Finite Element Model Reduction for the Determination of Accurate Conductive Links and Application to MTG IRS BTA

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Requirements reach classical method limits

Interferometer

Imager (IBI)

Solar Baffle

T° gradient < 1 °C

Conductive IF @ 20+/-5°C

Radiator @ [-70, -30]°C

IRS

Ka-band antenna

S4/UVN

Infrared sounder
## Finite Element vs. Lumped Parameter Model

<table>
<thead>
<tr>
<th></th>
<th>FEM</th>
<th>LPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of nodes</td>
<td>$10^4 - 10^6$</td>
<td>$10^1 - 10^3$</td>
</tr>
<tr>
<td>1. Conductive links computation</td>
<td>✔ Automatic</td>
<td>✗ Manual, error-prone</td>
</tr>
<tr>
<td>2. Radiative links computation</td>
<td>✗ Prohibitive</td>
<td>✔ Affordable</td>
</tr>
<tr>
<td>3. Surface accuracy for ray-tracing</td>
<td>✗ FE facets</td>
<td>✔ Primitives</td>
</tr>
<tr>
<td>4. User-defined components</td>
<td>✗ Difficult</td>
<td>✔ Easy</td>
</tr>
<tr>
<td>5. Thermo-mech. analysis</td>
<td>✔ Same mesh</td>
<td>✗ Mesh extrapolation</td>
</tr>
</tbody>
</table>
Global approach & proposed solutions

(2) Radiative links computation

- Reduce # of rays: quasi-Monte Carlo method (isocell, Halton)
- Reduce # of facets: super-face concept (mesh clustering)
- Parallelization: GPUs

(3) Surface accuracy for ray-tracing

- Quadrics fitting

(1,4,5) Conductive links, thermo-mech. analysis and user-defined compts.

- Reduce detailed FE mesh (keep conductive info. of the detailed geometry)
- Able to recover detailed $T^\circ$ from reduced
- Transform reduced FE model to LP model to enable user-defined comp.
Today’s topic

(2) Radiative links computation

- Reduce # of rays: quasi-Monte Carlo method (isocell, Halton)
- Reduce # of facets: super-face concept (mesh clustering)
- Parallelization: GPUs

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Outline

Mesh clustering

Mathematical reduction

Step by step procedure

Benchmarking

Conclusions
How to reduce the system accurately?

2 step process:
- FE mesh partitioning matching ESARAD mesh
- FE mesh reduction to determine the GLs
Merging meshes

Superimpose ESARAD and FE meshes

Assign skin FE to ESARAD shells

Greedy region growing

Cluster boundary smoothing
From FE clusters to GLs

\[ \mathbf{K}_D \ (65k \times 65k) \quad \leftrightarrow \quad \mathbf{K}_R \ (340 \times 340) \]
Guyan (static) condensation

Split the system

$$KT = Q$$

With retained and condensed nodes:

$$
\begin{bmatrix}
K_{RR} & K_{RC} \\
K_{RC}^T & K_{CC}
\end{bmatrix}
\begin{bmatrix} T_R \\ T_C \end{bmatrix} = \begin{bmatrix} Q_R \\ Q_C = 0 \end{bmatrix}
$$

Reduced system:

$$K'T_R = Q'$$

With

$$K' = K_{RR} - K_{RC}K_{CC}^{-1}K_{RC}^T = R^TKR$$

$$Q' = Q_R - K_{RC}K_{CC}^{-1}Q_C = R^TQ = Q_R$$

$$R = \begin{bmatrix}
I_{RR} \\
-K_{RC}K_{CC}^{-1}
\end{bmatrix}$$

Condensed temperatures can be recovered: $$T = RT_R$$
Problem of Guyan condensation

Need to select particular nodes to be retained

No (or known) heat load on condensed nodes

Heat load on selected node ≠ heat load on cluster represented by node
Create new “super-nodes”

