Noname manuscript No.
(will be inserted by the editor)

Vehicle routing problems with loading constraints: State-of-the-art and future directions

Hanne Pollaris · Kris Braekers · An Caris · Gerrit K. Janssens · Sabine Limbourg

Received: date / Accepted: date

Abstract Distributors are faced with loading constraints in their route planning. These loading constraints include multi-dimensional packing constraints, unloading sequence constraints, stability constraints and axle weight limits. Not taking into account these loading constraints would make the planning often not feasible in practice and gives rise to last-minute changes in planning which may result in additional costs. The development of vehicle routing models that incorporate loading constraints is vital for a more efficient planning of routes. The number of contributions to this field of research has increased enormously in the last couple of years. Almost sixty percent of the papers that study the integration of loading constraints in routing models are published after 2009. The contribution of this paper is twofold. First, an overview of recent developments in this research field is provided. Literature on all transport modes in which loading constraints play a key role is discussed (trucks, airplanes, ships, and automated guided vehicles). To identify the loading constraints considered in each paper, a state-of-the-art classification scheme on loading problems is used. Second, research gaps and opportunities for future research are identified.

Keywords Vehicle routing problem · Loading problem · Pickup and delivery · Rich VRP · Two-dimensional packing · Three-dimensional packing

H. Pollaris · K. Braekers · A. Caris · G. K. Janssens
Hasselt University, Agoralaan - Gebouw D, 3590 Diepenbeek, Belgium
Tel.: +32-11-268784
E-mail: hanne.pollaris@uhasselt.be

A. Caris
Reseach Foundation Flanders (FWO), Egmontstraat 5, 1000 Brussels, Belgium

S. Limbourg
University of Liege (ULg), HEC Management School, QuantOM, Rue Louvrex 14 (N1), 4000 Liege, Belgium

1 Introduction

The most studied combinatorial optimization problem in transport and logistics is the vehicle routing problem. The vehicle routing problem concerns the distribution of goods between depots and customers [98]. Its goal is to find a set of routes for a fleet of vehicles where the objective function (e.g. total distance, routing costs) is optimized. Every demand needs to be fulfilled and vehicle capacities need to be respected. A solution for a basic vehicle routing problem contains two elements: the assignment of each customer to a trip and the sequence in which the customers will be visited in the trip. The basic version of the vehicle routing problem is the Capacitated Vehicle Routing Problem (CVRP). The CVRP considers a homogeneous vehicle fleet with a fixed capacity (in terms of weight or number of items) which delivers goods from a depot to customer locations. Split deliveries are not allowed. The CVRP can be extended to VRP with time windows (VRPTW) by specifying time windows in which deliveries need to take place. Another variant is the VRP with Pickups and Deliveries (VRPPD) in which orders may be picked up and delivered at customer places. For each order, an origin (pickup location) and a destination (delivery location) is specified [87]. It is possible to have both deliveries and pickups at a given location. When only one vehicle is considered, the VRPPD reduces to a Traveling Salesman Problem with Pickup and Delivery (TSPPD). A third common extension of the basic CVRP is the VRP with backhauls (VRPB) in which again pickups and deliveries may be combined in a single tour, but first all delivery requests need to be performed, and afterwards the empty vehicle may pick up goods at customer locations. [98]

The *classic* vehicle routing problem described in the previous paragraph, has been extensively studied in the last decades. A review of solution methods can be found in [64]. In real-life, companies are faced with several additional constraints which greatly increase the complexity of the problem. Examples of such complicating constraints or attributes are maximum route length and duration, incompatibilities between goods and vehicles and loading constraints. *Rich* vehicle routing problems (RVRP) are routing problems that take into account some of these additional realistic constraints [8]. The reader is referred to Vidal et al [102] for a synthesis and analysis of solution methods dealing with rich vehicle routing problems.

This paper focuses on the integration of loading constraints in vehicle routing problems and provides a literature review on this topic. A survey conducted by the authors among several Belgian logistics service providers pointed out that they are faced with complex loading problems in their route planning (e.g. multi-dimensional packing constraints, unloading sequence constraints, stability constraints and axle weight limits). Not taking into account these loading constraints makes the planning often not feasible in practice and gives rise to last-minute changes in planning which may result in additional costs. The development of vehicle routing models that incorporate loading constraints is therefore vital for a more efficient planning of routes. The packing scheme of the vehicle changes each time a load is picked up or delivered at a customer which implies that loading constraints should not only be monitored at the moment of departure but also during the rest of the trip. Not only in road transport, but also in maritime and air transport, loading constraints play an important role in the planning.

The combination of routing problems and loading problems is a fairly recent domain of research. A review up to 2010 of 31 papers concerning vehicle routing and loading constraints may be found in Iori and Martello [58]. The number of contributions to this field of research has however increased enormously in the last couple of years. The purpose of this paper is to extend the overview of Iori and Martello [58]. In total, 76 papers dealing with vehicle routing problems with loading loading constraints are discussed in this paper. Only 31 (or 40%) of these papers are included in the overview of Iori and Martello [58]. In addition, a more in-depth discussion of the loading constraints is provided. The classification of Bortfeldt and Wäscher [12] is used to identify the loading constraints. When rich constraints (other than loading constraints) are included, this is mentioned in the description of the models. Furthermore, this paper takes a broad perspective by not only focusing on road transport, but also considering maritime transport, air transport and automated guided vehicles. Finally, a comparison between the papers is provided and future research directions are identified.

In Section 2, relevant problem characteristics for the VRP are described. Loading problems that may be considered in combination with routing problems are identified in Section 3. In Section 4, an overview of the literature concerning vehicle routing problems combined with loading problems is provided. In Section 5, conclusions and opportunities for further research are discussed.

2 Problem characteristics of VRP

For a general discussion of the VRP, the reader is referred to [98], [25] and [56]. In this section, the main characteristics that may influence the solution of a vehicle routing problem are described. Characteristics of the vehicle fleet, characteristics of the cargo, (time dependent) travel times, legal framework, transportation requests and objective function are discussed in the following paragraphs.

Characteristics of the vehicle fleet such as vehicle capacity, configuration of the loading space and unloading possibilities play an important role in the problem solution. The capacity of vehicles may be specified in terms of weight, number of items or volume. The loading space of the vehicle often influences the capacity. The loading space is determined by the measurements of the vehicle in three dimensions (length, width and height) and may have a specific configuration. For example vehicles may be divided into multiple compartments which allows the transport of goods that need to be kept segregated. A tank truck may be divided into compartments to avoid all the liquid being shifted to the front of the truck when it stops (due to *mass in motion*). The configuration of the loading space may also allow loading of goods into several piles. Lastly, vehicles differ in the ways in which they can be loaded or unloaded. Loading of the vehicle may happen via the rear (rear loading), the long side, and/or via the top side of the vehicle. A homogeneous vehicle fleet consists of vehicles which have the same vehicle characteristics. In a heterogeneous fleet, vehicles may differ in terms of capacity, loading space or other relevant vehicle characteristics. **Characteristics of the cargo** include the measurements and fragility of the items and orientation issues. The measurements may determine if an item fits into a container or not. Often items are assumed to have a rectangular shape in two dimensions and a cuboid shape in three dimensions to make the loading process easier. Items can be fragile (e.g. porcelain) or non-fragile (e.g. newspapers) which may have an influence on the loading possibilities of the items into a container. Items may have specific orientation constraints. For example several items have a fixed orientation with respect to the height. This means the items cannot be placed upside-down but have a fixed top. Cargo may consist of homogeneous or heterogeneous items. In case of heterogeneous items, compatibility issues of product pairs may arise. More specifically, it is possible that certain products may not be transported together in the same vehicle or vehicle compartment. Furthermore it is possible that some product types (e.g. frozen or refrigerated items) need to be transported in adapted containers or container compartments. The **travel time** on a certain route may vary at different instances in time. Travel time depends on the level of congestion on the road, which usually changes throughout the day in congested areas. The **legal** driving hours specify the maximum amount of hours a truck driver may drive each day as well as the minimum duration and frequency of breaks during his shift. Next, rules concerning the loading of the vehicle (e.g. European Best Practice Guidelines on Cargo Securing for Road Transport [1]) may be specified. Road speed limits are used to regulate the speed of the trucks and may therefore influence the solution of the VRP. **Transportation requests** can involve a pickup or a delivery of items or both. Most of the time, split deliveries or split pickups are not allowed. This implies that each customer is visited only once. Customers may specify time windows within which the visits (delivery or pickup) of the vehicle must be made. These time windows may be *hard* or *soft*. Soft time windows imply that deliveries can take place outside the time windows, in which case a penalty cost will be incurred by the transportation company. Hard time windows do not allow a delivery to take place outside the time windows. As already mentioned, when time windows are specified, the problem is called a VRP with time windows (VRPTW). Several **objectives** are relevant when considering the VRP with loading constraints. Minimization of number of vehicles, total cost, total route length and total time are often considered. Furthermore, equal route lengths and maximization of volume utilisation may be objectives.

[1] `http://ec.europa.eu/transport/road_safety/vehicles/doc/cargo_securing_guidelines_en.pdf`

3 Loading constraints

Loading problems arise when goods cannot be placed freely in a container or vehicle because several constraints have to be taken into account. An overview of packing problems discussed in literature can be found in Wäscher et al [103]. In a state-of-the-art review of container loading problems, Bortfeldt and Wäscher [12] identify several types of loading constraints which are container-related, item-related, cargo-related or load-related. Container-related constraints concern the container or vehicle in which the items are placed. Item-related constraints refer to individual items, where cargo-related constraints address a subset of items. Load-related constraints are related to the result of the packing process. In the following paragraphs, loading constraints that may be relevant in combination with vehicle routing problems are briefly discussed. The classification is mainly based on the taxonomy of Bortfeldt and Wäscher [12].

3.1 Classical (multi-) dimensional packing constraints

This constraint entails that items cannot overlap and should be completely packed inside the vehicle. In a three-dimensional problem the three dimensions (length, width and height) of the vehicle are considered to verify this constraint. In a one-dimensional and a two-dimensional problem respectively a single or two dimensions are taken into consideration. In a Bin Packing Problem (BPP), items are placed into a minimum number of identical bins (=vehicles). In a Strip Packing Problem (SPP), items are placed in an open ended rectangle with infinite height with the objective to minimize the total height.

3.2 Cargo-related constraints

Complete-Shipment constraints

When the vehicle capacity is not sufficient to accommodate all items, some items need to be left behind. Complete-shipment constraints may be specified when items from a certain subset need to stay together. Either all items of the subset have to be loaded, or none of them can be loaded [12]. Shipping companies that operate in the tramp market face complete-shipments constraints in their ship scheduling. Tramp shipping companies select cargoes at the spot market and construct routes to maximize profit [45]. A single order on the spot market may consist of several cargoes from different origins. Either all cargoes in the order are serviced by the shipping company or none of them may be serviced.

