dimension of the three semi-axes (a, b, c) of the ellipsoid which suits at best the observed asteroid lightcurves as well as the Bowell and Lumme multiple-scattering factor Q. Considering only four of the best published lightcurves of 22 Kalliope, we find the following results: $\lambda_0 = 201° \pm 2°$, $\beta_0 = 22° \pm 6°$, $a = 82 \pm 4$ km, $b = 62 \pm 3$ km, $c = 55 \pm 2$ km and $Q = 0.164 \pm 0.009$ leading to a value of $\beta = 0.028 \pm 0.001$ mag deg^{-1} for the phase coefficient. This technique is only seriously limited by the accuracy of the published photometric zero points as well as by the symmetry of the asteroid lightcurve and it should therefore always be applied with much care.

Key words: asteroids – photometry – solar system: general – pole orientation – 22 Kalliope

1. Introduction

Between 1953 and 1983, nine sets of photoelectric lightcurves of the minor planet 22 Kalliope have been obtained at distinct oppositions (see references in Table 2). From these photometric data, we know that 22 Kalliope is an M-type asteroid having colors $B-V = 0.71$ and $U-B = 0.28$ mag (Gehrels and Owings, 1962), a phase coefficient $\beta = 0.031 \pm 0.001$ mag deg^{-1} and a synodic rotation period $P = 4^h08^m52^s \pm 1^s$ (Scaltriti et al., 1978). The most recent pole determination has been quoted by Zappalà and Knežević (1984) who derived two possible solutions:

$$P_1: \lambda_0 = 13° \pm 4°, \quad \beta_0 = 17° \pm 7°$$

and $$P_2: \lambda_0 = 214° \pm 7°, \quad \beta_0 = 42° \pm 5°,$$

with the following semi-axes ratios $a/b = 1.34$ and $b/c = 1.18$. Taking the value $p_v = 0.123$ for the geometrical albedo given by Chapman et al. (1975), Scaltriti et al. (1978) have inferred the dimensions $215 \times 160 \times 130$ km^3 for the ellipsoid which accounts at best for the observed light variations of 22 Kalliope. A radiometric diameter of 165 km and 175 km has been inferred for this minor planet by Hansen (1977) and Bender et al. (1978), respectively.

Applying the revisited amplitude-aspect relation (Pospieszalska-Surdej and Surdej, 1985, Paper I) to the previous sets of photometric lightcurves, we have determined a unique solution for the rotation axis of 22 Kalliope: $\lambda_0 = 204° \pm 2°$, $\beta_0 = 31° \pm 6°$, $a/b = 1.34 \pm 0.02$, and $b/c = 1.30 \pm 0.12$, from which it was straightforward to predict the full amplitude Δm [cf. Eq. (I.10), i.e. Eq. (10) in Paper I] of the ellipsoidal lightcurve that would be observed around mid-august 1985: $\Delta m = 0.152$ mag. The full amplitude of the lightcurve observed by H.J.S. on August 17, 1985 with the European Southern Observatory (ESO) 50-cm telescope (see Sect. 2) has been derived to be $\Delta m = 0.154$ mag, in excellent agreement with the predicted value. This new lightcurve of 22 Kalliope has been combined with all previous ones in order to compute, by means of the revisited amplitude-aspect relation, a better pole solution (see Sect. 3). In Sect. 4, we establish the two magnitude-aspect equations which allow ones to derive a pole solution (λ_0, β_0, a/b, b/c) as well as the absolute dimensions $(a \times b \times c)$ of the fitted ellipsoid and the Bowell and Lumme multiple scattering factor Q on the basis of a minimum of three accurately calibrated photometric lightcurves observed at suitable oppositions. This method is applied to the case of 22 Kalliope in Sect. 5. Finally, a discussion and conclusions form the last section.

