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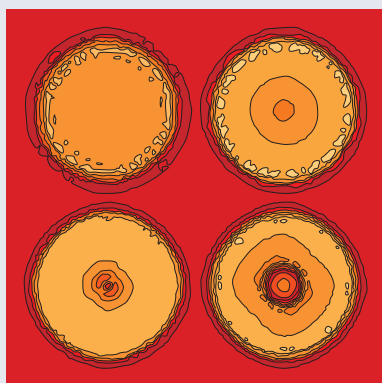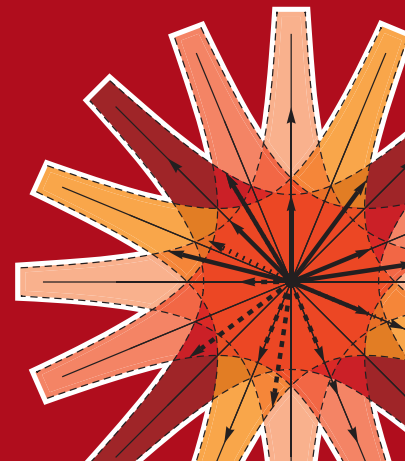

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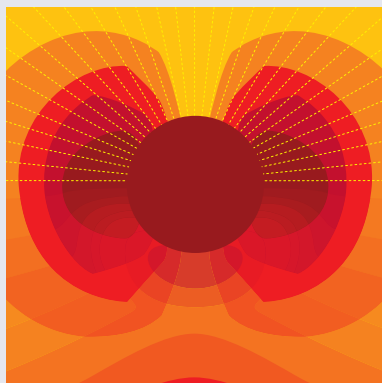
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Multiple rotations of a drop rolling inside a horizontal circular hydraulic jump

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Abstract – We explore the complex dynamics of a non-coalescing drop of moderate size inside a circular hydraulic jump of the same liquid formed on a horizontal disk. In this situation the drop is moving along the jump and one observes two different motions: a periodic one (it orbits at constant speed) and an irregular one involving reversals of the orbital motion. Modeling the drop as a rigid sphere exchanging friction with liquid across a thin film of air, we rationalize both the orbital motion and the internal rotation of the drop. This internal rotation is evidenced by tracer visualization and exhibits an unexpected dependence upon drop size.

Usually when a drop comes into contact with the same liquid or a solid surface it coalesces with liquid or spreads on the solid. There are however exceptions, which could be called situations of "non-wetting", when a very thin layer of air or vapor remains trapped between the drop and the substrate such as in the well-known Leidenfrost effect [1]. In these levitation situations, the disappearance or reduction of friction with the substrate leads to remarkable dynamics of fluids with (nearly) no contact: unusual shapes (Poincaré's shapes) of drops rolling down a plane [2], particle-wave duality of drops bouncing on a vibrated bath [3–5], drop motions induced by its own harmonic modes on vibrated viscous bath [6,7], chaotic behavior of a droplet on a soap film [8,9], etc.

Another case of mobile drops in a "non-wetting" situation has been reported by Sreenivas *et al.* [10] and Pirat *et al.* [11], when a drop is deposited inside a circular hydraulic jump of the same liquid [12–15]. A thin layer of air is entrained underneath the drop by the supercritical flow of liquid feeding the jump, that prevents coalescence, the drop remaining trapped at the shock front with a strong internal rotation. In a previous letter [11] we have shown that this "non-wetting" situation was also associated with remarkable dynamics in the case of a slightly inclined jump. In a well-defined range of flow rate, a drop

Fig. 1: (Colour on-line) A drop of radius $a = 1$ mm of silicone oil in non-coalescence state inside a horizontal hydraulic jump of the same liquid ($R = 5$ mm).

of moderate size (typically close to the capillary length) undergoes a gyroscopic instability, with surprising motions along the jump perimeter, leading to oscillations around the lowest equilibrium position.