Allocation constraints

Allocation constraints may be specified when multiple vehicles or containers are considered. Two types of allocation constraints have been identified: connectivity constraints and separation constraints [12]. **Connectivity constraints** require that items of a certain subset are shipped in the same container or vehicle. In VRP literature it is common that each customer is visited only once and by a single vehicle (split deliveries are not allowed). It is therefore necessary that all items demanded by a customer are shipped in the same vehicle. As a result, connectivity constraints are incorporated in most VRP models [e.g. 54, 97, 52, 94]. Secondly, **separation constraints** may be specified to prevent that certain types of products are shipped in the same container or vehicle. Separation constraints may be relevant when different types of goods (e.g. food and toxic items) may not be transported together in the same vehicle. An example may be found in Battarra et al [8] where a distinction is made between three types of commodities: vegetables, fresh products (e.g. milk and meat) and non-perishable items. A variation of this constraint has been investigated in the multi-compartment VRP. The multi-compartment VRP allows the transport of different types of goods in separate compartments in the same vehicle. Applications of VRPs with multiple compartments can be found in the distribution of petrol (different types of petrol transported in one vehicle) [e.g. 14, 28], distribution of food (e.g. a refrigerated compartment and a regular compartment in one vehicle) [19], waste collection [84], on-farm milk collection [38] and ship scheduling [43].

Positioning constraints

The location of the items inside the vehicle may be restricted by positioning constraints. Absolute as well as relative positioning restrictions may be specified [12]. Absolute constraints refer specifically to a place inside the vehicle where the items may or may not be placed. Relative constraints allow or restrict the placement of the item relative to the positions of other items. An example of relative constraints may be found in Lurkin and Schyns [72]. The authors present an airline container loading problem in which they specify a minimum distance required between dangerous goods and other goods. In multi-drop situations a vehicle has multiple drop-off points in one trip. These situations usually require sequence based loading, which can be seen as a combination of relative and absolute constraints. Sequence based loading ensures that when arriving at a customer, no items belonging to customers served later, block the removal of items of the current customer. This constraint is commonly used in VRPs [e.g. 59, 54, 82, 36] and is in literature sometimes referred to as a Last-In-First-Out (LIFO) constraint. It is however important to remark that only when a single dimension is considered, it is truly LIFO, since in a two- and three- dimensional problem items can be placed beside each other.

3.3 Container-related constraints

Weight limits

The total weight of items in the vehicle or container should not exceed the weight capacity of the vehicle. Weight limits are a standard feature in VRPs. In several transportation modes (truck, airplane, ship) the weight capacity may be an important restriction when transporting heavy cargo.

Weight distribution constraints

To ensure the stability of the vehicle, it is important that there is a balanced distribution of the weight of the cargo in the vehicle. Several authors propose to achieve an even weight distribution by demanding that the center of gravity (CG) of the load is close to the midpoint of the container [e.g. 2, 53, 34, 12, 86]. Limbourg et al [70] propose an approach for loading ULDs (Unit Loading Devices) into an aircraft. To ensure the balance of the plane, the authors do not only take the center of gravity into consideration but also minimize the moment of inertia. The minimization of the moment of inertia leads to a more dense packing of the load around the CG which reduces stress on the aircraft structure and leads to a better aircraft manoeuvrability [70]. Although weight distribution is an important issue in practice [34], to the authors' knowledge it has only been considered once in combination with routing problems. Øvstebø et al [85] introduce weight distribution constraints in a maritime transportation problem. To ensure the stability of the ship, the torque from the cargo on the ship that makes the ship lean sideways and the distance between the bottom of the ship and its center of gravity are considered.

Closely related to an even weight distribution inside the vehicle, is the distribution of the cargo over the axles of the vehicle. A truck has several axles (at least two: one of the tractor and one of the trailer). The axle weight is the total weight (weight of the cargo and weight of the truck) that is placed on the axle. This is illustrated in figure 1. When item j is placed onto a vehicle, the weight of the item is divided over the axle of the tractor and the axle of the trailer. F_K^j represents the weight of item j placed on the axle of the tractor. F_A^j represents the weight of item j on the axle of the trailer. Axle weight limits impose a great challenge for transportation companies. Transporters face high fines when violating these limits, while current planning programs do not incorporate axle weight constraints. Legislation about axle weight limits varies by country (for an overview of the axle weight limits in Europe, the reader is referred to the International Transport Forum (2011)). Lim et al [69] address axle weight constraints in a container loading problem. They develop a heuristic method to tackle the single container loading problem with axle weight constraints. To the authors' knowledge, Pollaris et al [91] are the only authors that consider axle weight limits in a VRP. They propose a mixed integer linear programming model to solve the problem exactly.

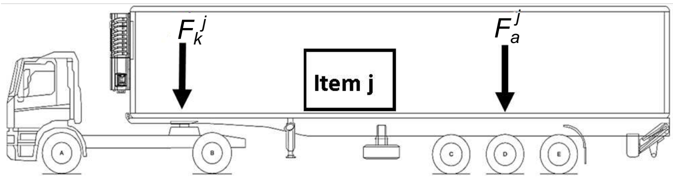

Fig. 1 Axle weight tractor and trailer (figure adapted from TruckScience)

3.4 Item-related constraints

Loading priorities

Loading priorities may play a role in the packing process when vehicle capacity is not sufficient to accommodate all items. The decision on which items are shipped and which are left behind may depend on factors as product shelf life and delivery deadlines [9]. Several papers in literature on aircraft loading [e.g. 50, 20, 100] take into account loading priorities to select the items to be loaded.

The incorporation of priorities in vehicle routing problems is considered in orienteering problems, where a score or priority is assigned to each location. Since the literature concerning orienteering problems does not consider any other loading constraints, papers dealing with the orienteering problem are not considered in the remaining of this paper. For a recent survey of the orienteering problem, the reader is referred to Vansteenwegen et al [101].

Orthogonality constraints

In packing literature, it is often assumed that items have a rectangular shape. In most papers [e.g. 54, 83, 59, 52], the edges of the items are assumed to be packed orthogonal or parallel with the edges of the vehicle. This constraint is often used in combination with two- and three-dimensional loading constraints.

Orientation constraints

The orientation of items may be fixed with respect to the height, width and length of the vehicle. The vertical orientation is often fixed to prevent the item from being damaged when put upside down in the container. A fixed vertical orientation constraint is also denoted as a "this-way-up!" constraint, referring to items that are marked with a "this-way-up!" sign [11]. The horizontal orientation of the items can be fixed as well [e.g. 60]. This may be necessary when items can only be accessed via a particular side (e.g. pallets that need to be accessed by forklifts) [12]. However, in most papers that incorporate orientation constraints, it is allowed to rotate the items 90 degrees on the width-length (horizontal) plane [e.g. 54, 97, 52, 111, 94]. This constraint is frequently used in VRPs with two- and three-dimensional loading constraints.

Stacking constraints

When items are placed on top of each other in the vehicle, items may be damaged by the pressure of items placed above them. Stacking constraints (also denoted as load-bearing strength constraints or fragility constraints) prevent this from happening. The load-bearing strength of an item is the maximum pressure that can be applied on this item [61]. The load bearing strength may vary across different vertical orientations of this item [92]. The box contents (solid contents vs. less solid contents) and loading conditions (humidity, duration of loading, way of stacking ...) may also influence the load bearing strength of an item [12]. Fragile items can be defined as items that cannot bear any pressure from other items, indicating that no item can be placed upon this item. Some models in literature [e.g. 54, 97, 52, 94] allow to place other fragile items upon fragile items, but forbid non-fragile items to be placed upon fragile ones. Stacking constraints have been considered in several papers concerning three-dimensional loading VRPs [e.g. 54, 97, 52, 94, 61].

3.5 Load-related constraints

Stability constraints

When items are stacked on top of each other in the vehicle, the items have to be supported by other items or by the floor to ensure vertical (or static) stability of the cargo. Vertical stability constraints specify the minimum supporting area of each item (for example as a percentage of the base area of the item). Horizontal (or dynamic) stability of the cargo refers to the support of the lateral faces of items in the container to avoid items from moving around in the container [61]. Literature concerning three-dimensional VRPs often take vertical stability constraints into account [e.g. 54, 52, 10, 111, 94]. According to the authors, horizontal stability constraints have not yet been considered explicitly in routing models in literature.

4 Integration of loading constraints in vehicle routing problems

The integration of loading constraints in VRPs is a fairly recent domain of research. The two problems are separately already NP hard. Combining these problems is therefore very challenging but leads to a better overall logistic solution. A survey conducted among several Belgian logistics service providers pointed out that they are faced with important loading problems in their route planning. Pollaris et al [90] point out that if a planning does not take into account axle weight constraints, it is likely that it contains axle weight violations for some trucks and ad hoc changes need to be made in the planning to make it feasible. The development of VRP models that incorporate loading constraints is therefore vital for a more efficient planning of routes. In this section, literature on the integration of vehicle routing problems and loading problems is reviewed. Since loading constraints also appear in a maritime transport context, papers introducing these constraints in routing problems for maritime transport are discussed as well. To the authors' knowledge, there is no literature on the integration of loading constraints in a routing model in an air transport context.

Papers dealing with the combination of routing problems and loading problems may be placed in following categories based on the type of routing problem and the loading characteristics that are dealt with: Two-Dimensional Loading CVRP (2L-CVRP), Three-Dimensional Loading CVRP (3L-CVRP), multi-pile VRP, multi-compartments VRP, Pallet Packing VRP (PPVRP), Minimum Multiple Trip VRP (MMTVRP) with incompatible commodities, Traveling Salesman Problem with Pickups and Deliveries (TSPPD) with LIFO/FIFO constraints, Double TSP with Pickups and Deliveries with Multiple Stacks (DTSPMS) and Vehicle Routing Problem with Pickups and Deliveries (VRPPD) with additional loading constraints. This is a similar classification as used by Iori and Martello [58]. For each category, an overview of the loading constraints that are considered is provided. For this purpose, the classification of Bortfeldt and Wäscher [12] is used. In Table 1 an overview is provided from the papers concerning 2L-CVRP and 3L-CVRP. An overview from the papers concerning the multi-pile VRP, multi-compartments VRP, PPVRP and the MMTVRP with incompatible commodities is provided in Table 2. For the categories concerning the PDPs, an overview is provided in Table 3. With the exception of one paper [45], complete-shipment constraints and loading priorities are not applicable in the models since the capacity of the vehicle fleet is assumed to be sufficient to accommodate all items. Connectivity constraints on the other hand are standard features in routing models with multiple vehicles since it is often assumed that all items of a customer have to be shipped in the same vehicle. Vertical stability constraints and stacking constraints are only relevant when the height dimension is taken into account. Orthogonality and orientation constraints are only applicable when at least two dimensions are considered. A discussion of the papers in each section will be provided in the following paragraphs. A general remark is that few other rich constraints (besides loading constraints) are included in the current VRP models with loading constraints. When models do include other real-life constraints (such as time windows or a heterogeneous vehicle fleet), these will be mentioned. In most papers described in this survey, the objective function aims at minimizing total routing costs or travel distance. If the objective function is different, this will be mentioned in the description of the problem. Another observation is that problems in which more than one dimension is considered (2L-CVRP, 3L-CVRP, pallet packing VRP) are mostly solved with a two-stage approach. The routing problem acts as the main problem and iteratively calls exact or heuristic methods to solve the packing subproblem [96]. The methods for solving the packing problem are mostly based on bin packing literature [e.g. 7, 71, 76]. Maximum touching perimeter (or touching area in the three-dimensional case) and bottom-left-fill are often used to solve two- and three dimensional packing problems heuristically [e.g. 59, 54, 97, 96, 37], while branch-and-bound methods and lower bounds are usually employed to deal with the packing problem exactly [e.g. 59, 51, 55]. For each category with multi-dimensional loading, a paragraph describes how the packing problem is generally dealt with. For the other categories, the loading part is usually not that complex, which does not make it necessary to employ heuristics merely for the packing problem. In the latter case, the loading constraints are usually incorporated in the vehicle routing problem [e.g. 26, 88, 22].

