Finite Element Model Reduction for the Determination of Accurate Conductive Links and Application to MTG IRS BTA

Lionel Jacques[1,2], Luc Masset[1], Tanguy Thibert[2], Pierre Jamotton[2], Coraline Dalibot[2], Gaetan Kerschen[1]

[1] Space Structures and Systems Laboratory, University of Liège
[2] Centre Spatial de Liège

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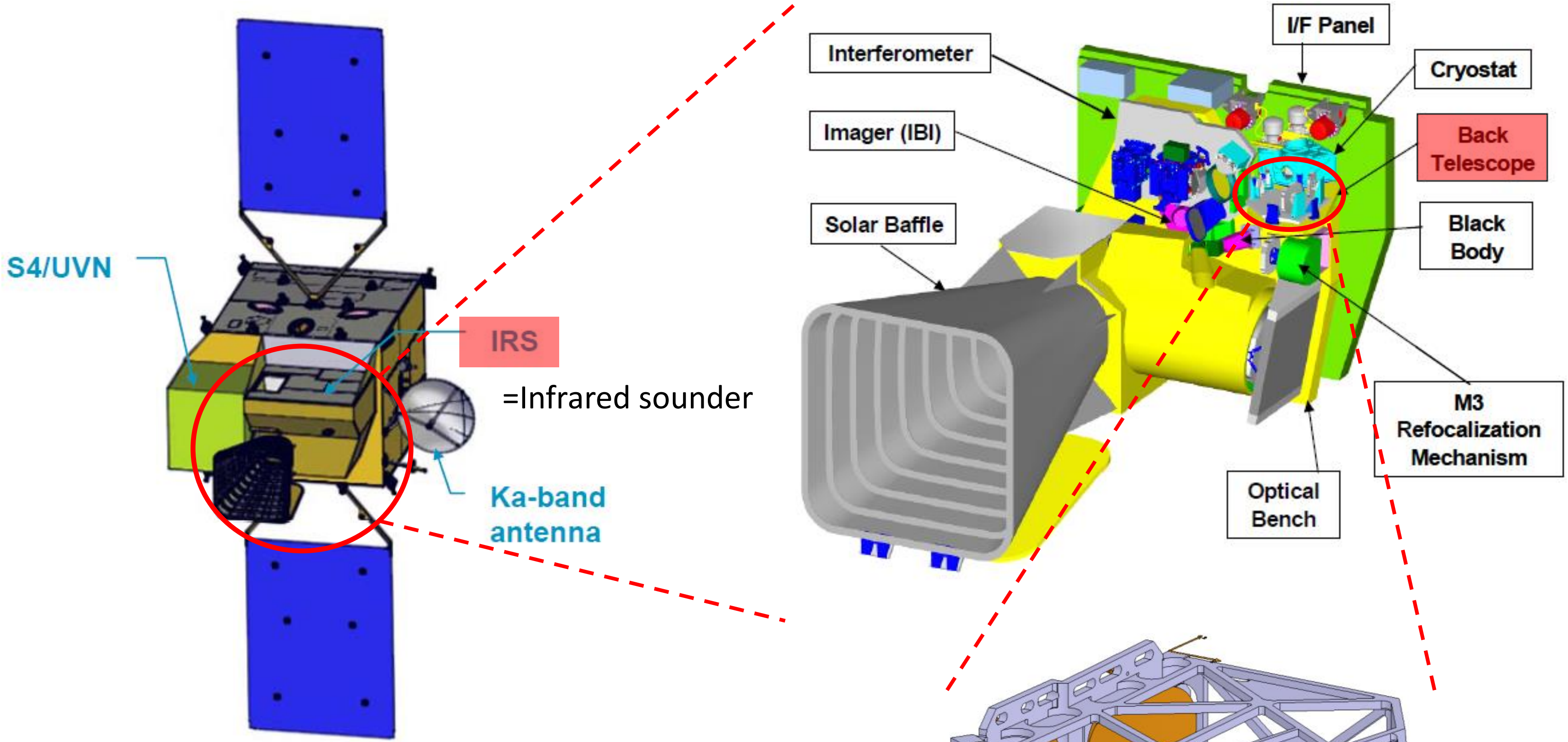
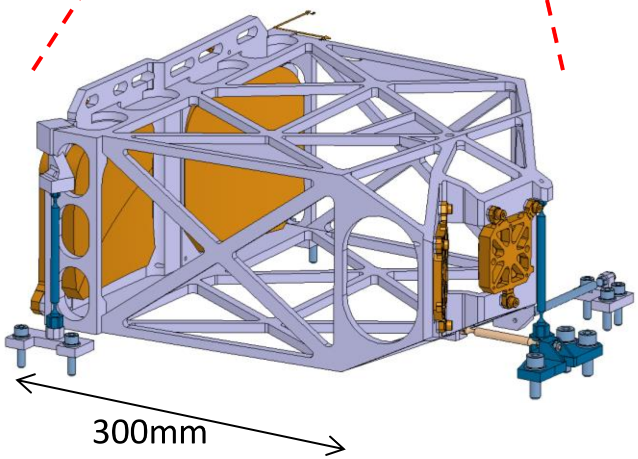
Requirements reach classical method limits