In the present letter, the authors investigate experimentally a new situation: the drop is now deposited inside a jump, formed on a perfectly horizontal circular disk, taking care to have uniform boundary conditions at large scale (*i.e.*, circular and horizontal geometry). Because of the disappearance of any reference equilibrium position, the drop is in fact always moving around the jump, with at least two different possible states: clockwise or anti-clockwise regular orbital motion and a more complex state in which the drop "hesitates" between these two possible motions, leading to an irregular behavior involving complex mechanisms of reversal. These reversals of motions are reminiscent of those observed in more complex hydrodynamical systems [16–18], such as Rayleigh-Bénard convection, Kolmogorov flow or the "dynamo instability" (geomagnetic reversals). They can also be observed in a meteorological context (quasi-biennial oscillations of high altitude winds) [19]. These phenomena are presently attracting a great deal of interest from a large community ranging from hydrodynamics and non-linear physics to geophysics and meteorology. Our system constitutes perhaps one of the simplest experiment that one can build to observe these reversals in fluid dynamics, and in particular without turbulence.

A model of this new situation is proposed and provides a reasonable description of the regular orbital motion. As this model involves an internal rotation of the drop that has never been characterized, experiments were developed and allowed us to observe and study this internal rotation. As predicted by our model, this rotation exists, but its dependence upon drop radius is more complex than expected from an analogy with a rigid sphere supported by an air film under Couette flow, we suggest that drop shape distortions (as well as jump shape distortions) have to be considered to get a better description.

A picture of the experiment is reproduced in fig. 1. A jet of silicone oil (viscosity 20 cS, surface tension 20.6 mN/m, density 0.95) issued from a vertical tube of internal diameter 3 mm, hits the center of a transparent glass disk placed 4 cm below the outlet and of radius $R_0 = 15$ cm. With these boundary conditions (absence of a wall fixing the outer height of the jump), and following surface tracers, we observed that the studied hydraulic jump was of type I [14] (*i.e.*, unidirectional surface flow with no vortex able to reverse the flow at the free surface). Also the radius of the jump R increased with the flow rate Q while remaining very close to a power law $R = \alpha Q^\beta$, where $\alpha = 55 \pm 10$ and $\beta = 0.77 \pm 0.02$ (units used for Q and R are, respectively, here $m^3 \cdot s^{-1}$ and m). In these conditions the hydraulic jump is well known to be steady [20]. A constant level tank supplied with liquid by a gear pump is used in order to suppress any flow rate pulsation. Millimeter-sized drops of the same fluid were deposited directly inside the jump and small enough ones remained trapped at the shock front [10].

When a drop does not touch the impinging jet (*i.e.*, when the distance between the jump and the jet is larger
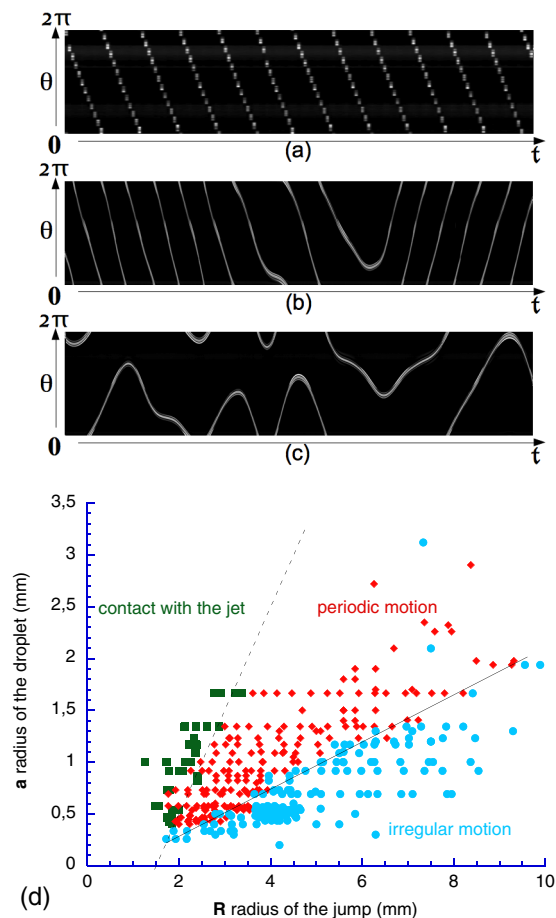