4.1 Two-dimensional loading CVRP

In the Two-Dimensional Loading CVRP (2L-CVRP), the demand of the customers and the measurements of the vehicles are expressed in two dimensions. Usually the width and the length dimension are taken into account while the height dimension is not considered. In real-life applications, this problem arises in distribution logistics when items cannot be stacked on top of each other because of their weight, fragility or large dimensions [95]. Examples of applications may be found in the distribution of large kitchen appliances such as refrigerators, large mechanical components or fragile items such as porcelain. Two papers propose an exact method [59] [78]. Sequence based loading is assumed in most papers as well as multiple vehicles as can be seen in Table 1. When the height dimension is not considered, stacking constraints and vertical stability constraints are not applicable in the problems. A single paper assumes a heterogeneous fleet [66] and three papers consider time windows [5] [62] [78]. Martinez and Amaya [78], Dominguez et al [37] and Pollaris et al [91] present a mathematical formulation for a 2L-CVRP.

Iori et al [59] are the first to address a 2L-CVRP. They develop a branch-and-bound and solve the problem to optimality for up to 35 customers. The 2L-CVRP has also been solved heuristically with Tabu Search (TS) [55], guided TS [107], extended guided TS [65] and a local search metaheuristic [110].Fuellerer et al [51] employ an Ant Colony Optimisation (ACO) method for a similar problem, with a small alteration in the loading constraints. The items are allowed to rotate 90 degrees on the horizontal plane.

Attanasio et al [5] consider a variant of the 2L-CVRP based on a consolidation and dispatching problem of a multinational chemical company. Each shipment must take place within a multi-day time window, spanning from the manufacturing date to a given deadline. Only two dimensions are considered because all items and bins have the same height. Attanasio et al [5] develop a heuristic based on a cutting plane framework in which a simplified Integer Linear Program (ILP) is solved. Items are allowed to rotate and sequence based loading is assumed. Strodl et al [95] develop a Variable Neighborhood Search (VNS) to address the routing problem and formulate a heuristic and an exact procedure for the two-dimensional loading problem. Items have a fixed orientation and sequence based loading is not considered. Duhamel et al [39] address the 2L-CVRP without sequence based loading. They solve the problem using a two-stage approach. First, the 2L-CVRP is converted into a Resource

Table 1 Papers on 2L-CVRP and 3L-CVRP

	Exact solution method	Heterogenous vehicle fleet	Time windows	Classical packing	Complete - Shipment	Connectivity	Separation	Positioning (*)	Weight limits	Weight distribution	Loading priorities	Orthogonality	Orientation	Stacking (fragility)	Vertical stability	Horizontal stability
2L-CVRP																
Iori et al [59]	x			x	-	x		x	x		-	x	x	-	-	
Attanasio et al [5]			x	x	-	x		x	x		-		x	-	-	
Gendreau et al [55]				x	-	x		x	x		-	x	x	-	-	
Fuellerer et al [51]				x	-	x		x	x		-	x	x	-	-	
Zachariadis et al [107]				x	-	x		x	x		-	x	x	-	-	
Strodl et al [95]				x	-	x			x		-	x	x	-	-	
Leung et al [65]				x	-	x		x	x		-	x	x	-	-	
Duhamel et al [39]				x	-	x			x		-	x	x	-	-	
Leung et al [66]		x		x	-	x		x	x		-	x	x	-	-	
Khebbache-Hadji et al [62]			x	x	-	x			x		-	x	x	-	-	
Zachariadis et al [110]				x	-	x		x	x		-	x	x	-	-	
Martinez and Amaya [78] (1)	x		x	x	-	x					-			-	-	
Martinez and Amaya [78] (2)			x	x	-	x					-			-	-	
Dominguez et al [37]				x	-	x			x		-	x	x	-	-	
Pollaris et al [91]				x	-	x		x	x		-	x	x	-	-	
3L-CVRP																
Gendreau et al [54]				x	-	x		x	x		-	x	x	x	x	
Aprile et al [3]				x	-	x					-					
Moura [82]			x	x	-	x		x			-	x	x	x		
Moura and Oliveira [83]			x	x	-	x		x			-	x	x	x		
Tarantilis et al [97]				x	-	x		x	x		-	x	x	x	x	
Fuellerer et al [52]				x	-	x		x	x		-	x	x	x	x	
Ren et al [93]				x	-	x		x	x		-	x	x	x	x	
Massen et al [79]				x	-	x		x	x		-	x	x	x	x	
Bortfeldt [10]				x	-	x		x	x		-	x	x	x	x	
Wisniewski et al [105]				x	-	x		x	x		-	x	x	x	x	
Zhu et al [111]				x	-	x		x	x		-	x	x	x	x	
Miao et al [81]				x	-	x		x	x		-	x	x	x	x	
Ruan et al [94]				x	-	x		x	x		-	x	x	x	x	
Bortfeldt and Homberger [11]			x	x	-	x		x	x		-	x	x	x	x	
Ceschia et al [18]		x		x	-	x		x	x		-	x	x	x	x	
Tao and Wang [96]				x	-	x		x	x		-	x	x	x	x	
Junqueira et al [61]	x			x	-	x			x		-	x	x	x	x	

x = considered in the reference, - = not applicable in the reference, ?= not mentioned in the reference
(*) positioning constraints refer in most papers to sequence based loading (or LIFO loading)

Table 2 Papers on multi-pile VRP, multi-compartments VRP, Pallet-Packing VRP and MMTVRP with incompatible commodities

	Exact solution method	Heterogenous vehicle fleet	Time windows	Classical packing	Complete - Shipment	Connectivity	Separation	Positioning (*)	Weight limits	Weight distribution	Loading priorities	Orthogonality	Orientation	Stacking (fragility)	Vertical stability	Horizontal stability
Multi-pile VRP																
Doerner et al [36]				x	-	x		x			-	-	-			
Tricoire et al [99] (1)				x	-	x		x			-	-	-			
Tricoire et al [99] (2)	x			x	-	x		x			-	-	-			
Massen et al [79]				x	-	x		x			-	-	-			
Multi-compartments VRP																
Brown and Graves [14]		x		x	-	x	x	-	x		x	-	-	-	-	-
Avella et al [6] (1)		x		x	-	x	x	-			-	-	-	-	-	-
Avella et al [6] (2)	x	x		x	-	x	x	-			-	-	-	-	-	-
Cornillier et al [28]	x	x		x	-	x	x	-			-	-	-	-	-	-
Cornillier et al [29]		x		x	-	x	x	-			?	-	-	-	-	-
Cornillier et al [30]		x	x	x	-	x	x	-			-	-	-	-	-	-
Cornillier et al [31]		x	x	x	-	x	x	-			-	-	-	-	-	-
Fagerholt and Christiansen [43]	x	x	x	x	x	x	x	-	x		-	-	-	-	-	-
Fagerholt and Christiansen [44]	x		x	x	x	x	x	-	x		-	-	-	-	-	-
Chajakis and Guignard [19]	x		x	x	-	x	x	-	x		-	-	-	-	-	-
Dooley et al [38]				x	?	?	x	-			?	-	-	-	-	-
Fallahi et al [46]				x	-		x	-	x		-	-	-	-	-	-
Mendoza et al [80]				x	-	x	x	-			-	-	-	-	-	-
Muyldermans and Pang [84]				x	-	x	x	-			-	-	-	-	-	-
Pallet Packing VRP																
Zachariadis et al [108]			x	x	-	x					-	x	x		x	
Zachariadis et al [109]			x	x	-	x					-	x	x		x	
MMTVRP with incompatible commodities																
Battarra et al [8]			x	x	-	x	x				-	-	-	-	-	-

x = considered in the reference, - = not applicable in the reference, ?= not mentioned in the reference
(*) positioning constraints refer in most papers to sequence based loading (or LIFO loading)

Constraint Project Scheduling Problem - CVRP (RCPSP-CVRP) by relaxing the bin packing constraints. The items in the packing problem are represented by activities in the RCPSP. Each activity has a duration (length of item) and requirement of resource (width of item). A route is feasible if the makespan of the RCPSP does not exceed the length of the vehicle [39]. The RCPSP-CVRP is solved with a Greedy Randomized Adaptive Search Procedure (GRASP) in an Evolutionary Local Search (ELS) framework. In the second step, the feasibility of the best RCPSP-CVRP solutions with the 2L-CVRP constraints are checked by transforming the RCPSP-CVRP solutions into 2L-CVRP solu-

Table 3 Papers on TSPPD, DTSPMS, VRPPD

	Exact solution method	Heterogenous vehicle fleet	Time windows	Classical packing	Complete - Shipment	Connectivity	Separation	Positioning (*)	Weight limits	Weight distribution	Loading priorities	Orthogonality	Orientation	Stacking (fragility)	Vertical stability	Horizontal stability
TSPPD with LIFO/FIFO constraints																
Ladany and Mehrez [63]	x	-		x	-	-		x			-	-	-	-	-	-
Pacheco (in [49])		-		x	-	-		x			-	-	-	-	-	-
Levitin and Abezgaouz [67]	x	-		x	-	-		x			-	-	-	-	-	-
Carrabs et al [15]		-		x	-	-		x			-	-	-	-	-	-
Carrabs et al [16] (1)	x	-		x	-	-		x			-	-	-	-	-	-
Carrabs et al [16] (2)	x	-		x	-	-		x$^{(a)}$			-	-	-	-	-	-
Erdoğan et al [42]		-		x	-	-		x$^{(a)}$			-	-	-	-	-	-
Arbib et al [4]	x	-		x	-	-		x			-	-	-	-	-	-
Cordeau et al [26]	x	-		x	-	-		x			-	-	-	-	-	-
Cordeau et al [27]	x	-		x	-	-		x$^{(a)}$			-	-	-	-	-	-
Li et al [68]		-		x	-	-		x			-	-	-	-	-	-
Øvstebø et al [85](1)	x	-	x	x	-	-		x		**x**	-	-	-	-	-	-
Øvstebø et al [85](2)		-	x	x	-	-		x		**x**	-	-	-	-	-	-
Côté et al [32]		-		x	-	-		x			-	-	-	-	-	-
Côté et al [33]	x	-		x	-	-		x			-	-	-	-	-	-
DTSPMS																
Petersen and Madsen [88]		-		x	-	-		x			-	-	-	-	-	-
Felipe et al [47]		-		x	-	-		x			-	-	-	-	-	-
Lusby et al [74]	x	-		x	-	-		x			-	-	-	-	-	-
Petersen et al [89]	x	-		x	-	-		x			-	-	-	-	-	-
Lusby and Larsen [73]	x	-		x	-	-		x			-	-	-	-	-	-
Alba et al [1]	x	-		x	-	-		x			-	-	-	-	-	-
Felipe et al [49]		-		x	-	-		x			-	-	-	-	-	-
Carrabs et al [17]	x	-		x	-	-		x			-	-	-	-	-	-
VRPPD with loading constraints																
Xu et al [106]		x	x	x	-	-	x	x			-	-	-	-	-	
Malapert et al [75]				x	-	x		x	x		-	x	x			
Cheang et al [21]				x	-	-		x			-	-	-	-	-	
Fagerholt et al [45]		x	x	x	x	x			x		-	-	-	-	-	
Cherkesly et al [22]	x		x	x	-	x		x	x		-	-	-	-	-	
Cherkesly et al [23]			x	x	-	x		x	x		-	-	-	-	-	