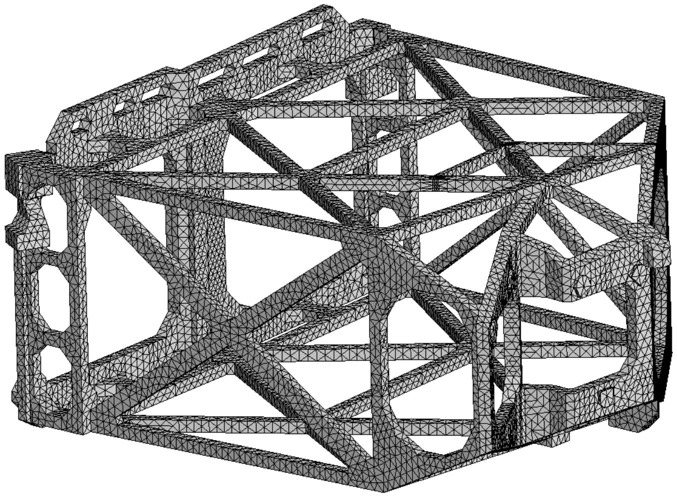
T° gradient < 1 °C

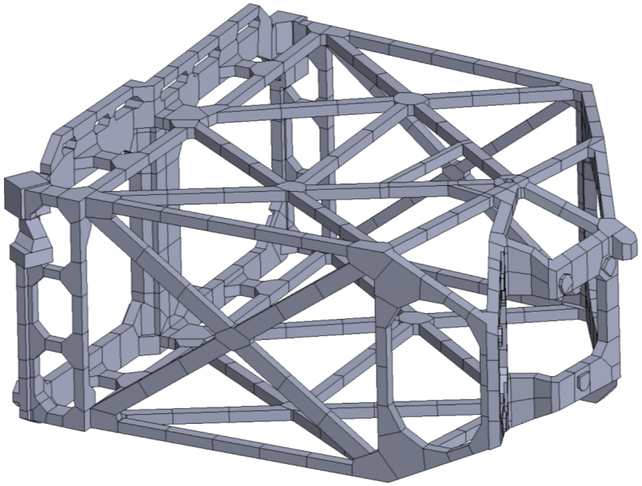
Conductive IF @ 20+/-5°C

Radiator @ [-70, -30]°C



Finite Element vs. Lumped Parameter





	FEM	LPM
Number of nodes	10^4 - 10^6	10^1 - 10^3
1. Conductive links computation	✔ Automatic	✘ Manual, error-prone
2. Radiative links computation	✘ Prohibitive	✔ Affordable
3. Surface accuracy for ray-tracing	✘ FE facets	✔ Primitives
4. User-defined components	✘ Difficult	✔ Easy
5. Thermo-mech. analysis	✔ Same mesh	✘ Mesh extrapolation

Global approach & proposed solutions

(2) Radiative links computation

- Reduce # of rays: quasi-Monte Carlo method (isocell, Halton)
- Reduce # of facets: super-face concept (mesh clustering)
- Parallelization: GPUs

(3) Surface accuracy for ray-tracing

- Quadrics fitting

(1,4,5) Conductive links, thermo-mech. analysis and user-defined compts.

- Reduce detailed FE mesh (keep conductive info. of the detailed geometry)
- Able to recover detailed T° from reduced
- Transform reduced FE model to LP model to enable user-defined comp.

Today's topic

(2) Radiative links computation

- Reduce # of rays: quasi-Monte Carlo method (isocell, Halton)
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Outline