2. Observations of 22 Kalliope during the 1985 opposition

Following standard observing procedures (cf. Surdej and Schober, 1980), H.J.S. has observed photometrically the minor planet

Send offprint requests to: J. Surdej

* Based on observations collected at the European Southern Observatory, La Silla, Chile
** Also, chercheur qualifié au Fonds National de la Recherche Scientifique, Belgium

© European Southern Observatory • Provided by the NASA Astrophysics Data System

1986A&A...170..167S

Table 1. Aspect and photometric data of 22 Kalliope and its comparison star observed on August 17, 1985

Object	R.A.(1950.0)	Decl.(1950.0)	r(A.U.)	Δ(A.U.)	V	B−V	U−B	n° of meas.
22 Kalliope	$22^h14^m23^s$	−32°42'49"	1.9461	2.9169	see Fig.1	0.71±0.01	0.26±0.01	67
CD−33°15992	-	-	-	-	9.61±0.01	1.01±0.01	0.81±0.01	19

Table 2. Date, reference, ecliptic coordinates, phase angle, and characteristics of the $x-y$ relation for different light curves of the minor planet 22 Kalliope (see text)

Date of observations	Reference	λ (1950.0)	β (1950.0)	α	t_M	B	C	D	R
15 February 1953	Fig.9(A)	151°.73	+20°.51	7°.1	$10^h.524$	———	———	2.352(−1)±0.107(−1)	0.936
24 February 1958	Fig.11(GO)	153.47	+20.59	6.9	7.828	4.713(−7)±0.173(−7)	2.358(−6)±0.011(−6)	1.999(−1)±0.082(−1)	0.949
4 September 1965	Fig.2(ZH)	304.01	−18.15	13.0	21.511	6.607(−7)±0.261(−7)	1.106(−6)±0.016(−6)	5.972(−1)±0.324(−1)	0.967
12 June 1974	Fig.3(ZH)	236.21	+ 0.74	7.7	22.014	4.528(−7)±0.397(−7)	2.402(−6)±0.025(−6)	1.885(−1)±0.184(−1)	0.845
19 November 1976	(Sal)	64.20	− 1.72	2.6	23.930	———	———	2.068(−1)±0.067(−1)	0.886
25 November 1976	(Sal)	62.87	− 1.18	0.5	20.996	11.926(−7)±0.327(−7)	5.096(−6)±0.021(−6)	2.340(−1)±0.074(−1)	0.895
30 November 1981	Fig.1(BD)	67.38	+ 0.64	0.4	21.163	———	———	2.678(−1)±0.147(−1)	0.922
1 December 1981	Fig.6(Zal)	67.19	+ 0.71	0.7	13.606	———	———	2.577(−1)±0.174(−1)	0.902
7 March 1983	Fig.3(DC)	171.37	+19.82	6.7	25.200	2.313(−7)±0.174(−7)	2.958(−6)±0.009(−6)	0.782(−1)±0.061(−1)	0.744
17 August 1985	Fig.1(PW)	323.42	−20.30	6.9	2.879	7.460(−7)±0.211(−7)	2.273(−6)±0.013(−6)	3.282(−1)±0.111(−1)	0.974

A : Ahmad, 1954
GO : Gehrels and Owings, 1962
ZH : Zappalà and van Houten-Groeneveld, 1979
Sal : Scaltriti et al., 1978
BD : Barucci and Dipaoloantonio, 1983
Zal : Zhou et al., 1983
DC : Di Martino and Cacciatori, 1984
PW : present work

NOTE : The sign of the ordinates in Fig.3 of DC should be reversed.

© European Southern Observatory • Provided by the NASA Astrophysics Data System

1986A&A...170..167S

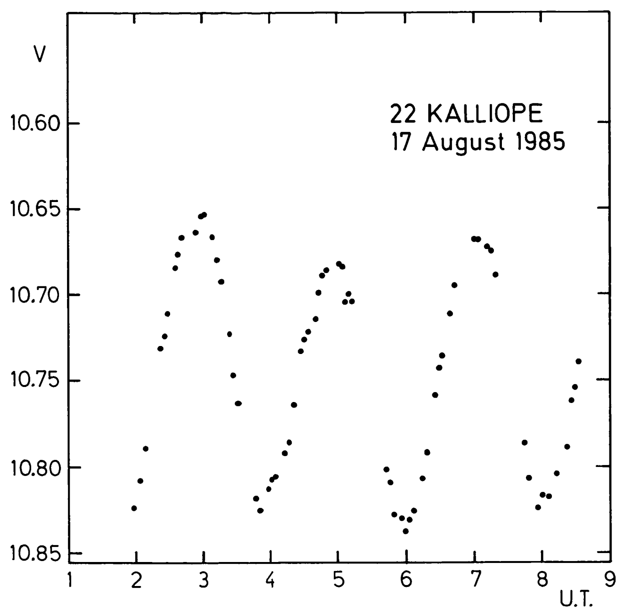

Fig. 1. V photoelectric lightcurve of 22 Kalliope observed with the ESO 50 cm telescope on 17 August, 1985 (see text)