x = considered in the reference, - = not applicable in the reference
(*) positioning constraints refer in most papers to sequence based loading (or LIFO loading)
(a) : FIFO

tions. According to the authors, this approach saves a lot of computation time because a packing plan is only computed for the best RCPSP-CVRP solutions. Leung et al [66] develop a Simulated Annealing (SA) model to solve the 2L-CVRP with heterogeneous fleet. The packing constraints that are considered in this model are the same as in Iori et al [59]. The vehicles have different weight capacities and different measurements.

Martinez and Amaya [78] consider a VRP with multi-trips, time windows and two-dimensional circular loading constraints. A homogeneous fleet is considered and sequence based loading is not assumed. The problem is based on a real-life problem faced by a home-delivery service transporting perishable circular shaped products. A mixed integer non-linear programming mathematical model is developed to solve small-size problems (up to 17 customers) exactly. Furthermore, a two-step heuristic method is proposed to handle real-size instances. In the first step, an initial solution is built using a sequential insertion heuristic. In the second step this solution is improved with a TS algorithm.

Pollaris et al [91] present a mixed ILP model for the CVRP with sequence based pallet loading and axle weight constraints. This is a special case of 2L-CVRP in which all items are homogeneous pallets and may be placed in two horizontal rows in the vehicles. The model takes into account weight restrictions on the axles of the tractor and trailer of the vehicle at all times (i.e. at the depot as well as after each delivery). The authors compare the model to the CVRP with sequence based pallet loading without axle weight restrictions and conclude that not including axle weight restrictions may induce major violations of axle weight limits.

Dominguez et al [37] develop a biased-randomized algorithm for the 2L-CVRP with and without item rotations. The problem assumes a homogeneous vehicle fleet and sequence based loading is not considered. The algorithm uses a multi-start approach and combines at each restart a biased randomization of a savings-based routing algorithm as proposed by Clarke and Wright [24] for the routing part with a multi-start biased-randomized version of the best fit packing heuristic to check loading feasibility. In the first biased randomization process, the savings list of the edges is randomized using a biased probability distribution (geometric distribution). For the loading feasibility check, first a biased randomization is applied on the list of items to be loaded. Next, the best fit heuristic is used, beginning with the items at the top of the list. If after several iterations, the best fit heuristic does not find a feasible loading scheme, the proposed route will be assumed to be infeasible and a new randomization is applied on the savings list of the edges which will again be followed by a loading feasibility check.

Khebbache-Hadji et al [62] develop a heuristic solution method to solve the 2L-CVRP with Time Windows (2L-CVRPTW) without sequence based loading.

The packing feasibility check in the above papers consists of a mix between several types of solution methods (heuristic as well as exact). The most common methods are the bottom-left-fill heuristic [e.g. 59, 107, 51], maximum touching perimeter [e.g. 107, 95, 62], lower bounds [e.g. 59, 54, 51] and branch-and-bound [e.g. 59, 54, 51, 95]. If a combination of heuristic and exact algorithms is used, first the heuristics are applied and when they do not find a feasible solution, the exact method is used to solve the packing problem.

4.2 Three-dimensional loading CVRP

In the Three-Dimensional Loading CVRP (3L-CVRP), the three dimensions of the vehicle are taken into account and the demand of the customer also consists of three-dimensional items. Since the height dimension is considered, additional loading constraints concerning fragility and vertical stability of the cargo may be specified. This problem is frequently encountered in distribution logistics when items may be stacked on top of each other in the container. Examples of applications of the 3L-CVRP may be found in the distribution of furniture, household appliances, soft drinks and staple goods [94]. Sequence based loading is incorporated in most models as can be seen in Table 1. Most papers assume a homogeneous fleet, while only three papers consider time windows [82], [83], [11]. An exact solution method and a formulation of the 3L-CVRP is provided by Junqueira et al [61].

Gendreau et al [54] are the first to address the 3L-CVRP. Their model includes sequence based loading, stacking and vertical stability constraints and a fixed vertical orientation of the items in the vehicles (it is allowed to rotate the items 90 degrees on the width-length plane). The same problem is solved heuristically with ACO [52], a combination of TS and guided local search [97], Honey Bee Optimization [94], TS [10] [105] [111] and a combination of a Genetic Algorithm (GA) with a TS method (GATS) [81]. Ren et al [93]develop a hierarchical method to solve the 3L-CVRP. In the subordinated module, a branch-and-bound method is applied to find a solution for the modified 3L-CVRP in which the loading constraints are relaxed and replaced by a volume-ratio constraint. Next, a container loading algorithm is used to check if the items of the customers of each minimum cost route generated by the branch-and-bound algorithm can be feasibly loaded into the container. A superior module repeats this process and varies the volume-ratio until all items are feasibly loaded. Aprile et al [3] develop a Simulating Annealing heuristic (SA) to solve the 3L-CVRP. With regard to the loading constraints, only the classical three-dimensional packing constraints are included in their model. Tao and Wang [96] use a TS method to solve the 3L-CVRP heuristically. To the best of our knowledge, this is currently one of the best working heuristics in terms of solution quality and computational efficiency for the 3L-CVRP defined by Gendreau et al [54]. While the TS for the routing part is quite simple, the authors employ two mechanisms from 3D bin packing literature to help exploiting the loading space better. First, a least waste packing heuristic [104] is employed which aims at minimizing the space waste when packing an item into a vehicle. Second, the mechanism for updating new potential points in the container at which items may be loaded is a combination of normal points and corner points. While normal positions are widely used, corner points have not yet been used in the 3L-CVRP literature. Corner points follow the concept of envelope and are introduced by Martello et al [77] for 3D bin packing.

Junqueira et al [61] are to the authors' knowledge the first to propose an exact method to solve the 3L-CVRP. They assume a homogeneous vehicle fleet, sequence based loading, stacking constraints, orientation constraints and stability constraints. The authors take into account the unloading pattern of the items at customer places. By specifying a reach length of the worker or forklift, they avoid that items placed on top of items of other customers cannot be reached. An ILP is proposed to solve small-sized instances (number of customers < 15).

Bortfeldt and Homberger [11] develop a two-stage method, called *Packing first - Routing second* for the 3L-CVRP with Time Windows (3L-CVRPTW). In the first stage, the packing problem is solved for each customer separately. The resulting packing plans minimize the total loading length of the boxes of each customer in the vehicle. In the second stage, the vehicle routes are constructed with the constraint that the sum of the loading lengths (calculated in the first stage) may not exceed the length of the loading space of the vehicle. After these stages, a packing plan is determined for the previously generated routes. Moura [82] develops a multi-objective GA to solve the 3L-CVRPTW. The presented problem has three objectives: minimization of number vehicles, minimization of total distance traveled and maximization of volume utilization. The model considers sequence based loading, orientation constraints and stability constraints. In 2009, Moura and Oliveira [83] develop a sequential and a hierarchical approach to solve the 3L-CVRPTW. The objective is to minimize the number of vehicles and the total route time. In the hierarchical approach, the loading problem is seen as a subproblem of the routing problem. The routes are planned first and afterwards, for each route, the items are packed into the vehicles. As in Moura [82], the model considers sequence based loading, orientation constraints and stability constraints. In the sequential approach, the container loading and the vehicle routes are planned at the same time. The unloading sequence constraint is relaxed in this solution approach.

Massen et al [79] develop a column generation based heuristic method for vehicle routing problems with black box feasibility (VRPBB). In the VRPBB the routes of the basic VRP need to satisfy a number of unknown constraints. A black box algorithm is used to verify the feasibility of a route. Their approach is tested on the 3L-CVRP as well as on the multi-pile VRP.

Ceschia et al [18] consider the 3L-CVRP with sequence based loading and a (weakly) heterogeneous vehicle fleet. They consider stacking and stability constraints, orientation constraints, the maximum reach length of a worker or forklift as well as the possibility of split deliveries. Ceschia et al [18] solve the problem in one stage using a local search approach that combines SA and Large Neighborhood Search (LNS).

Maximum touching area and bottom-left-fill are often employed to check the loading feasibility in the 3L-CVRP literature [e.g. 54, 52, 111, 105, 94]. These heuristics are extensions of the bottom-left-fill and maximum touching perimeter methods from 2D bin packing literature. Tao and Wang [96] employ

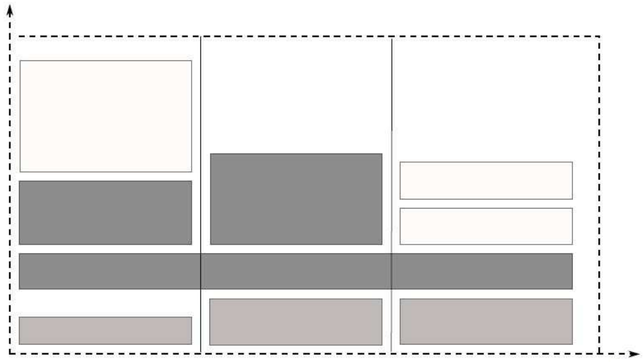

Fig. 2 Example of a multi-pile vehicle (figure adapted from [79])

in combination with maximum touching area, a least waste algorithm. Junqueira et al [61] solve the 3L-CVRP with an ILP in which they incorporate the 3D loading feasibility check.

4.3 Multi-pile VRP

The Multi-Pile Vehicle Routing Problem (MP-VRP) is introduced by Doerner et al [36]. They develop a TS method and an ACO heuristic to solve a real-world transportation problem regarding the transport of wooden chipboards. For every order, chipboards of the same type (small or large) are grouped into a unique item, which is placed onto a single pallet. The vehicle is divided into three piles on which pallets can be stacked. Pallets containing large chipboards can extend over multiple piles. The other pallets can be placed into a single pile. An example of a loading plan of a multi-pile vehicle can be found in Figure 2 where each color represents the items of one customer. Because of this specific configuration of pallets placed into multiple piles, the original three-dimensional problem can be reduced to a one-dimensional one. In all papers, a homogeneous vehicle fleet is assumed. A single paper proposes an exact solution method [99].