Mesh clustering

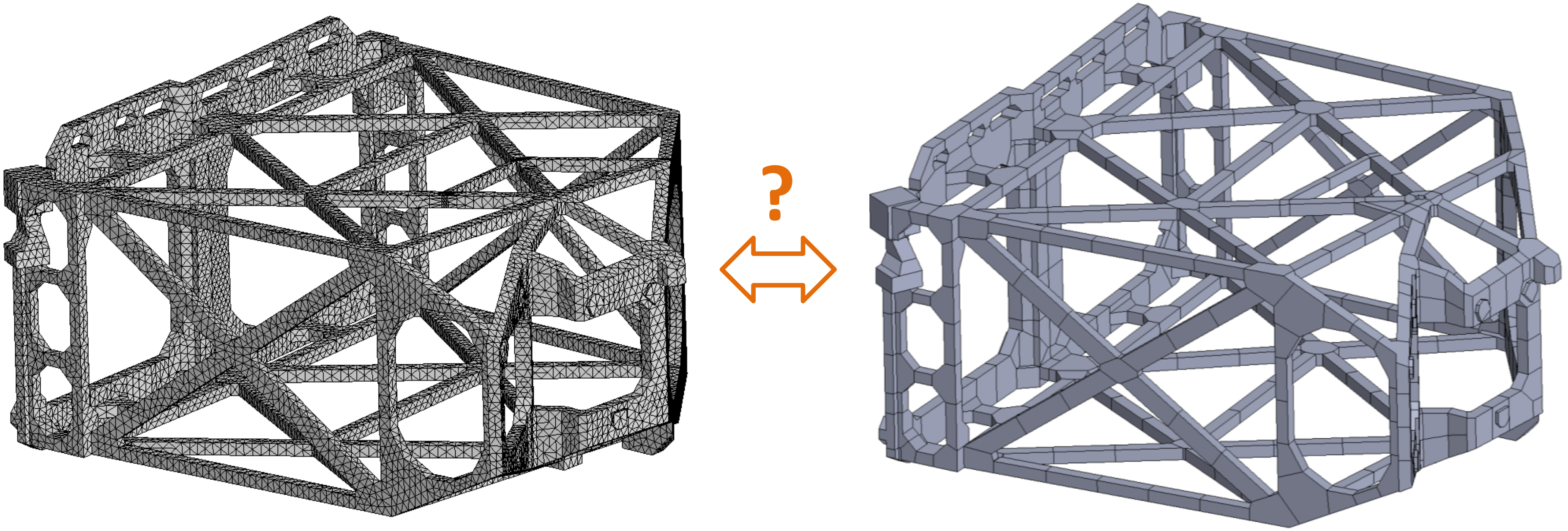
Mathematical reduction

Step by step procedure

Benchmarking

Conclusions

How to reduce the system accurately?

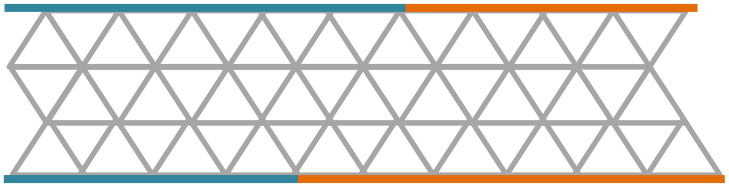


2 step process:

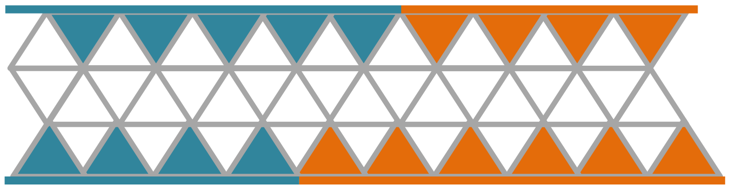
- FE mesh partitioning matching ESARAD mesh
- FE mesh reduction to determine the GLs