22 Kalliope on August 17, 1985 with the 50-cm telescope of ESO (La Silla, Chile). A single-channel photometer equipped with an EMI 6256 photomultiplier, Schott standard filters for the UBV magnitudes and a dry-ice cooling system was used for the photoelectric measurements. Light was collected within a 19″ diaphragm, using a basic integration time of one second, repeated usually 20–40 times until an internal accuracy of 0.4% was reached for the average count rate. The data were reduced to the standard UBV system taking into account the first and second order extinction as well as a linear color transformation. Aspect and photometric data referring to 22 Kalliope and the observed comparison star are summarized in Tables 1 and 2. Figure 1 displays the V photometric measurements of 22 Kalliope on August 17, 1985. The lightcurve has not been corrected for the phase and distance effects and the abscissae are UT without correction for light time. The $B-V$ and $U-B$ color indices of 22 Kalliope (see Table 1) are similar to those published by Gehrels and Owings (1962) and did not show any significant trend of variation with the rotation phase. The synodic rotation period measured on August 17, 1985 is found to be $4^h06^m \pm 4^m$.

3. The pole orientation of 22 Kalliope based on the revisited "amplitude-aspect" relation

We refer the reader to Paper I for a full description of the revisited "amplitude-aspect" method used for deriving the pole orientation of a minor planet. Adopting a synodic rotation period $P = 4^h08^m52^s$ for 22 Kalliope (Scaltriti et al., 1978), we have first computed the quantity $D_i = B_i/C_i$ for each of the ten lightcurves ($i = 1-10$) presented in Table 2. We recall that D_i is related to the slope of the $x-y$ relation, where $y = 10(-0.8\,V)$ and $x = \cos^2(\psi)$, V being the magnitude of a photometric measurement recorded at the rotation phase ψ. Column 6 of Table 2 lists the derived epochs of the light maxima and column 10 contains the Pearson product-moment factors R obtained for each of the $x-y$ least-squares linear fits.

Table 3. The pole solutions of 22 Kalliope based upon the revisited amplitude-aspect relation

1) Unweighted calculations

λ_0	β_0	a/b	b/c
204°1±2°2	30°5±6°8	1.33±0.02	1.29±0.12

2) Weighted calculations (see text)

λ_0	β_0	a/b	b/c
203°1±1°8	28°5±5°4	1.33±0.01	1.24±0.08

By means of the FIT numerical program (see MIDAS, 1984) adapted to our purposes and considering Eqs. (I.11) and (I.12), we have then solved a system of ten non-linear equations for which the ecliptic coordinates λ_i, β_i of 22 Kalliope are given in Table 2. We find that the only set of sign ($\cos(A_i)$), A_i being the aspect angle for lightcurve No. i, for which a solution can be found is $+, +, -, +, -, -, -, -, +, -$ ($i = 1-10$). It is probably because the minor planet has covered a wide range of ecliptic latitudes ($\beta_i \in [-20°, 20°]$) that a unique pole solution is found. The results of these unweighted calculations as well as the relevant covariance matrix are reported in Tables 3 and 4, respectively.

Noticing that only six distinct intervals of ecliptic longitudes have been suitably covered along the trajectory of the asteroid, we assign the following weights: 1/12, 1/12, 1/6, 1/6, 1/24, 1/24, 1/24, 1/24, 1/6, and 1/6 to each of the photometric lightcurves in Table 2 and have repeated the calculations of the pole orientation (see Tables 3 and 4). Although there are no significant differences between the unweighted and weighted solutions, we adopt the latter as the most representative one. Let us remark that a very slight correlation between the λ_0 and β_0 and, β_0 and b/c parameters is apparent in Table 4. It is likely that a photoelectric lightcurve of 22 Kalliope observed near its pole longitude $\lambda \sim 203°$ would help very much in suppressing these correlations.