Fig. 2: (Colour on-line) Spatio-temporal diagrams obtained for a drop ($a = 1.1$ mm) in the periodic regime (a) ($R = 4.6$ mm and the recorded time is $t_{tot} = 7.4$ s) and in the irregular regime (b), (c). Case (b) ($R = 5.5$ mm and $t_{tot} = 28.8$ s) displays a single reversal of the sense of rotation just above the transition between periodic and irregular motion, while case (c) ($R = 5.9$ mm and $t_{tot} = 22.4$ s) is obtained well above this one. (d) Phase diagram for the different kind of drop motions. (●) irregular motion, (♦) periodic motion and (■) drop in contact with the jet.

than the drop width) one observes two different drop dynamics: periodic and aperiodic. To characterize these phenomena, movies of the drop were recorded from below, through the glass plate. From the obtained frames the gray level evolution is extracted on a circle in order to obtain spatio-temporal diagrams giving the law $\theta(t)$, where θ is the angular position of the drop. Three examples are given in fig. 2:

i) Figure 2(a) illustrates the periodic motion: the drop orbits periodically along the jump with a frequency ranging between a few tenths of Hz and a few Hz. This frequency decreases when the jump radius increases.

ii) Figure 2(c) illustrates the "irregular" motion: unlike the periodic one this regime is not characterized by a single orbital period. Speed variations are observed but also reversals of the sense of rotation.

iii) In the irregular case, just beyond the transition between periodic and irregular motion, one observes sudden reversals of the drop rotation sense, separating sequences of quasi-regular motion in opposite directions (see fig. 2(b)).

In principle, when the geometry of the experiment is fixed (nozzle radius, impact distance, ...) one only needs two parameters to characterize the different regimes: the drop radius a and the jump radius R. A phase diagram is presented in fig. 2(d) where three different symbols are used for periodic and irregular motions, and for a drop in contact with the jet. In this last case one also observes complex motions (static, regular and irregular motions) but not well defined and with a lower drop velocity. As expected the dashed line separating the contact with the jet and the periodic motion is given by solving the equation $R = 2a + R_J$, where R_J is the radius of the jet that we find to be almost constant and equal to approximately 1.5 mm. The continuous line separating the periodic and aperiodic motions is empirical but can be fitted by the following law $R = \beta a + R_c$, where $\beta = 4.4$ and $R_c = 0.84$ mm.

A model of the drop motion was suggested in ref. [11] and we have adapted it to the case of a perfectly horizontal jump ($\alpha = 0$). Here we first remind its essential features before developing the results obtained. Entrained by the radial flow of the bath, the drop acquires an internal rotation with a kinetic momentum parallel to the shock front. If a perturbation slightly shifts the drop, the kinetic momentum conservation leads to the appearance of a radial component of rotation (see fig. 3(b)). In such a situation, an active torque should appear, amplifying the initial perturbation and leading to a self-sustained orbital motion. To recover this, one models the drop as a rigid sphere of radius a with two contact points A and B. The situation is described in fig. 3(a). We assume that the exchanged forces at these points are simply viscous frictions through a sheared air film: $\overrightarrow{F_A} = \eta_a(S_A/d_A)(\overrightarrow{U_A} - \overrightarrow{V_G} - \overrightarrow{\Omega} \times \overrightarrow{GA})$ and $\overrightarrow{F_B} = \eta_a(S_B/d_B)(\overrightarrow{U_B} - \overrightarrow{V_G} - \overrightarrow{\Omega} \times \overrightarrow{GB})$, where η_a is the air dynamic viscosity and the quantities S_A, S_B, $d_A, d_B, \overrightarrow{U_A}, \overrightarrow{U_B}$ designate in this order: the contact surfaces, the local air layer thickness and the surface speed of the flow at the points A and B. G is the center of mass of the drop, $\overrightarrow{V}_G$ is the speed of the center of mass of the drop and $\overrightarrow{\Omega}$ designates the angular velocity vector of the drop around G in the laboratory frame. The kinetic momentum conservation equations lead to

$$\frac{d\Omega_r}{dt} + \frac{1}{\tau_A}\left(\Omega_r + \frac{L}{a}\omega\right) = \omega\Omega_\theta,$$

$$\frac{d\Omega_\theta}{dt} + \left(\frac{1}{\tau_A} + \frac{1}{\tau_B}\right)\Omega_\theta = -\omega\Omega_r - \frac{1}{a}\left(\frac{U_A}{\tau_A} + \frac{U_B}{\tau_B}\right), \quad (1)$$