Merging meshes

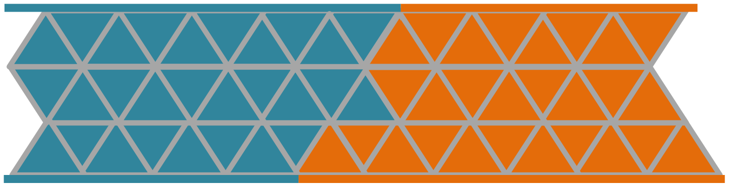
Superimpose ESARAD and FE meshes



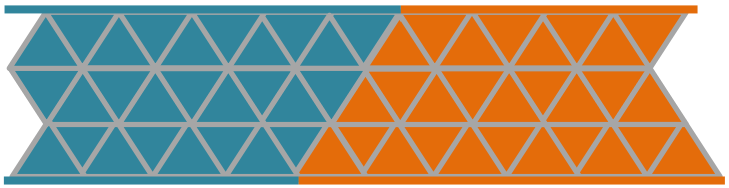
Assign skin FE to ESARAD shells



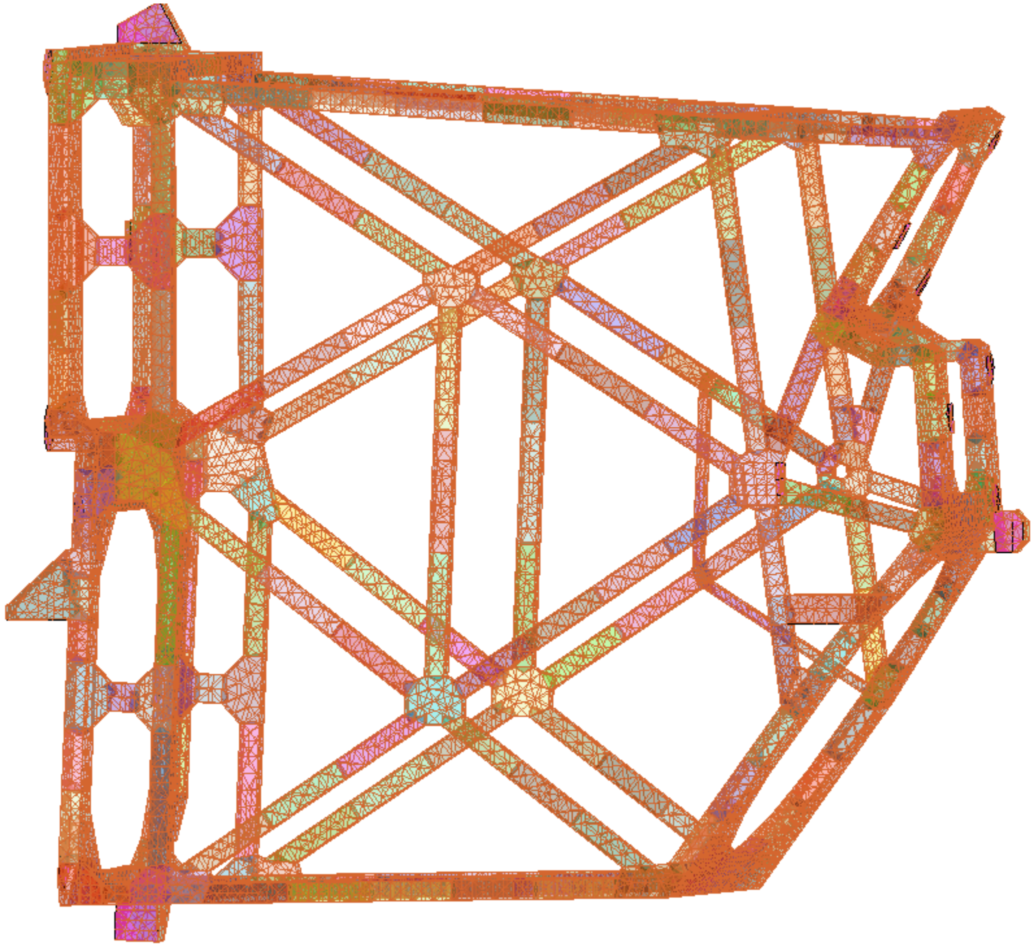
Greedy region growing



Cluster boundary smoothing



From FE clusters to GLs



$\mathbf{K_D}$ (65k × 65k) ⇔ ? $\mathbf{K_R}$ (340 × 340)

Guyan (static) condensation

Split the system

$$\mathbf{K}\mathbf{T} = \mathbf{Q}$$

With retained and condensed nodes:

$$\begin{bmatrix} \mathbf{K}_{\mathrm{RR}} & \mathbf{K}_{\mathrm{RC}} \\ \mathbf{K}_{\mathrm{RC}}^{\mathrm{T}} & \mathbf{K}_{\mathrm{CC}} \end{bmatrix} \begin{Bmatrix} \mathbf{T}_{\mathrm{R}} \\ \mathbf{T}_{\mathrm{C}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{Q}_{\mathrm{R}} \\ \mathbf{Q}_{\mathrm{C}} = 0 \end{Bmatrix}$$

Reduced system:

$$\mathbf{K}'\mathbf{T}_{\mathrm{R}} = \mathbf{Q}'$$

With

$$\mathbf{K}' = \mathbf{K}_{\mathrm{RR}} - \mathbf{K}_{\mathrm{RC}}\mathbf{K}_{\mathrm{CC}}^{-1}\mathbf{K}_{\mathrm{RC}}^{\mathrm{T}} = \mathbf{R}^{\mathrm{T}}\mathbf{K}\mathbf{R}$$

$$\mathbf{Q}' = \mathbf{Q}_{\mathrm{R}} - \mathbf{K}_{\mathrm{RC}}\mathbf{K}_{\mathrm{CC}}^{-\mathbf{1}}\mathbf{Q}_{\mathrm{C}} = \mathbf{R}^{\mathrm{T}}\mathbf{Q} = \mathbf{Q}_{\mathrm{R}}$$

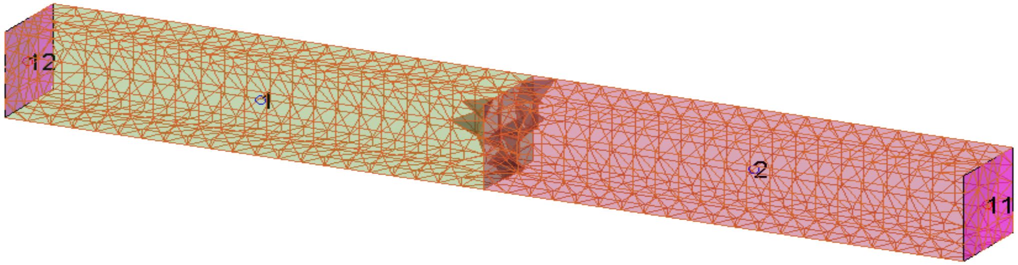
$$\mathbf{R} = \begin{bmatrix} \mathbf{I}_{\mathrm{RR}} \\ -\mathbf{K}_{\mathrm{RC}}\mathbf{K}_{\mathrm{CC}}^{-1} \end{bmatrix}$$