4. The revisited "magnitude-aspect" relations

Whereas the classical "magnitude-aspect" equations only relate the variations of the maximum and/or minimum brightness of an asteroid as a function of the pole position (cf. Sather, 1976; Zappalà, 1981), we show in the present section that it is possible to take into account the "full information" of a lightcurve when expressing the light changes of a minor planet along its trajectory with respect to the ecliptic coordinates of its rotation axis.

Within the framework of an ellipsoid model and referring to Eqs. (I.3)–(I.8), the quantities B and C may be written in the form

$$B = (\pi abc)^2\,(1-\cos^2(A))\left(\left(\frac{1}{b}\right)^2 - \left(\frac{1}{a}\right)^2\right), \qquad (1)$$

and

$$C = (\pi abc)^2\left(\frac{\cos^2(A)}{c^2} + \frac{(1-\cos^2(A))}{a^2}\right), \qquad (2)$$

$a > b > c$ representing the semi-axes of the ellipsoid and A, the angle between the rotation axis and the line-of-sight (aspect).

© European Southern Observatory • Provided by the NASA Astrophysics Data System

1986A&A...170..167S

Table 4. Correlation factors between the parameters $\lambda_0, \beta_0, a/b$ and b/c (revisited amplitude-aspect relation)

1) Unweighted calculations : covariance matrix

	λ_0	β_0	a/b
β_0	0.962		
a/b	0.426	0.548	
b/c	0.853	0.927	0.783

2) Weighted calculations (see text) : covariance matrix

	λ_0	β_0	a/b
β_0	0.946		
a/b	0.371	0.539	
b/c	0.841	0.918	0.760

In terms of the $V(0°)$ magnitude of a measurement made at unit distances ($r = \Delta = 1$ AU), zero phase angle ($\alpha = 0°$)− and implicitly rotation phase ψ−, Eq. (I.8) transforms into

$$10^{-0.8\,V(0°)} = Bx + C. \tag{3}$$

From the multiple-scattering theory of light in atmosphereless bodies (Bowell and Lumme, 1979; Lumme and Bowell, 1981a, b) we know that the $V(1,\alpha)$ magnitude of a measurement performed at a phase angle α may be related to $V(0°)$ via the following equation:

$$10^{-0.8V(0°)} = 10^{-0.8V(1,\alpha)} g^2(Q,\alpha), \tag{4}$$

with

$$g(Q,\alpha) = 1/(Q + (1-Q)\ 10^{-0.4V_1(\alpha)}), \tag{5}$$

and where for $0° \leqq \alpha \leqq 30°$

$$V_1(\alpha) = 0.067\,\alpha^{0.785} + \alpha/(1.36\alpha + 14.73), \tag{6}$$

Q being the multiple scattering factor which fully characterizes the shape – including the opposition effect – of phase curves.

Since photometric observations can just be reduced to $V(1,\alpha)$ measurements, it is more convenient to rewrite Eq. (1) in the form

$$10^{-0.8V(1,\alpha)} = Bx + C, \tag{7}$$

with

$$B = \frac{L}{g^2(Q,\alpha)} \cdot \left(\left(\frac{a}{b}\right)^2 - 1\right)(1 - \cos^2(A)), \tag{8}$$

and

$$C = \frac{L}{g^2(Q,\alpha)}\left(1 + \cos^2(A)\left(\left(\frac{a}{c}\right)^2 - 1\right)\right), \tag{9}$$

with

$$L = (\pi bc)^2. \tag{10}$$

Inserting Eqs. (8) and (9) into the well known relation between the aspect angle A_i, the ecliptic coordinates λ_i, β_i and λ_0, β_0, we obtain the two "magnitude-aspect" equations which should enable one to determine the pole solution, i.e. $\lambda_0, \beta_0, a/b, b/c, L$, and Q, as a function of the observed quantities $\lambda_i, \beta_i, \alpha_i, B_i, C_i$ and sign $(\cos(A_i))$:

$$\sin(\beta_i)\sin(\beta_0) + \cos(\beta_i)\cos(\beta_0)\cos(\lambda_i - \lambda_0) + \mathrm{sign}(\cos(A_i)) \sqrt{1 - \frac{B_i g^2(Q,\alpha_i)}{\left(\left(\frac{a}{b}\right)^2 - 1\right)L}} = 0, \tag{11}$$

and

$$\sin(\beta_i)\sin(\beta_0) + \cos(\beta_i)\cos(\beta_0)\cos(\lambda_i - \lambda_0) + \mathrm{sign}(\cos(A_i)) \sqrt{\left(\frac{C_i g^2(Q,\alpha)}{L} - 1\right) \Big/ \left(\left(\frac{a}{b}\right)^2\left(\frac{b}{c}\right)^2 - 1\right)} = 0. \tag{12}$$

Let us note that any linear combination of Eqs. (11) and (12) altogether with Eq. (I.12) would also be adequate in order to solve our problem.

Since two "magnitude-aspect" equations are available for each single lightcurve, we only need "in principle" three photometric lightcurves of an asteroid observed at suitable ecliptic longitudes in order to derive the six unknown parameters $\lambda_0, \beta_0, a/b, b/c, L$, and Q. We immediately stress that very accurately calibrated photometric lightcurves are necessary in order to calculate a consistent pole orientation. In this context, we feel that a homogeneous *UBV* photometric monitoring of previously observed comparison stars of minor planets would constitute a very rewarding observing programme for checking the accuracy of the photometric zero points of published asteroid lightcurves used for the determination of a pole orientation.

5. The pole orientation of 22 Kalliope based on the revisited "magnitude-aspect" relations

For only six of the ten photoelectric lightcurves reported in Table 2 it was possible to find or to derive the photometric zero points. For these, the values of the B_i and C_i quantities appearing in Eq. (7) are listed in columns 7 and 8 of Table 2. Adapting the FIT numerical program (see MIDAS, 1984) to the case of Eqs. (11) and (12), we have computed the pole solutions of 22 Kalliope for various combinations of the available oppositions (see Tables 5 and 6). Considering first the calculations performed for the six lightcurves observed at distinct oppositions we find that the $O-C$ deviations calculated from Eq. (12) are about five times as large for the 1958 and 1985 oppositions as for the 1965, 1974, 1976, and 1983 oppositions. These abnormally high observed deviations can be directly accounted for by a possible error (weather conditions?, variability of the comparison star?, etc.) in the relevant photometric zero points. The pole solution computed on the basis of the lightcurves from the 1965, 1974, 1976, and 1983 oppositions (see case No. 3 in Tables 5 and 6) is considered to be the most reliable one. Here also, the pole solution is found to be unique. Within the calculated uncertainties, this pole solution lies very near that computed by means of the revisited "amplitude-aspect" relation (see Table 3). Except for a slight correlation between the ecliptic coordinates λ_0 and β_0 (cf. Sect. 3), the covariance matrix for

© European Southern Observatory • Provided by the NASA Astrophysics Data System

1986A&A...170..167S

Table 5. The unweighted pole solutions of 22 Kalliope based upon the revisited magnitude-aspect relations

1°) Lightcurves from the six oppositions : 1958,1965,1974,1976,1983,1985 (see Table 1)

λ_0	β_0	a/b	b/c	L	Q
198°4±15°1	16°0±54°6	1.33±0.03	1.12±0.04	3.533(-6)±0.316(-6)	0.158±0.026

2°) Lightcurves from five selected oppcsitions : 1958,1965,1974,1976 and 1983

λ_0	β_0	a/b	b/c	L	Q
198°8±9°8	16°6±34°4	1.32±0.02	1.12±0.03	3.511(-6)±0.238(-6)	0.164±0.020

3°) Lightcurves from four selected oppositions : 1965,1974,1976 and 1983

λ_0	β_0	a/b	b/c	L	Q
200°7±2°1	21°6±5°6	1.32±0.01	1.13±0.01	3.532(-6)±0.104(-6)	0.164±0.009