Tricoire et al [99] develop a combination of VNS and branch-and-cut to solve the MP-VRP exactly for instances with up to 44 customers and heuristically for large-sized instances. Tricoire et al [99] propose a general formulation for the VRP but do not formulate the packing problem. The authors use a pool of feasible packing solutions in their branch-and-cut algorithm. Those solutions are generated with a packing heuristic or a dynamic programming method. Massen et al [79] test a column generation method for vehicle routing problems with black box feasibility (VRPBB) on the MP-VRP.

Doerner et al [36] and Tricoire et al [99] both use a heuristic algorithm as well as dynamic programming to check the loading feasibility. The heuristic algorithm computes in a preprocessing phase the minimum height of the items of every customer and of the combined loading of the items of any pair of customers. Whenever a route is processed, this information is used to compute an upper bound for the total height of the load in the vehicle.

4.4 Multi-compartments VRP

The multi-compartments VRP is related to the multi-pile VRP. Vehicles with multiple compartments allow the transport of heterogeneous products in separate compartments in the same vehicle. The compartments are not always compatible with every type of product and certain product pairs cannot be loaded together into the same compartment [35]. Vehicle routing problems with compartments are encountered in several industries like the distribution of petrol, the distribution of food, waste collection, on-farm milk collection and ship scheduling. In this section, papers dealing with multi-compartments VRP are discussed. In several papers, a heterogeneous vehicle fleet and/or time windows are considered and various exact solution methods have been developed as shown in Table 2. El Fallahi et al [41] present a formulation for the multi-compartments VRP. Cornillier et al [29], Cornillier et al [30] and Cornillier et al [31] provide formulations for respectively the Petrol Station Replenishment Problem (PSRP), the PSRP with Time Windows (PSRP-TW) and the multi-depot PSRP-TW.

To our knowledge, Brown and Graves [14] are the first to consider the dispatching of petroleum tank trucks. Each tank truck has several compartments which may carry different types of petroleum. They develop an automated real-time dispatch system for the distribution of petroleum products for a major US oil company. Each order includes several gasoline products, jointly constituting a full truckload. Avella et al [6] also consider a real-life case of a company that supplies petrol to fuel pumps. Several less than truckload orders may be shipped in the same truck. They propose a solution method that uses a savings based routing algorithm for the generation of routes and a best fit decreasing heuristic for the packing problem. They also develop an exact method that uses a branch-and-price algorithm, based on a set partitioning formulation and can solve instances with up to 60 stations. The PSRP have been studied by Cornillier et al [28], Cornillier et al [29], Cornillier et al [30], Cornillier et al [31]. The aim of the PSRP is to optimize the delivery of several petroleum products to petrol stations. Compartments can only hold one type of product and since the compartments do not have flow meters, the content of one compartment may not be split between petrol stations. Cornillier et al [29] consider the multi-period PSRP while Cornillier et al [31] consider the PSRP-TW with multiple depots. The exact algorithm of Cornillier et al [30] may solve instances with up to 200 stations.

Fagerholt and Christiansen [43] consider the Ship Scheduling and Allocation Problem (SSAP) derived from a real-life case of the transport of mineral fertilizers by a bulk ship. The problem is comparable with a pickup and delivery problem with time windows and multiple compartments. The compartments are flexible and made by partitioning the loading space. They present a set partitioning approach to solve the problem exactly for instances with up to 70 customers. Fagerholt and Christiansen [44] focus on a subproblem of the SSAP studied by Fagerholt and Christiansen [43]. More precisely, they consider the Traveling Salesman Problem with Allocation, Time Windows and Precedence Constraints (TSP-ATWPC). They develop a dynamic programming algorithm to solve the problem exactly for instances with up to 70 customers.

Chajakis and Guignard [19] consider the distribution of goods to convenience stores in vehicles with multiple compartments. The authors develop two integer programming models for two possible cargo space layouts. Approximation schemes based on Langrangean Relaxation are presented to solve these problems exactly for instances with up to 240 customers. Dooley et al [38] use a GA software for the on-farm collection problem of milk. The model may be used to evaluate alternative transport management strategies with regards to milk collection.

El Fallahi et al [41] construct a memetic algorithm with a post-optimization phase based on path-relinking and a TS algorithm to solve the VRP with multiple compartments. Note that a memetic algorithm is a GA combined with a local search procedure to intensify the search. The authors assume that each compartment is dedicated to a single product. The demand of a customer for a given product type cannot be split between vehicles, but different product types of the same customer order can be split between several vehicles. Since order splitting is allowed, connectivity constraints are not included in the model. The results are compared with cases in literature in which order splitting is not allowed and conclude that order splitting improves the results on average. Secondly, the authors conclude that TS provides slightly better results than the memetic algorithm, but also requires more computation time. Mendoza et al [80] also construct a memetic algorithm to solve the VRP with multiple compartments and take into account stochastic demands.

Muyldermans and Pang [84] construct a guided local search metaheuristic to solve the VRP with multiple compartments. Their research is based on a one-dimensional co-collection problem of waste. Homogeneous vehicles with multiple compartments are used to co-collect different types of waste. Derigs et al [35] implement a portfolio of different heuristics to solve the VRP with multiple compartments.

4.5 Pallet packing VRP

The Pallet Packing VRP (PPVRP) is introduced by Zachariadis et al [108]. The demand of customers is in the form of three-dimensional rectangular boxes which are first feasibly stacked onto pallets. These pallets are then loaded onto the vehicle. The items demanded by a single customer must be stacked into the same pallet. Many real-world applications of the PPVRP arise in distribution logistics. Examples may be found in the grocery and pharmaceutical industry.

Distribution centers receive orders from grocery stores and manually pick and palletize the items of the orders for each store and send them to the store locations [108]. In the pharmaceutical industry, items are grouped into cartons that are palletized and transported from the production or distribution center to the pharmacies [108]. To the authors' knowledge, a formulation for the pallet packing VRP has not yet been provided.

Zachariadis et al [108] develop a local search metaheuristic strategy to solve the basic PPVRP and the PPVRP with time windows (PPVRPTW). They assume that every pallet can be unloaded at all times from the vehicle, without having to move other pallets. Because of this assumption, sequence based loading of the pallets onto the vehicle is not required. Sequence based loading of the boxes onto the pallets is also not assumed. Orientation, orthogonality as well as vertical stability constraints are considered for the loading of the boxes onto the pallets. Zachariadis et al [109] consider a variant of the PPVRP: the Pickup and Delivery Routing Problem with Time Windows and Pallet loading (PDRP-TWP). The key difference with the PPVRPTW is that there are two types of requests considered in the PDRP-TWP namely plane delivery requests as well as paired pickup and delivery requests. Zachariadis et al [109] extend the metaheuristic developed in Zachariadis et al [108] in order to deal with the paired pickup and delivery requests. The model takes into account the same routing and loading constraints as in Zachariadis et al [108].

For the 3D loading feasibility check for the packing of boxes onto pallets, the above papers employ in addition to the packing heuristics used in 3L-CVRP literature (bottom-left-fill and maximum touching area), also a heuristic that packs each box in the minimum volume cuboid that can accommodate this box [108]. This heuristic aims at finding a high degree of pallet volume utilization. The models also make use of a memory structure that keeps track of feasible and infeasible packing structures to avoid making the same feasibility check twice.

4.6 Minimum multiple trip VRP with incompatible commodities

Battarra et al [8] consider the minimum multiple trip VRP (MMTVRP) with time windows and incompatible commodities. Vehicles may perform multiple routes within a single trip (i.e. working shift) which is limited in total duration. The objective is to minimize the total number of multiple trips. Three types of products (vegetables, fresh products and non-perishable items) are considered which are incompatible with each other. This means that they cannot be transported together in a vehicle. One-dimensional loading is considered. Battarra et al [8] propose a two-phase heuristic that presents a solution by decomposing the problem into two subproblems. In the first subproblem, a set of routes is determined by using a VRPTW heuristic. In the second subproblem, the routes are aggregated into multiple trips by means of a packing heuristic. To the authors' knowledge, an exact method or a problem formulation have not yet been developed for the MMTVRP with incompatible commodities.

4.7 Traveling salesman problem with pickups and deliveries with LIFO/FIFO constraints

In a VRPPD, items can be picked up and delivered at customers visited by the vehicle, as opposed to the general VRPs in which items are only delivered at customer locations. In the TSPPD a single route needs to be determined. Applications of the TSPPD may be found in the routing of automated guided vehicles which move items between workstations, in dial-a-ride systems where passengers are transported between different pickup and delivery locations and in less-than-truckload transportation [40]. Papers that have appeared in literature concerning the TSPPD provide exact methods as well as heuristics to solve the problem and all consider, to the authors' knowledge, one-dimensional loading. The sequence based loading constraint can therefore be reduced to a LIFO constraint. First-in-first-out (FIFO) is also sometimes assumed as can be seen in Table 3. Furthermore, various models include time windows. Orthogonality constraints, orientation constraints and stacking constraints are not relevant since only one-dimensional models have been developed. Formulations for the TSPPD with LIFO loading are presented by Arbib et al [4] and Cordeau et al [26], while a formulation for the TSPPD with FIFO loading is presented by Erdoğan et al [42] and Cordeau et al [27]. Côté et al [33] present a formulation for the TSPPD with multiple stacks and LIFO loading. Øvstebø et al [85] give a formulation for the TSP on Roll-on/Roll-of (RoRo) ships.

Ladany and Mehrez [63] make the first contribution to the TSPPD with LIFO constraints. The motivation for their study is a real-world delivery problem in which reshuffling of goods inside a container causes cost and time losses. They are the first to deal with this problem of *reshuffling* in optimal routing design and are able to solve instances exactly with up to 3 requests. Later, Pacheco (1997, in [58]) develops a heuristic method to solve the TSPPD with LIFO constraints. Carrabs et al [15] develop a VNS to solve the TSPPD with LIFO loading. In 2007, Carrabs et al develop an additive branch-and-bound method to solve the same problem exactly for instances with up to 43 vertices. In the same paper, a branch-and-bound algorithm is applied to the TSPPD with FIFO loading. Erdoğan et al [42] and Cordeau et al [27] also consider the TSPPD with FIFO loading. Cordeau et al [27] tackle the problem with a branch-and-cut method and are able to solve instances with up to 43 vertices. Arbib et al [4] present a linear programming formulation of the TSPPD with LIFO loading. The authors solve the problem with up to 21 vertices using CPLEX 9.0. Cordeau et al [26] develop a branch-and-cut method to solve the TSPPD with LIFO for instances with up to 25 requests. Li et al [68] build upon and improve the VNS of Carrabs et al [15] to solve the problem heuristically.

Levitin and Abezgaouz [67] consider the routing of an Automated Guided Vehicle (AGV) which is used for carrying multiple pallets between workstations. Each new picked up pallet is placed on top of the pallets that are already carried by the AGV. To avoid rearranging the pallets at the workstations, LIFO is assumed. They develop an exact algorithm to solve the problem with up to 100 vertices.