Condensed temperatures can be recovered: $\mathbf{T} = \mathbf{R}\mathbf{T}_{\mathrm{R}}$

Problem of Guyan condensation

Need to select particular nodes to be retained

No (or known) heat load on condensed nodes



Heat load on selected node ≠ heat load on cluster represented by node

Create new “super-nodes”

Not picking a representative node of the cluster but creating new nodes

A super-node = weighted (area, volume) average each node cluster

$$\mathbf{T_{SN}} = \mathbf{AT}$$

$$T_{SN_i} = \sum_{j=1}^{N} A_{ij} T_j \qquad \sum_{j=1}^{N} A_{ij} = 1$$

Combining the relations

As done at element level in MSC Thermica®:

$$\begin{cases} \mathbf{KT} = \mathbf{Q} \\ \mathbf{T_{SN}} = \mathbf{AT} \end{cases} \Leftrightarrow \begin{bmatrix} \mathbf{K} & \mathbf{A^T} \\ \mathbf{A} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{T} \\ \mathbf{0} \end{Bmatrix} = \mathbf{M} \begin{Bmatrix} \mathbf{T} \\ \mathbf{0} \end{Bmatrix} = \begin{Bmatrix} \mathbf{Q} \\ \mathbf{T_{SN}} \end{Bmatrix}$$

$$\begin{Bmatrix} \mathbf{T} \\ \mathbf{0} \end{Bmatrix} = \mathbf{M^{-1}} \begin{Bmatrix} \mathbf{Q} \\ \mathbf{T_{SN}} \end{Bmatrix} = \begin{bmatrix} \mathbf{X} & \mathbf{Y^T} \\ \mathbf{Y} & \mathbf{Z} \end{bmatrix} \begin{Bmatrix} \mathbf{Q} \\ \mathbf{T_{SN}} \end{Bmatrix}$$

$$\mathbf{YA^T} = \mathbf{I} = \mathbf{AY^T}$$

$$\mathbf{0} = \mathbf{YQ} + \mathbf{ZT_{SN}}$$

If the load is uniform over each super-node ($\mathbf{Q} = \mathbf{A^T Q_{SN}}$): $\mathbf{YQ} = \mathbf{Q_{SN}}$

$$-\mathbf{ZT_{SN}} = \mathbf{Q_{SN}}$$

$$\boxed{\mathbf{K_{SN}} = -\mathbf{Z}}$$

And the detailed T° can be recovered:

$$\mathbf{T} = \mathbf{XQ} + \mathbf{Y^T T_{SN}}$$

You need to invert $\mathbf{M}$ to get $\mathbf{K}_{SN}$!

size($\mathbf{M}$) > size($\mathbf{K}$) → very expensive + $\mathbf{M}$ is not sparse!

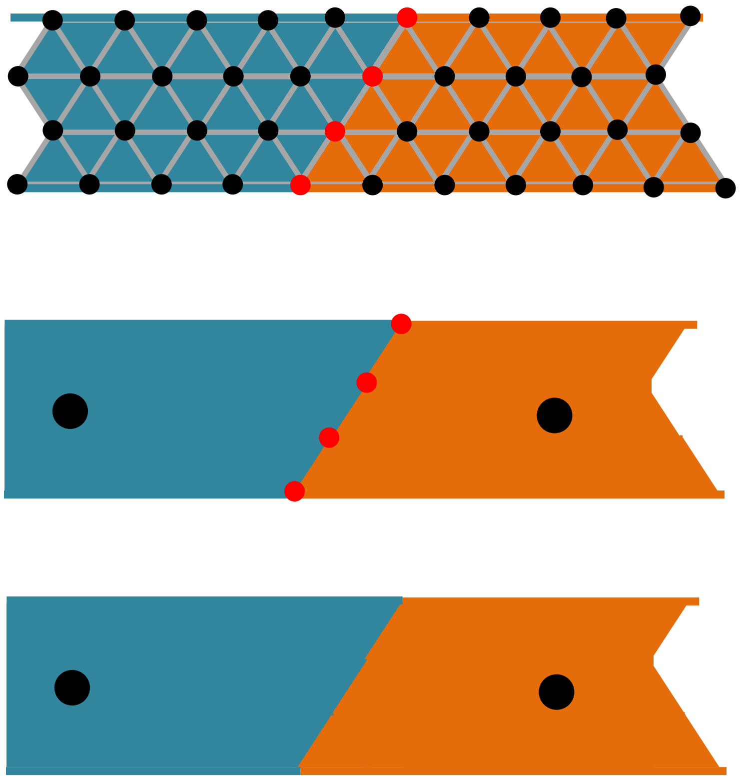
Detailed T° not needed:

- LDL decomposition of $\mathbf{M}$ → selective inversion of sparse matrix and only $\mathbf{K}_{SN}$ is computed.