4°) Lightcurves from three selected oppositions : 1965,1976 and 1983

λ_0	β_0	a/b	b/c	L	Q
199°5±0°6	16°0±2°3	1.32±0.01	1.12±0.01	3.585(-6)±0.005(-6)	0.160±0.001

Table 6. Correlation factors between the parameters $\lambda_0, \beta_0, a/b, b/c, L$ and Q (revisited magnitude-aspect relations)

1°) Covariance matrix for the 1958,1965,1974, 1976,1983 and 1985 oppositions :

	λ_0	β_0	a/b	b/c	L
β_0	0.992				
a/b	-0.400	-0.341			
b/c	0.149	0.168	0.423		
L	0.095	0.055	-0.743	-0.836	
Q	0.152	0.161	0.231	0.726	-0.786

2°) Covariance matrix for the 1958,1965,1974, 1976 and 1983 oppositions :

	λ_0	β_0	a/b	b/c	L
β_0	0.988				
a/b	-0.462	-0.395			
b/c	0.139	0.173	0.381		
L	0.122	0.070	-0.711	-0.828	
Q	0.161	0.179	0.182	0.734	-0.788

3°) Covariance matrix for the 1965,1974,1976 and 1983 oppositions :

	λ_0	β_0	a/b	b/c	L
β_0	0.934				
a/b	-0.655	-0.535			
b/c	0.019	0.133	0.355		
L	0.311	0.180	-0.718	-0.807	
Q	0.090	0.164	0.162	0.736	-0.773

4°) Covariance matrix for the 1965,1976 and 1983 oppositions :

	λ_0	β_0	a/b	b/c	L
β_0	0.998				
a/b	-0.053	-0.011			
b/c	0.214	0.230	0.451		
L	-0.114	-0.144	-0.733	-0.843	
Q	0.177	0.186	0.273	0.764	-0.829

case No. 3 in Table 6 looks quite acceptable. As it was emphasized in Sect. 4, we see that the pole solution derived in Table 5 on the basis of only three observed lightcurves (case No. 4) essentially constitutes a first good approximation. This result could be expected on the grounds that the ecliptic longitude of the asteroid is about equally distributed ($\Delta\lambda \sim 120°$) between the 1965, 1976, and 1983 oppositions.

From Eq. (7), we find that for $x=0$, $\alpha=0°$ and $A=90°$, the zero-phase absolute magnitude of the lightcurve minimum is expressed by

$$V_m(0°, 90°) = -2.5 \log_{10}(\sqrt{L}), \tag{13}$$

such that $V_m(0°, 90°) = 6.815 \pm 0.016$ mag for case No. 3 of 22 Kalliope in Table 5. Using Eq. (7) of Bowell and Lumme (1979)

© European Southern Observatory • Provided by the NASA Astrophysics Data System

1986A&A...170..167S

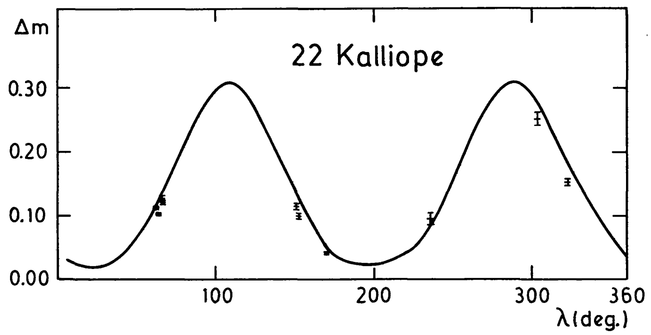

Fig. 2. Predicted full amplitude Δm of the ellipsoidal lightcurve of 22 Kalliope as a function of its position along the ecliptic (see text). Previous photometric observations are indicated with a cross (cf. Table 2)

we can estimate a value for the zero-phase geometrical albedo $P(0°)=0.2444\pm0.0110$ and rewriting their Eq. (8) for the case of an ellipsoid model, i.e.

$$\tfrac{1}{2}\log_{10}(b\cdot c)=2.821-0.5\log_{10}(P(0°))-0.2\,V_m(1,0)\,, \qquad (14)$$

we derive straightforwardly the "photometric" dimensions of the ellipsoid which suits at best the shape of 22 Kalliope: $a=82\pm4$ km, $b=62\pm3$ km, and $c=55\pm2$ km. These semi-axis values are quite comparable to the radiometric diameters of 165 and 175 km quoted by Hansen (1977) and Bender et al. (1978), respectively. Finally, Eq. (10) of Bowell and Lumme (1979) allows one to calculate the phase coefficient of 22 Kalliope. Also for case No. 3 in Table 5, we find $\beta=0.028\pm0.001$ mag deg^{-1}.