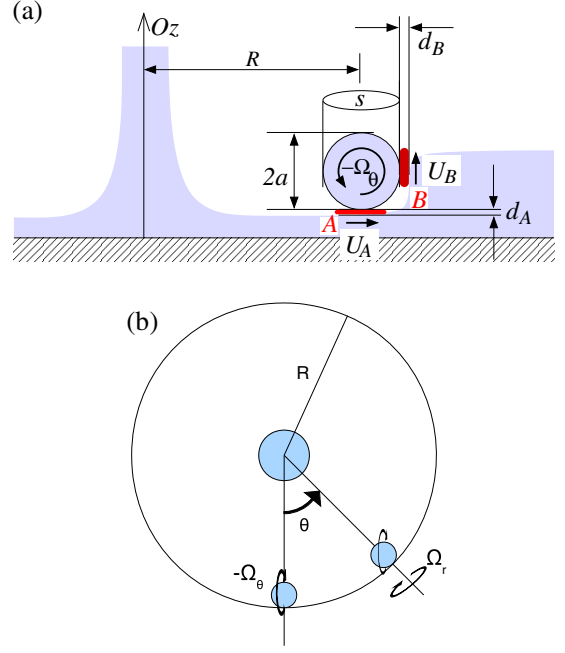$$\frac{d\Omega_z}{dt} + \frac{1}{\tau_B}\left(\Omega_z + \frac{L}{a}\omega\right) = 0.$$

Fig. 3: (Colour on-line) (a) Notations and structure of the problem expected in a radial vertical plane containing the jump central axis. The drop is rotating very fast because of the shear stress transmitted across the air film. (b) Principle of the instability (top view). The drop tries to keep constant its kinetic momentum, which develops an active radial component of the rotation vector that tends to amplify the drop displacement.

These equations must be coupled with the evolution equation for θ that can be deduced from the fundamental principles of dynamics:

$$\frac{d\omega}{dt} + \frac{2}{5}\left(\frac{1}{\tau_A} + \frac{1}{\tau_B}\right)\omega = -\frac{2}{5}\frac{a}{L}\left(\frac{\Omega_r}{\tau_A} + \frac{\Omega_z}{\tau_B}\right), \quad (2)$$

where $\omega = \frac{d\theta}{dt}$ designates the orbital speed and $L = R - a$ is the radius of the orbit described by the drop. The two characteristic times are $\tau_A = \frac{8}{15}\pi a^3 \frac{\rho_l}{\eta_a}\frac{d_A}{S_A}$ and $\tau_B = \frac{8}{15}\pi a^3 \frac{\rho_l}{\eta_a}\frac{d_B}{S_B}$, where ρ_l designates the mass density of liquid. There are only two stationary solutions to these equations. A first trivial one is

$$\omega = \Omega_r = \Omega_z = 0,$$

$$\Omega_\theta = -\frac{1}{a}\frac{\frac{U_A}{\tau_A} + \frac{U_B}{\tau_B}}{\frac{1}{\tau_A} + \frac{1}{\tau_B}}. \quad (3)$$