Detailed T° needed: $\mathbf{X}$ and $\mathbf{Y}$ are required (size($\mathbf{X}$)=size($\mathbf{K}$), not sparse)

- Local inversion of M for each super-node
- Global inversion for small problems.

Local inversion of $\mathbf{M}$



Local inversion of M:

super-node + keep all detailed IF nodes

Guyan condensation to eliminate the

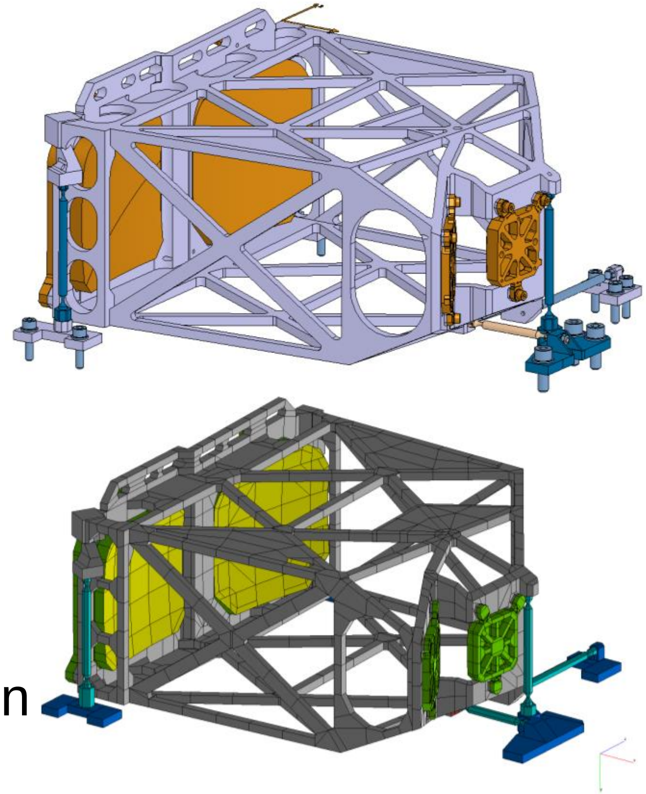
detailed IF nodes

Detailed T° recovery by inverse procedure:

no need to store the full $\mathbf{X}$ and $\mathbf{Y}$

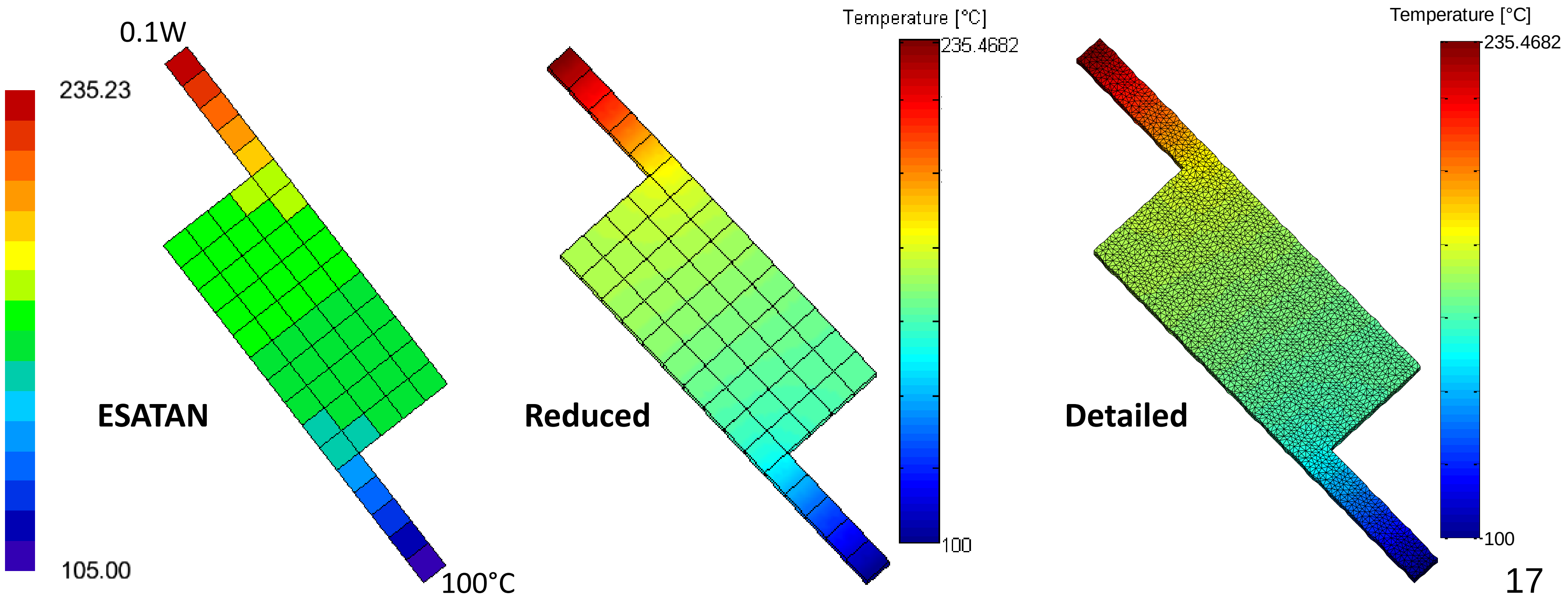
Overall procedure

- CAD cleaning + ESARAD shells drawing
- Import .step to ESARAD
- LPM nodes numbering in ESARAD
- FE meshing cleaned CAD
- Superimposition of FE & ESARAD meshes
- FE mesh partitioning
- FE assembly and detailed $\mathbf{K}$ matrix computation
- Reduction of $\mathbf{K}$ to $\mathbf{K}_{\mathbf{SN}}$
- Export $\mathbf{K}_{\mathbf{SN}}$ and super-nodal capacitances to ESATAN
- Compute the radiative links (with ESARAD or other)
- Combine radiative + conductive links and others → solve for $\mathbf{T}_{\mathbf{SN}}$



Benchmarking

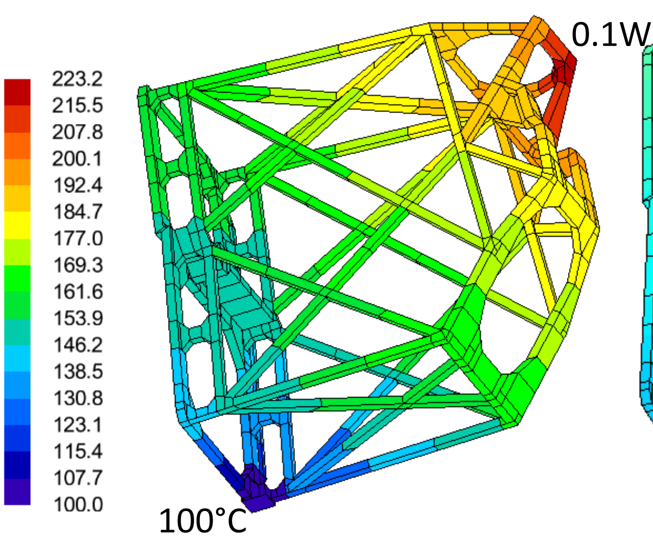
	Detailed	ESATAN	Reduced
ΔT	235.47K	240.23 K	235.47 K
# nodes	11897	62	62
# GLs	71033	97	1891



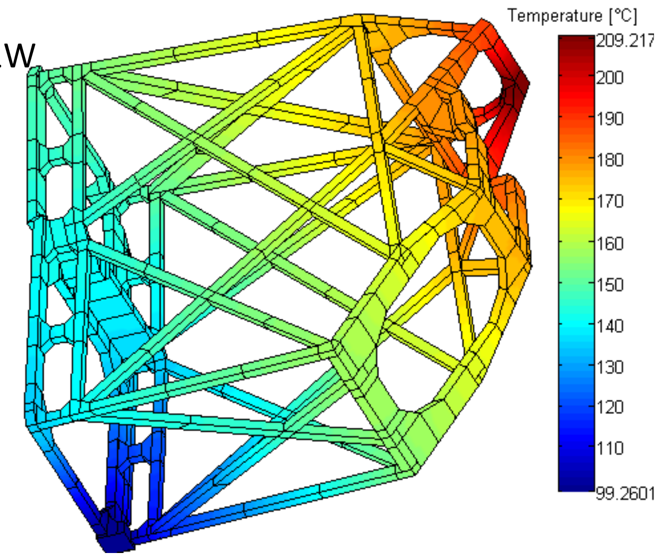
MANUAL GLS LEAD TO 10% ERROR

	Detailed (Solidworks)	ESATAN (manual)	Reduced
ΔT	107.4	123.2	107.7
# nodes	46405	280	280
# GLs	253004	402	39060