6. Discussion and conclusions

Both the "amplitude-aspect" and "magnitude-aspect" poles derived in the present work for 22 Kalliope compare relatively well with previous determinations. Scaltriti et al. (1978) report: $\lambda_0=215°\pm10°$, $\beta_0=45°\pm15°$ and $a\times b\times c=108\times80\times65$ km^3 and Zappalà and Kneževič (1984) quote: $\lambda_0=214°\pm7°$, $\beta_0=42°\pm5°$, $a/b=1.34$, and $b/c=1.18$ for the second of their possible solutions. We feel confident (see Paper I) that the revisited "amplitude-aspect" pole calculated from ten photoelectric lightcurves observed at distinct oppositions still provide an improved and more consistent unique solution: $\lambda_0=203°\pm2°$, $\beta_0=29°\pm5°$, $a/b=1.33\pm0.01$ and $b/c=1.24\pm0.08$.

As far as the absolute dimensions of the best fitted ellipsoid are concerned, we recall that the "magnitude-aspect" equations derived in Sect. 4 do include a coherent description of the phase relation via the multiple-scattering theory of light in atmosphereless bodies developed by Lumme and Bowell (1981a, b) (see also Bowell and Lumme, 1979). For the case of 22 Kalliope, we find: $a=82\pm4$ km, $b=62\pm3$ km, $c=55\pm2$ km and that the multiple scattering factor $Q=0.164\pm0.009$.

On the basis of the pole solutions listed in Table 3 (case No. 2) and Table 5 (case No. 3), we have constructed diagrams in Figs. 2 and 3 which represent, respectively, the predicted full amplitude Δm [cf. Eq. (I.10)] and the zero-phase absolute magnitude $V_m(0°, A)$ [set $x=0$ and $\alpha=0°$ in Eq. (7)] of the brightness minimum of the ellipsoidal lightcurve which will be observed for 22 Kalliope at various locations along its trajectory. In the second diagram, a correction for the phase effect can be directly accounted for by simply subtracting from the observed magnitude $V_m(\alpha, A)$ the required amount ΔV_m represented as a function of the phase angle α at the upper right hand corner of Fig. 3.

In order to better improve the revisited "magnitude-aspect" pole of 22 Kalliope, we intend to reobserve all comparison stars which have been used for the construction of the photoelectric lightcurves compiled in Table 2. This should allow us to check for any photometric inconsistency of the published lightcurve zero points as well as to put more confidence on the accuracy of the derived pole.

Acknowledgements. Both the model and numerical applications of the revisited "amplitude-aspect" method were developed while J.S. was a paid associate at the European Southern Observatory (Munich, FRG). J.S. and A.P.-S. wish to thank again Prof. Woltjer and Prof. Setti for making this stay possible. T.M. thanks Dr. J. Danziger for the invitation and support of his stay at the European Southern Observatory (Munich, FRG) in July