Integrating numerically eqs. (1) and (2) with a fourth-order Runge-Kutta method, we have checked that whatever were the initial conditions this solution was unstable, in accordance with our experimental observations, the system always evolving to another solution. Also, we have
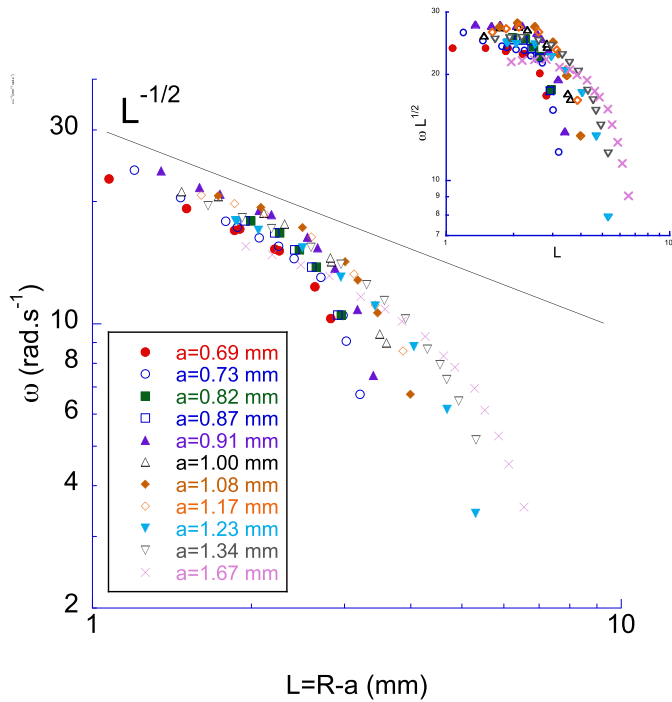
Fig. 4: (Colour on-line) Orbital speed of the drop $\omega = \mathrm{d}\theta/\mathrm{d}t$ *vs.* the orbit radius followed by the drop: $L = R - a$. A large range of drop radii is reported: $0.69\,\mathrm{mm}$–$1.67\,\mathrm{mm}$. Insert: the ratio $\omega L^{1/2}$ *vs.* L, that clearly saturates for low L. Error bars have the same order of magnitude as the symbols, consequently they are not shown here.

Fig. 5: Coal particles inside a drop viewed from above ($a = 1.6\,\mathrm{mm}$ and $R = 1.3\,\mathrm{cm}$). Image sequence (the time between two successive pictures is about $10\,\mathrm{ms}$). The black traces are clouds of coal particles and the pointer indicates a particular cloud that is passing below the drop. The jump is visible on the left bottom corner.

checked this analytically by seeking for eigenstates and eigenvalues of these equations, which confirmed this result. There is also a second solution defined by

$$\Omega_\theta = 0,$$

$$\Omega_r = \Omega_z = -\omega\frac{L}{a}, \tag{4}$$

$$\omega^2 = \frac{U_A}{L\tau_A} + \frac{U_B}{L\tau_B},$$

which corresponds to the periodic orbital state of our system. Complementary experiments (not described here) showed that the velocity U_A and U_B have almost the same magnitude (about $0.3\,\mathrm{m\cdot s^{-1}}$) and are nearly independent of the flow rate Q (for a typical range of 5–$60\,\mathrm{cm^3\cdot s^{-1}}$). This absence of dependence upon flow rate will be checked later, when discussing fig. 6. It seems also reasonable to suppose that d_A and d_B, the local air layer thickness at the points A and B, are independent of the flow rate too (they could only be fixed by the surface speed and by the drop properties). So one can assume that τ_A and τ_B are independent of the flow rate. We thus obtain that the orbital speed should scale as $\omega \propto L^{-1/2}$. This conjecture is tested in fig. 4 by increasing the radius (through a modification of the flow rate) for different drop radii. The right behavior is obtained for low jump radii, but the larger decrease for large values of L (at the approach of the
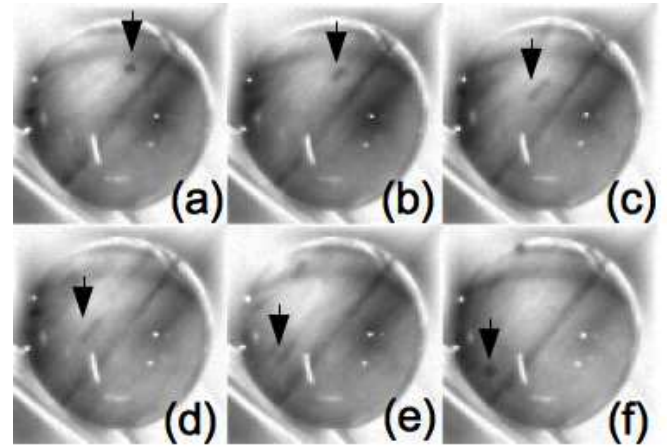
reversal transition) still needs to be explained. In addition, this model is unable to capture the aperiodic motion.