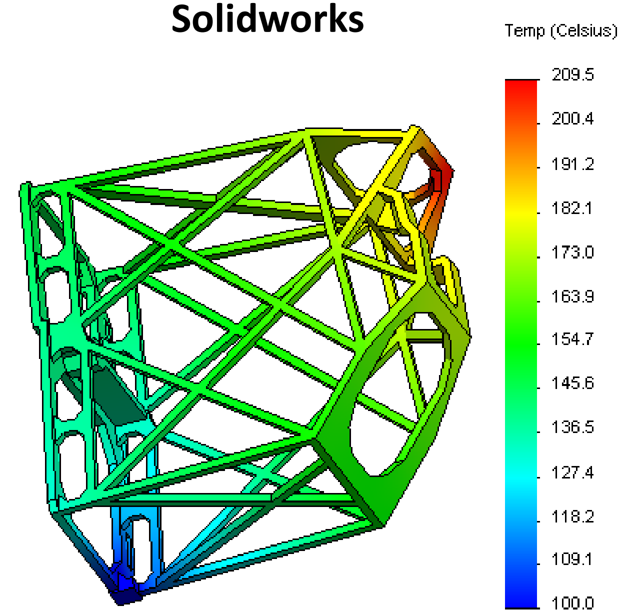
ESATAN (manual)



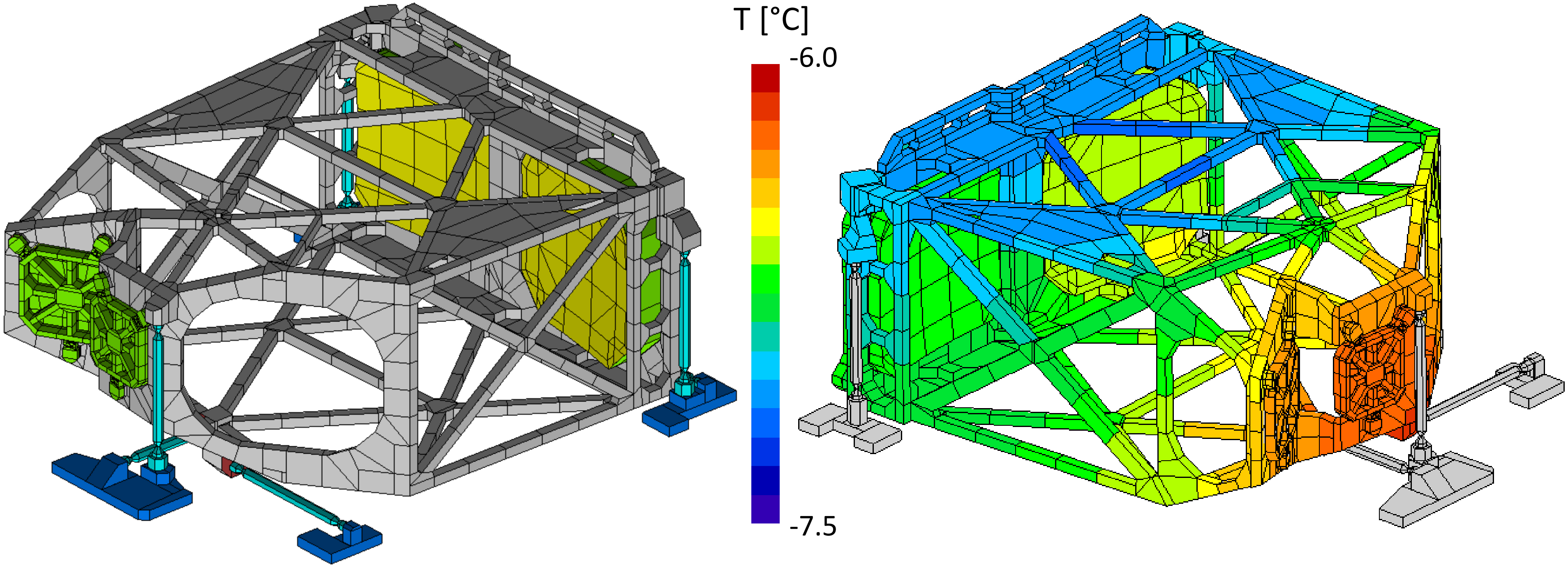
Reduced



Solidworks



Integration of all components & run



CONCLUSIONS

Conductive reduction method offers:

- better accuracy
- automatic GLs computation in complex 3D nodes
- detailed T° map recovery for thermo-mech. analyses

FEM vs. LPM: *Unity makes strength (Belgian motto)*

2nd step to bridge the gap between structural and thermal analysis

Thank you for your attention…

Any question?

References

[1] T.D. Panczak, The failure of finite element codes for spacecraft thermal analysis, Proceedings of the International Conference on Environmental Systems, Monterey, USA, 1996.

[2] MSC THERMICA User Manual, Version 4.5.1, 2012, ASTRI.UM.757138.ASTR

Contact

Lionel Jacques, ljacques@ulg.ac.be

Thermal Engineer & PhD student

- **University of Liège**

  Space Structures and Systems Lab

  1, Chemin des Chevreuils (B52/3)
  Liege, B-4000, Belgium

  http://www.ltas-s3l.ulg.ac.be/

- **Centre Spatial de Liège**
  Liège Science Park
  Avenue Pré-Aily
  B-4031 Angleur Belgium
  http://www.csl.ulg.ac.be