Côté et al [32] consider the TSPPD with multiple stacks with LIFO loading. A LNS is proposed to solve the problem heuristically. Côté et al [33] propose a branch-and-cut algorithm for the same problem (TSPPD with multiple stacks and LIFO loading) and are able to solve instances with up to 43 vertices.

Øvstebø et al [85] examine a similar problem on Roll-on/Roll-of (RoRo) ships that transport cargo on wheels. The ship contains several decks and each deck may be divided into several lanes in which the cargo may be placed. The lanes may be compared to stacks in a truck. Sequence based loading, stability constraints as well as time windows are considered. Sequence based loading is modeled as a soft constraint. A penalty cost is incurred if the constraint is violated. According to the authors this corresponds to reality because although reshuffling of cargo represents an inconvenience, this may be allowed in RoRo setting if more cargo can be carried. Two types of stability measures concerning weight distribution are considered. The first one is the torque from the cargo on the ship that makes the ship lean side-ways which should be within limits at all times. The second stability measure is the distance of the ship's bottom deck to the center of gravity of the ship which should be less than some specified ceiling at all times. The aim of the problem is to maximize the revenue from cargo carried from optional nodes minus a penalty for cargo not carried from mandatory nodes, a penalty for violating the sequence based loading constraint, travel cost, and cost of ship usage. A mixed integer programming model is used to solve the problem exactly for instances with up to 8 requests. A heuristic method which consists of a TS and a squeaky wheel optimization construction heuristic is developed to solve larger (realistic) instances.

4.8 Double traveling salesman problem with pickups and deliveries with multiple stacks

The Double Traveling Salesman Problem with Multiple Stacks (DTSPMS) is proposed by Petersen and Madsen [88]. Pickup and delivery of goods are performed in two separate networks. All pickups must be made before any delivery can take place. The goods cannot be repacked, nor vertically stacked. The goods can be placed in several rows (horizontal stacks). In each row the LIFO principle has to be obeyed. It is assumed that each order consists of a single item. The problem is based on a real-world application in which in a first phase a container is loaded onto a truck to perform pickup operations and returned by that truck to a depot or terminal. In a second phase, the container is loaded onto a train, ship, plane or another truck and transported to another depot or terminal. In the depots or terminals, there are no facilities to repack the items inside the container. In the final phase, the container is again transferred to a truck which performs the delivery operations [88]. A solution for the DTSPMS consists of a pickup and a delivery tour with a corresponding feasible packing plan for the items in the container. The total combined distance of the pickup and delivery tour is minimized. In Figure 3 an example of a simple DTSPMS with four items and two stacks is displayed. Items are picked up in the pickup tour (a) and delivered in the delivery tour (b). A possible feasible packing plan can be found in the last picture (c). The vehicle starts at the pickup depot at node 0, loads items h, i, j, k and returns to the pickup depot. Then the vehicle goes to the delivery depot and delivers items i, k, h, j and returns to the delivery depot. The loading of the items in the stack is done from bottom to top and the unloading from top to bottom. In the loading plan can be seen that the LIFO constraints in both stacks are satisfied. All models take into account one-dimensional packing constraints and LIFO loading in each stack. Several exact solution methods have been developed as may be seen in Table 3. A formulation of the DTSPMS is presented by Petersen and Madsen [88]. To our knowledge, none of the papers tackling the DTSPMS include time windows.

Petersen and Madsen [88] develop four metaheuristics to tackle the problem: Iterated Local Search (ILS), TS, SA and LNS. In the ILS, the method of the steepest descent is used as local search strategy. This means that after each random restart, the solution providing the best improvement is chosen. According to the authors, results indicate that the LNS performs much better than the other three methods. Felipe et al [48] develop four new neighborhood structures for the DTSPMS which are implemented in a VNS and a SA method. Lusby et al [74] propose an exact algorithm to solve the DTSPMS for instances with up to 18 requests. They first generate a set of pickup tours and a set of delivery tours. In a second step, combinations of delivery and pickup tours are matched in the TSP matching problem which verifies whether the combinations generate a feasible packing plan. Only the k-best delivery and pickup tours in terms of length are considered. Petersen et al [89] propose several different modeling approaches for an exact solution of the DTSPMS. First, a branch-and-cut approach is used on the mathematical programming formulation of the problem introduced in Petersen and Madsen [88] which is called the 'precedence' model. Next, a variation of the precedence model is proposed and solved with a branch-and-cut approach. Finally, two new different mathematical formulations (the flow model and the TSP with Infeasible Paths (TSPIP)) are developed. To solve the flow model, again a branch-and-cut approach is used. For the TSPIP a decomposition approach is used to solve the problem. The solution of the TSPIP with a decomposition approach turned out to be the most successful approach in which the problem is solved exactly for instances with up to 25 requests. Lusby and Larsen [73] improve the exact method developed by Lusby et al [74] by including an additional preprocessing technique: the longest common subsequence between the pickup and the delivery tour. This preprocessing technique significantly decreases the number of matching problems that need to be solved. This allows to consider more matching problems in a shorter amount of time and dramatically improves the efficiency of the solution method. The authors are able to solve instances with up to 28 requests. Alba et al [1] develop a branch-and-cut algorithm to solve the DTSPMS exactly for instances with up to 25 requests. Felipe et al [49] improve the previously developed VNS in Felipe et al [48] by allowing

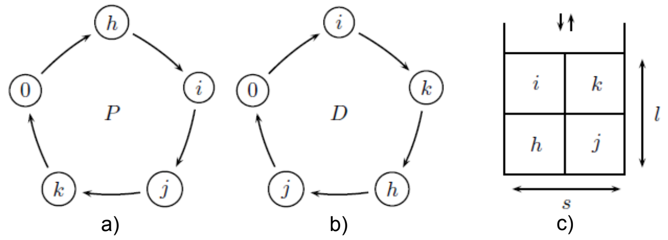

Fig. 3 A simple DTSPMS example with a pickup tour (a), a delivery tour (b) and a loading plan (c) (figure from [1])

intermediate infeasible solutions. Carrabs et al [17] consider the double TSP with two stacks. They develop a branch-and-bound algorithm to solve this problem exactly for instances with up to 29 requests.

4.9 VRP with pickups and deliveries with additional loading constraints

To the authors' knowledge, seven papers in literature so far consider pickup and delivery problems with multiple vehicles combined with loading constraints. Five of them consider one-dimensional loading. Time windows as well as a heterogeneous vehicle fleet are sometimes included as shown in Table 3. A single paper proposes an exact solution [22]. Fagerholt et al [45] present a formulation for the VRPPD with time windows, complete-shipment constraints and connectivity constraints. Cherkesly et al [22] present a formulation for the VRPPD with time windows and LIFO loading. The VRPPD with multiple vehicles is a generalization of the TSPPD. As a consequence all applications (AGVs, dial-a-ride-problems, less-than-truckload transportation ...) of the TSPPD may be considered by the VRPPD with the additional possibility of using more than a single vehicle, which is often encountered in real-life [13].

Xu et al [106] present a practical pickup and delivery problem in which they consider multiple time windows, heterogeneous vehicles, compatibility constraints between items and vehicle types, separation constraints, driver's work rules and LIFO loading. The authors solve this problem with a hybrid approach in which heuristics are integrated in a column generation framework. Cheang et al [21] consider the multiple vehicle pickup and delivery problem with LIFO loading and distance constraints. A homogeneous fleet is assumed. A two-stage method is proposed to solve the problem. In the first stage the number of vehicles required is minimized using a SA and an ejection pool approach. The second stage minimizes total travel distance using a VNS and a probabilistic TS.

Fagerholt et al [45] present a VRPPD with time windows and loading constraints to solve a real-life ship routing and scheduling problem that arises in tramp shipping. Complete-shipment constraints, connectivity constraints and a heterogeneous vehicle fleet are taken into account. The objective function maximizes the revenue from the optional spot cargoes minus the variable sailing and port costs through the planning period. A TS heuristic is proposed to solve the problem.

Cherkesly et al [22] consider the VRPPD with time windows and LIFO loading. They develop three branch-price-and-cut algorithms to solve the problem exactly for instances with up to 75 requests. Cherkesly et al [23] develop a population based metaheuristic to solve larger instances of the same problem heuristically. In both papers the number of vehicles is first minimized before minimizing the total traveled distance. Zachariadis et al [109] consider the Pickup and Delivery Routing Problem with Time Windows and Pallet loading (PDRP-TWP) which is discussed in Section 4.5.

Malapert et al [75] propose a framework to handle the two-dimensional VRPPD with multiple vehicles and sequence based loading. Items have to be packed orthogonal to the sides of the loading surface and the orientation of the items is fixed. A constraint programming model is formulated and a simple commitment heuristic is applied but turned out not to be efficient to solve the problem. According to the authors, most packing techniques use reduction procedures which are not compatible with the sequence based loading constraint.

4.10 Benchmark instances

In Table 4, an overview of benchmark instances on routing problems with loadings constraints is provided. A distinction is made between different types of problems. For each benchmark instance, the references of papers that use the instances, the number of vertices, the number of instances and the link to the website is provided.

Table 4 Benchmark instances

	Ref	V	I	Website
2L-CVRP				
Iori et al [59]	[55] [51] [107] [95] [65] [39] [62] [110] [37]	15-255	180	http://www.or.deis.unibo.it/research.html
Martinez and Amaya [78]		6-30	67	http://ftpprof.uniandes.edu.co/ pylo/inst/VRPM-TW-CL/instances.htm
3L CVRP				
Gendreau et al [54]	[97] [93] [79] [52] [10] [105] [111] [96] [81] [11] [18]	15-100	27	http://www.or.deis.unibo.it/research.html
Moura and Oliveira [83]	[82] [11]	25	46	www.fe.up.pt/ esicup
Bortfeldt and Homberger [11]		100-1000	120	http://www.fernuni-hagen.de/evis/service/downloads.shtml
Ceschia et al [18]		11-129	13	http://www.diegm.uniud.it/ceschia/index.php?page=vrclp
Multi-pile VRP				
Doerner et al [36]	[99] [79]	50-100	21	http://prolog.univie.ac.at/research/VRPandBPP/
Multi-compartments VRP				
Cornillier et al [30]		15-50	41	http://www.fsa.ulaval.ca/personnel/renaudj/Recherche/PSRPTW/
Pallet packing VRP				
Zachariadis et al [108]		50-200	70 (no TW) 68 (TW)	http://users.ntua.gr/ezach/
Zachariadis et al [109]		100	36	http://users.ntua.gr/ezach/
TSPPD				
Carrabs et al [15]	[42] [26] [27] [68] [32] [33]	24-750	42	http://neumann.hec.ca/chairelogistique/data
Li et al [68]		24-1000	96	http://www.tigerqin.com/publicatoins/tsppdl
DTSPMS				
Petersen and Madsen [88]	[48] [74] [89] [1] [49] [17]	12-66	60	Website no longer available
Felipe et al [48]	[49]	132	20	http://www.mat.ucm.es/ gregoriotd/dtspmsEn.htm
VRPPD				
Cheang et al [21]		24-750	126	http://www.computational-logistics.org/orlib/topic/MTSPPDL/