Not picking a representative node of the cluster but creating new nodes

A super-node = weighted (area, volume) average each node cluster

\[ T_{SN} = AT \]

\[ T_{SN_i} = \sum_{j=1}^{N} A_{ij} T_j \]

\[ \sum_{j=1}^{N} A_{ij} = 1 \]
Combining the relations

As done at element level in MSC Thermica®:

\[
\begin{align*}
\begin{cases}
KT = Q \\
T_{SN} = AT
\end{cases} \iff
\begin{bmatrix} K & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} T \\ T_{SN} \end{bmatrix} = M \begin{bmatrix} T \\ T_{SN} \end{bmatrix} = \begin{bmatrix} Q \\ 0 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix} T \\ 0 \end{bmatrix} &= M^{-1} \begin{bmatrix} Q \\ T_{SN} \end{bmatrix} = \begin{bmatrix} X & Y^T \\ Y & Z \end{bmatrix} \begin{bmatrix} Q \\ T_{SN} \end{bmatrix} \\
YA^T &= I = AY^T \\
0 &= YQ + ZT_{SN}
\end{align*}
\]

If the load is uniform over each super-node \((Q = A^TQ_{SN})\): \(YQ = Q_{SN}\)

\[-ZT_{SN} = Q_{SN}\]

\[
\boxed{K_{SN} = -Z}
\]

And the detailed \(T^\circ\) can be recovered:

\[
T = XQ + Y^T T_{SN}
\]
You need to invert $M$ to get $K_{SN}$!

size($M$) > size($K$) $\rightarrow$ very expensive + $M$ is not sparse!

Detailed $T^\circ$ not needed:

- LDL decomposition of $M$ $\rightarrow$ selective inversion of sparse matrix and only $K_{SN}$ is computed.

Detailed $T^\circ$ needed: $X$ and $Y$ are required (size($X$)=size($K$), not sparse)

- Local inversion of $M$ for each super-node
- Global inversion for small problems.
Local inversion of $M$

Local inversion of $M$:

super-node + keep all detailed IF nodes

Guyan condensation to eliminate the detailed IF nodes

Detailed $T^\circ$ recovery by inverse procedure:

no need to store the full $X$ and $Y$
Overall procedure

- CAD cleaning + ESARAD shells drawing
- Import .step to ESARAD
- LPM nodes numbering in ESARAD
- FE meshing cleaned CAD
- Superimposition of FE & ESARAD meshes
- FE mesh partitioning
- FE assembly and detailed $K$ matrix computation
- Reduction of $K$ to $K_{SN}$
- Export $K_{SN}$ and super-nodal capacitances to ESATAN
- Compute the radiative links (with ESARAD or other)
- Combine radiative + conductive links and others $\Rightarrow$ solve for $T_{SN}$
Benchmarking

<table>
<thead>
<tr>
<th></th>
<th>Detailed</th>
<th>ESATAN</th>
<th>Reduced</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta T$</td>
<td>235.47 K</td>
<td>240.23 K</td>
<td>235.47 K</td>
</tr>
<tr>
<td># nodes</td>
<td>11897</td>
<td>62</td>
<td>62</td>
</tr>
<tr>
<td># GLs</td>
<td>71033</td>
<td>97</td>
<td>1891</td>
</tr>
</tbody>
</table>

Detailed: 0.1W

Reduced: $\Delta T = 235.47 K$

Detailed: $\Delta T = 235.47 K$

ESATAN: $\Delta T = 240.23 K$

Reduced: $\Delta T = 235.47 K$

Detailed: $\Delta T = 235.47 K$
**MANUAL GLS LEAD TO 10% ERROR**

<table>
<thead>
<tr>
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<th>Detailed (Solidworks)</th>
<th>ESATAN (manual)</th>
<th>Reduced</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔT</td>
<td>107.4</td>
<td>123.2</td>
<td>107.7</td>
</tr>
<tr>
<td># nodes</td>
<td>46405</td>
<td>280</td>
<td>280</td>
</tr>
<tr>
<td># GLs</td>
<td>253004</td>
<td>402</td>
<td>39060</td>
</tr>
</tbody>
</table>

**ESATAN (manual)**

0.1W

100°C
Integration of all components & run

T [°C]
-7.5
-6.0
Conclusions

Conductive reduction method offers:
- better accuracy
- automatic GLs computation in complex 3D nodes
- detailed $T^\circ$ map recovery for thermo-mech. analyses

FEM vs. LPM: *Unity makes strength (Belgian motto)*

2$^{\text{nd}}$ step to bridge the gap between structural and thermal analysis
Thank you for your attention...

Any question?
REFERENCES


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