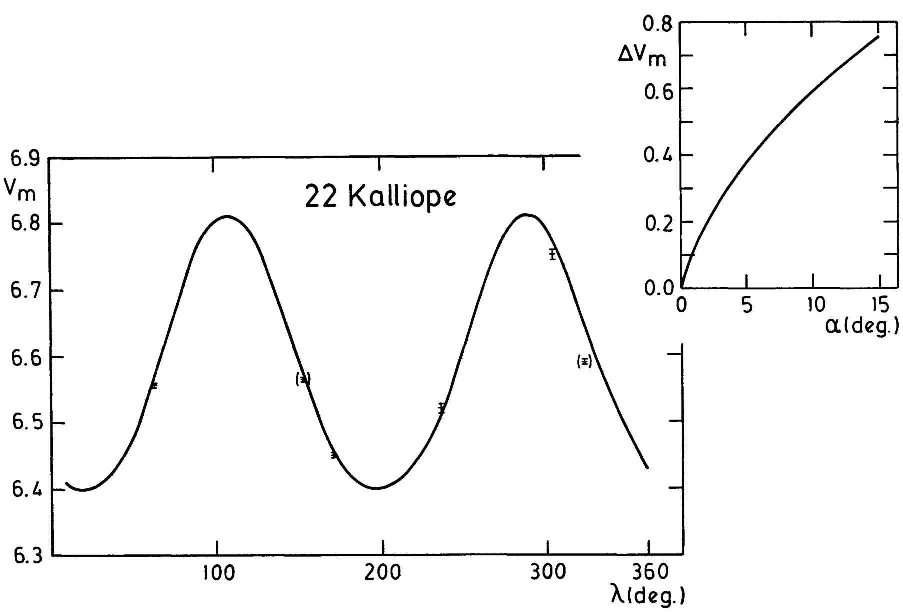

Fig. 3. Zero-phase absolute magnitude $V_m(0°, A)$ of the brightness minimum of the ellipsoidal lightcurve predicted for 22 Kalliope as a function of its position along the ecliptic (see text). Previous photometric observations are indicated with a cross (1965, 1974, 1976, and 1983 oppositions) and with a cross and parentheses (1958 and 1985 oppositions). In order to correct an observed magnitude $V_m(\alpha, A)$ for the phase effect, subtract the required amount ΔV_m represented as a function of the phase angle α at the upper right hand corner of this diagram

© European Southern Observatory • Provided by the NASA Astrophysics Data System

1986A&A...170..167S

1985 when part of the present work has been done. H.J.S. is indebted to ESO and its director Prof. Woltjer for the allocation of telescope time, though Austria is not a member-state of ESO. This project was financially supported by the Austrian "Fonds zur Förderung der Wissenschaftlichen Forschung", under project No. P4852. We should also like to thank M. Macours-Houssolongo and J. Bosseloirs for typing the manuscript and drawing the figures.

References

Ahmad, I.I.: 1954, *Astrophys. J.* **120**, 551

Barucci, M.A., Dipaoloantonio, A.: 1983, *Astron. Astrophys.* **117**, 1

Bender, D., Bowell, E., Chapman, C., Gaffey, M., Gehrels, T., Zellner, B., Morrison, D., Tedesco, E.: 1978, *Icarus* **33**, 630

Bowell, E., Lumme, K.: 1979, in *Asteroids*, ed. T. Gehrels, Univ. Arizona Press, Tucson, p. 132

Chapman, C.R., Morrison, D., Zellner, B.: 1975, *Icarus* **25**, 104

Di Martino, M., Cacciatori, S.: 1984, *Icarus* **60**, 75

Gehrels, T., Owings, D.: 1962, *Astrophys. J.* **135**, 906

Hansen, O.L.: 1977, *Icarus* **31**, 456

Lumme, K., Bowell, E.: 1981a, *Astron. J.* **86**, 1694

Lumme, K., Bowell, E.: 1981b, *Astron. J.* **86**, 1705

MIDAS: 1984, *Munich Image and Data Analysis Software*, Manual edited by the European Southern Observatory, p. 5–1

Popieszalska-Surdej, A., Surdej, J.: 1985, *Astron. Astrophys.* **149**, 186 (Paper I)

Scaltriti, F., Zappalà, V., Stanzel, R.: 1978, *Icarus* **34**, 93

Surdej, J., Schober, H.J.: 1980, *Astron. Astrophys. Suppl. Ser.* **41**, 335

Zappalà, V., van Houten-Groeneveld, I.: 1979, *Icarus* **40**, 289

Zappalà, V., Knežević, Z.: 1984, *Icarus* **59**, 436

Zhou, X.-h., Yang, X.-y., Wu, Z.-x.: 1983, *Chin. Astron. Astrophys.* **7**, 129

© European Southern Observatory • Provided by the NASA Astrophysics Data System