The previous model is based on the hypothesis of an internal rotation of the drop but this phenomenon remains to be checked. We therefore performed some experiments in order to observe and measure this rotation by injecting coal particles inside the drop. Pictures of this rotation are reproduced in fig. 5. The internal rotation is quite difficult to observe because of the drop motion inside the jump. To overcome this difficulty, we slightly inclined the jump plane (typically a few tenths of degrees), in order to maintain the drop globally motionless in the jump. Then, we proceeded to measurements by recording several drop rotations (between 3 and 20 rotation periods) and obtained reasonably accurate results with this method. Doing this, the solution (3) of eqs. (1) is selected, this solution being stabilized by the plate inclination [11]. Assuming that the speeds are proportional (or even equal), which seems rather reasonable, one obtains the scaling law

$$\Omega_\theta = -\frac{1}{a}\frac{\frac{U_A}{\tau_A} + \frac{U_B}{\tau_B}}{\frac{1}{\tau_A} + \frac{1}{\tau_B}} \propto -\frac{U_A}{a}. \tag{5}$$

Our experimental results, reproduced in fig. 6, highlighted a more complex situation: one can distinguish clearly two different zones by increasing the drop radius:

i) For "small" drops of radii $a < 1\,\mathrm{mm}$ (we recall that the capillary length l_c for silicone oil is about $1\,\mathrm{mm}$), in the limit case of the smallest radii, we recover asymptotically the $\frac{1}{a}$ law.

ii) For larger drops ($a > 1\,\mathrm{mm}$) one can observe a frequency law in $\Omega_\theta = U_A\frac{l}{a^2}$, where l is about $0.4\,\mathrm{mm}$.

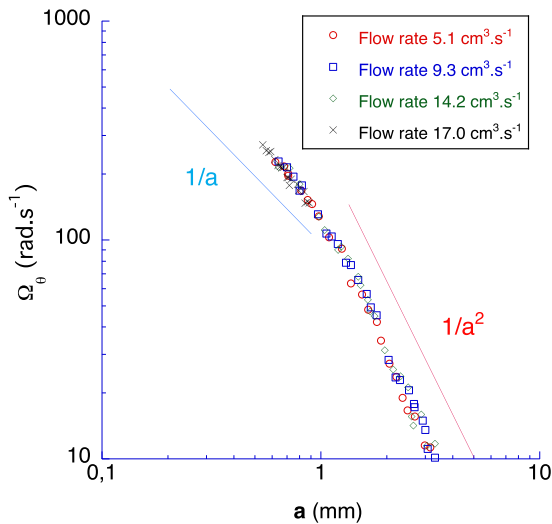Moreover, it is interesting to notice that all the curves presented here are collapsing onto a single master curve

Fig. 6: (Colour on-line) Ω_θ *vs.* drop radius observed for the stabilized inclined static case. Different flow rates are considered but all the curves are collapsing onto a single master curve without any rescaling, which suggests that the surface velocity at the jump could be the same for each flow rate. Error bars have the same order of magnitude as the symbols, consequently they are not shown here.

without any rescaling. In other words the rotation frequency does not depend on the flow rate. This is consistent with what we report above, *i.e.*, that, in our specific situation (boundary conditions, flow rate range, viscosity, . . .), U_A and U_B are nearly independent of flow rate.

The fact that the frequency law has a cross over around the capillary length suggests that the hypothesis of a rigid sphere on a sheared gas film is too rough and that the drop deformations have to be taken into account, with also possibly some local reversal of the air flow due to the lift applied on the drop. Indeed one can observe experimentally that the drops were slightly flattened by gravity. This is well known in the literature, in which the drop is often modeled by a truncated sphere [21], or a truncated sphere sustained by a non-axisymetric air pocket [22].

A new kind of "non-wetting" dynamics for a drop in non-coalescence state inside a horizontal hydraulic jump has been considered, with in particular two kinds of orbital motions: periodic and irregular. A model has been proposed and provides good agreement for the periodic orbital motion. An internal rotation of the drop has been highlighted in qualitative agreement with our modeling. The frequency law illustrates the need for further investigations on this fascinating object. The structure of the hydraulic jump has also been studied with an original mean (introducing drop at the shock front) and reveals some unexpected observations about the local surface speed at the jump (does it depend or not upon flow rate ?) that would deserve further investigations.

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