Ref = other papers using this dataset
V = number of vertices
I = number of instances

5 Discussion and future research

This paper provides a literature review of vehicle routing problems with loading constraints. Although a lot of research has been done on *classic* VRPs, these often do not reflect the problems distributors are currently facing. An important flaw of the *classic* VRP is the absence of several real-life loading constraints. An overview of loading constraints, mainly based on the classification of Bortfeldt and Wäscher [12], is provided. Recently, a number of papers have addressed the integration of loading constraints in vehicle routing problems. These papers may be placed in the following categories based on the type of routing problem and the loading characteristics: Two-Dimensional Loading CVRP (2L-CVRP), Three-Dimensional Loading CVRP (3L-CVRP), multi-pile VRP, multi-compartments VRP, Pallet Packing VRP (PPVRP), Minimum Multiple Trip VRP (MMTVRP) with incompatible commodities, Traveling Salesman Problem with Pickups and Deliveries (TSPPD) with LIFO/FIFO constraints, Double TSP with Pickups and Deliveries with Multiple Stacks (DTSPMS) and Vehicle Routing Problem with Pickups and Deliveries (VRPPD) with additional loading constraints. The last three categories consider pickup and delivery problems in which items may be picked up and delivered at customer places. For each category, the relevant loading constraints that are incorporated into the models are described and the available formulations are provided. Only a limited number of papers present a problem formulation. An explanation may be that including loading constraints in a routing problem usually makes the problem formulation much more complex. Adding a three-dimensional loading constraint does not imply adding a single extra row to the formulation, but affects the formulation as a whole. A second reason may be that due to the complexity of the problem mostly heuristic methods are developed which do not necessarily require a problem formulation.

The complexity of the problem not only depends on the complexity of the routing constraints and loading constraints separately, but is also influenced by the combination of the different constraints. For example, sequence based loading becomes much more complex in a three-dimensional loading problem than in a one-dimensional problem. The type of transportation request (pickup and delivery of items, or only a single type of request) influences in return the complexity of the sequence based loading constraint. A general observation from the literature survey is that in most models [e.g. 54, 36, 97, 10, 52, 94], loading constraints are handled as a subproblem of the routing model. First, solutions of the routing problem are computed, and afterwards, a feasibility check of the loading constraints is performed. Since loading constraints are often complex, a considerable amount of time may be saved by only checking the *best* solutions of the routing model. There are some exceptions to this method of incorporating loading constraints in VRP models, such as the sequential approach of Moura and Oliveira [83] in which the container loading and the vehicle routes are planned at the same time. Another example is the Packing First - Routing Second heuristic of Bortfeldt and Homberger [11] in which first a feasible packing scheme for each customer individually is computed and afterwards the routes are composed, followed by an optimization of the overall packing plan of all customers belonging to a single route.

As the combination of vehicle routing problems with loading constraints is a fairly recent domain of research, a number of opportunities for future research can be identified. An interesting research direction could be incorporating weight distribution constraints into VRPs. In packing literature, an even weight distribution of the cargo inside the vehicle is often achieved by placing the center of gravity of the load as close as possible to the midpoint of the container. Closely related to an even weight distribution inside the vehicle, is the distribution of the weight of the cargo over the axles of the vehicle. Axle weight limits impose a great challenge for transportation companies since they face high fines when violating these limits. Since the weight distribution changes each time a load is picked up or delivered at a customer, this should not only be monitored at the moment of departure but also during the rest of the trip. To the authors' knowledge, weight distribution constraints and axle weight restrictions have only been modeled once in combination with a routing problem by respectively [85] and [91].

Another line of future research could focus on pickup and delivery problems with loading constraints. Except for a single paper [75], current literature concerning PDPs only takes one dimension into account. Furthermore, few solutions methods for PDPs with loading constraints and multiple vehicles have been developed. In future research, PDPs with multiple vehicles and multiple dimensions may be analyzed. With regards to the multi-compartments VRP, future research might focus on planning over multiple periods or over multiple trips in a single tour where contamination from load residuals may be considered. If a product is transported in a certain compartment, it is possible that even after emptying the compartment, it cannot be used to transport another product unless it is thoroughly cleaned. With respect to solution methods, it is observed that at the moment few exact methods have been developed to solve VRPs with loading constraints. Future research could therefore focus on creating exact methods to solve VRPs with loading constraints to which heuristic solutions may be compared. A final observation is that rarely other *rich* constraints are incorporated into the current VRP models with loading constraints. Even time windows are not often included in the current models. It may be interesting to include time windows or other additional constraints such as a heterogeneous vehicle fleet, maximum route length and duration or drivers' regulations in current VRP models with loading constraints to make them more realistic.

Acknowledgements This work is supported by the Interuniversity Attraction Poles Programme initiated by the Belgian Science Policy Office (COMEX project: Combinatorial Optimization: Metaheuristics and Exact methods).

References

1. Alba M, Cordeau J, Dell'Amico M, Iori M (2011) A branch-and-cut algorithm for the double traveling salesman problem with multiple stacks. INFORMS Journal on Computing
2. Amiouny S, Bartholdi J, Vande Vate J, Zhang J (1992) Balanced loading. Operations Research 40(2):238–246
3. Aprile D, Egeblad J, Garavelli A, Lisi S, Pisinger D (2007) Logistics optimization: vehicle routing with loading constraints. In: Proceedings of the 19th International Conference on Production Research
4. Arbib C, Marinelli F, Servillio M (2009) On the pickup and delivery travelling salesman problem with LIFO loading. In: Proceedings of the International Network Optimisation Conference 2009, Pisa, Italy
5. Attanasio A, Fuduli A, Ghiani G, Triki C (2007) Integrated shipment dispatching and packing problems: a case study. Journal of Mathematical Modelling and Algorithms 6(1):77–85
6. Avella P, Boccia M, Sforza A (2004) Solving a fuel delivery problem by heuristic and exact approaches. European Journal of Operational Research 152(1):170–179
7. Baker B, Coffman E Jr, Rivest R (1980) Orthogonal packings in two dimensions. SIAM Journal on Computing 9(4):846–855
8. Battarra M, Monaci M, Vigo D (2009) An adaptive guidance approach for the heuristic solution of a minimum multiple trip vehicle routing problem. Computers & Operations Research 36(11):3041–3050
9. Bischoff E, MSW Ratcliff M (1995) Issues in the development of approaches to container loading. Omega 23(4):377 − 390
10. Bortfeldt A (2012) A hybrid algorithm for the capacitated vehicle routing problem with three-dimensional loading constraints. Computers & Operations Research 39(9):2248–2257
11. Bortfeldt A, Homberger J (2013) Packing first, routing second a heuristic for the vehicle routing and loading problem. Computers & Operations Research 40(3):873–885
12. Bortfeldt A, Wäscher G (2013) Constraints in container loading a state-of-the-art review. European Journal of Operational Research 229(1)
13. Braekers K, Caris A, Janssens G (2014) Exact and metaheuristic approach for a general heterogeneous dial-a-ride problem with multiple depots. Transportation Research Part B: Methodological 67:166 − 186
14. Brown GG, Graves GW (1981) Real-time dispatch of petroleum tank trucks. Management Science 27(1):19–32
15. Carrabs F, Cordeau J, Laporte G (2007a) Variable neighborhood search for the pickup and delivery traveling salesman problem with LIFO loading. INFORMS Journal on Computing 19(4):618–632
16. Carrabs F, Cerulli R, Cordeau J (2007b) An additive branch-and-bound algorithm for the pickup and delivery traveling salesman problem with LIFO or FIFO loading. INFOR: Information Systems and Operational Research 45(4):223–238

17. Carrabs F, Cerulli R, Speranza MG (2013) A branch-and-bound algorithm for the double travelling salesman problem with two stacks. Networks 61(1):58–75
18. Ceschia S, Schaerf A, Stützle T (2013) Local search techniques for a routing-packing problem. Computers & Industrial Engineering 66(4):1138 – 1149
19. Chajakis ED, Guignard M (2003) Scheduling deliveries in vehicles with multiple compartments. J of Global Optimization 26(1):43–78
20. Chan F, Bhagwat R, Kumar N, Tiwari M, Lam P (2006) Development of a decision support system for air-cargo pallets loading problem: A case study. Expert Systems with Applications 31(3):472 – 485
21. Cheang B, Gao X, Lim A, Qin H, Zhu W (2012) Multiple pickup and delivery traveling salesman problem with last-in-first-out loading and distance constraints. European Journal of Operational Research 223(1):60–75
22. Cherkesly M, Desaulniers G, Laporte G (2014) Branch-price-and-cut algorithms for the pickup and delivery problem with time windows and last-in-first-out loading. Transportation Science 0(0):null, DOI 10.1287/trsc.2014.0535, URL `http://dx.doi.org/10.1287/trsc.2014.0535`
23. Cherkesly M, Desaulniers G, Laporte G (2014) A population-based metaheuristic for the pickup and delivery problem with time windows and lifo loading. Technical report, Les Cahiers du GERAD, G-2014-66, GERAD, Montréal
24. Clarke G, Wright JW (1964) Scheduling of vehicles from a central depot to a number of delivery points. Operations Research 12(4):568–581
25. Cordeau J, Laporte G, Savelsbergh MW, Vigo D (2007) Chapter 6 vehicle routing. In: Barnhart C, Laporte G (eds) Transportation, Handbooks in Operations Research and Management Science, vol Volume 14, Elsevier, pp 367–428
26. Cordeau J, Iori G Mand Laporte, Salazar Gonzalez J (2010) A branch-and-cut algorithm for the pickup and delivery traveling salesman problem with LIFO loading. Networks 55(1):46–59
27. Cordeau J, Dell ´ Amico M, Iori M (2010b) Branch-and-cut for the pickup and delivery traveling salesman problem with FIFO loading. Computers & Operations Research 37(5):970–980
28. Cornillier F, Boctor FF, Laporte G, Renaud J (2008a) An exact algorithm for the petrol station replenishment problem. Journal of the Operational Research Society 59(5):607–615
29. Cornillier F, Boctor FF, Laporte G, Renaud J (2008b) A heuristic for the multi-period petrol station replenishment problem. European Journal of Operational Research 191(2):295–305
30. Cornillier F, Laporte G, Boctor FF, Renaud J (2009) The petrol station replenishment problem with time windows. Computers & Operations Research 36(3):919–935
31. Cornillier F, Boctor F, Renaud J (2012) Heuristics for the multi-depot petrol station replenishment problem with time windows. European Jour-

nal of Operational Research 220(2):361–369
32. Côté J, Gendreau M, Potvin J (2012a) Large neighborhood search for the pickup and delivery traveling salesman problem with multiple stacks. Networks 60(1):19–30
33. Côté J, Archetti C, Speranza M, Gendreau J Mand Potvin (2012b) A branch-and-cut algorithm for the pickup and delivery traveling salesman problem with multiple stacks. Networks 60(4):212–226
34. Davies A, Bischoff E (1999) Weight distribution considerations in container loading. European Journal of Operational Research 114(3):509–527
35. Derigs U, Gottlieb J, Kalkoff J, Piesche M, Rothlauf F, Vogel U (2011) Vehicle routing with compartments: applications, modelling and heuristics. OR Spectrum 33(4):885–914
36. Doerner K, Fuellerer G, Hartl R, Gronalt M, Iori M (2007) Metaheuristics for the vehicle routing problem with loading constraints. Networks 49(4):294–307
37. Dominguez O, Juan AA, Faulin J (2014) A biased-randomized algorithm for the two-dimensional vehicle routing problem with and without item rotations. International Transactions in Operational Research 21(3):375–398
38. Dooley A, Parker W, Blair H (2005) Modelling of transport costs and logistics for on-farm milk segregation in new zealand dairying. Computers and Electronics in Agriculture 48(2):75–91
39. Duhamel C, Lacomme P, Quilliot A, Toussaint H (2011) A multi-start evolutionary local search for the two-dimensional loading capacitated vehicle routing problem. Computers & Operations Research 38(3):617–640
40. Dumitrescu I, Ropke S, Cordeau J, Laporte G (2010) The traveling salesman problem with pickup and delivery: polyhedral results and a branch-and-cut algorithm. Mathematical Programming 121(2):269–305
41. El Fallahi A, Prins C, Wolfler Calvo R (2008) A memetic algorithm and a tabu search for the multi-compartment vehicle routing problem. Computers & Operations Research 35(5):1725–1741
42. Erdoğan G, Cordeau J, Laporte G (2009) The pickup and delivery traveling salesman problem with first-in-first-out loading. Computers & Operations Research 36(6):1800–1808
43. Fagerholt K, Christiansen M (2000a) A combined ship scheduling and allocation problem. Journal of the Operational Research Society 51(7):834–842
44. Fagerholt K, Christiansen M (2000b) A travelling salesman problem with allocation, time window and precedence constraints an application to ship scheduling. International Transactions in Operational Research 7(3):231–244
45. Fagerholt K, Hvattum L, Johnsen T, Korsvik J (2013) Routing and scheduling in project shipping. Annals of Operations Research 207(1):67–81
46. Fallahi AE, Prins C, Wolfler Calvo R (2008) A memetic algorithm and a tabu search for the multi-compartment vehicle routing problem. Com-

puters & Operations Research 35(5):1725–1741
47. Felipe A, Ortuño MT, Tirado G (2009) The double traveling salesman problem with multiple stacks: A variable neighborhood search approach. Computers & Operations Research 36(11):2983–2993
48. Felipe A, Ortuño MT, Tirado G (2009) The double traveling salesman problem with multiple stacks: A variable neighborhood search approach. Computers & Operations Research 36(11):2983–2993
49. Felipe A, Ortuño M, Tirado G (2011) Using intermediate infeasible solutions to approach vehicle routing problems with precedence and loading constraints. European Journal of Operational Research 211(1):66–75
50. Fok K, Chun A (2004) Optimizing air cargo load planning and analysis. In: Proceedings of the international conference on computing, communications and control technologies 2004, Austin Texas, USA
51. Fuellerer G, Doerner KF, Hartl RF, Iori M (2009) Ant colony optimization for the two-dimensional loading vehicle routing problem. Comput Oper Res 36(3):655–673
52. Fuellerer G, Doerner KF, Hartl RF, Iori M (2010) Metaheuristics for vehicle routing problems with three-dimensional loading constraints. European Journal of Operational Research 201(3):751–759
53. Gehring H, Bortfeldt A (1997) A genetic algorithm for solving the container loading problem. International Transactions in Operational Research 4(5/6):401–418
54. Gendreau M, Iori M, Laporte G, Martello S (2006) A tabu search algorithm for a routing and container loading problem. Transportation Science 40(3):342–350
55. Gendreau M, Iori M, Laporte G, Martello S (2008) A tabu search heuristic for the vehicle routing problem with two-dimensional loading constraints. Networks 51(1):4–18
56. Golden B, Raghavan S, Wasil E (2008) The vehicle routing problem: Latest advances and new challenges. Operations Research/Computer Science Interfaces, vol 43, Springer
57. International Transport Forum (2011) Permissible maximum weights of trucks in europe. URL `http://www.internationaltransportforum.org/IntOrg/road/pdf/weights.pdf`
58. Iori M, Martello S (2010) Routing problems with loading constraints. TOP 18(1):4–27
59. Iori M, Salazar-González JJ, Vigo D (2007) An exact approach for the vehicle routing problem with two-dimensional loading constraints. Transportation Science 41(2):253–264
60. Junqueira L, Morabito R, Sato Yamashita D (2012) Three-dimensional container loading models with cargo stability and load bearing constraints. Computers & Operations Research 39(1):74–85
61. Junqueira L, Oliveira J, Carravilla M, Morabito R (2013) An optimization model for the vehicle routing problem with practical three-dimensional loading constraints. International Transactions in Operational Research 20(5):645–666

62. Khebbache-Hadji S, Prins C, Yalaoui A, Reghioui M (2013) Heuristics and memetic algorithm for the two-dimensional loading capacitated vehicle routing problem with time windows. Central European Journal of Operations Research 21(2):307–336
63. Ladany SP, Mehrez A (1984) Optimal routing of a single vehicle with loading and unloading constraints. Transportation Planning and Technology 8(4):301–306
64. Laporte G (2009) Fifty years of vehicle routing. Transportation Science 43(4):408–416
65. Leung S, Zhou X, Zhang D, Zheng J (2011) Extended guided tabu search and a new packing algorithm for the two-dimensional loading vehicle routing problem. Computers & Operations Research 38(1):205–215
66. Leung SC, Zhang Z, Zhang D, Hua X, Lim MK (2013) A meta-heuristic algorithm for heterogeneous fleet vehicle routing problems with two-dimensional loading constraints. European Journal of Operational Research 225(2):199–210
67. Levitin G, Abezgaouz R (2003) Optimal routing of multiple-load AGV subject to LIFO loading constraints. Computers & Operations Research 30(3):397–410
68. Li Y, Lim A, Oon W, Qin H, Tu D (2011) The tree representation for the pickup and delivery traveling salesman problem with LIFO loading. European Journal of Operational Research 212(3):482–496
69. Lim A, Ma H, Qiu C, Zhu W (2013) The single container loading problem with axle weight constraints. International Journal of Production Economics 144:358–369
70. Limbourg S, Schyns M, Laporte G (2012) Automatic aircraft cargo load planning. Journal of the Operational Research Society 63(9):1271–1283
71. Lodi A, Martello S, Vigo D (1999) Heuristic and metaheuristic approaches for a class of two-dimensional bin packing problems. INFORMS Journal on Computing 11(4):345–357
72. Lurkin V, Schyns M (2013) The airline container loading problem with pickup and delivery. Working paper HEC-University of Liège
73. Lusby RM, Larsen J (2011) Improved exact method for the double TSP with multiple stacks. Networks 58(4):290–300
74. Lusby RM, Larsen J, Ehrgott M, Ryan D (2010) An exact method for the double TSP with multiple stacks. International Transactions in Operational Research 17(5):637–652
75. Malapert A, Guéret C, Jussien N, Langevin A, Rousseau L (2008) Two-dimensional pickup and delivery routing problem with loading constraints. In: Proceedings of the 1st CPAIOR Workshop on Bin Packing and Placement Constraints (BPPC0́8)
76. Martello S, Vigo D (1998) Exact solution of the two-dimensional finite bin packing problem. Management Science 44(3):388–399
77. Martello S, Pisinger D, Vigo D (2000) The three-dimensional bin packing problem. Operations Research 48(2):256–267

78. Martinez L, Amaya C (2013) A vehicle routing problem with multi-trips and time windows for circular items. Journal of the Operational Research Society 64:1630–1643
79. Massen F, Deville Y, Van Hentenryck P (2012) Pheromone-based heuristic column generation for vehicle routing problems with black box feasibility. In: International Conference on Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems (CPAIOR2012), Springer LNCS, Springer LNCS, Nantes, France, pp 260–274
80. Mendoza JE, Castanier B, Guéret C, Medaglia AL, Velasco N (2010) A memetic algorithm for the multi-compartment vehicle routing problem with stochastic demands. Computers & Operations Research 37(11):1886–1898
81. Miao L, Ruan Q, Woghiren K, Ruo Q (2012) A hybrid genetic algorithm for the vehicle routing problem with three-dimensional loading constraints. RAIRO - Operations Research 46(1):63–82
82. Moura A (2008) A multi-objective genetic algorithm for the vehicle routing with time windows and loading problem. In: Bortfeldt A, Homberger J, Kopfer H, Pankratz G, Strangmeier R (eds) Intelligent Decision Support, Current Challenges and Approaches, Gabler, Wiesbaden, Germany, pp 187–201
83. Moura A, Oliveira J (2009) An integrated approach to the vehicle routing and container loading problems. OR Spectrum 31(4):775–800
84. Muyldermans L, Pang G (2010) A guided local search procedure for the multi-compartment capacitated arc routing problem. Computers & Operations Research 37(9):1662–1673
85. Øvstebø B, Hvattum LM, Fagerholt K (2011) Routing and scheduling of roro ships with stowage constraints. Transportation Research Part C: Emerging Technologies 19(6):1225 – 1242
86. Paquay C, Schyns M, Limbourg S (2013) A mixed integer programming formulation for the three dimensional bin packing problem deriving from an air cargo application. International Transactions in Operational Research (forthcoming) DOI 10.1111/itor.12111
87. Parragh S, Doerner K, Hartl R (2008) A survey on pickup and delivery problems part ii: Transportation between pickup and delivery locations. Journal für Betriebswirtschaft 58(2):81–117
88. Petersen HL, Madsen O (2009) The double travelling salesman problem with multiple stacks formulation and heuristic solution approaches. European Journal of Operational Research 198(1):139–147
89. Petersen HL, Archetti C, Speranza MG (2010) Exact solutions to the double travelling salesman problem with multiple stacks. Networks 56(4):229–243
90. Pollaris H, Braekers K, Caris A, Janssens GK (2013) The capacitated vehicle routing problem with loading constraints. In: Bruzzone A, Gronalt M, Merkuryev Y, Piera M (eds) Proceedings of the International Conference on Harbor Maritime and Multimodal Logistics M & S, 2013, Athens,

Greece, pp 7–12
91. Pollaris H, Braekers K, Caris A, Janssens GK, Limbourg S (2014) Capacitated vehicle routing problem with sequence based pallet loading and axle weight constraints. EURO Journal on Transportation and Logistics (forthcoming)
92. Ratcliff M, Bischoff E (1998) Allowing for weight considerations in container loading. Operations Research Spektrum 20(1):65–71
93. Ren J, Tian Y, Sawaragi T (2011) A relaxation method for the three-dimensional loading capacitated vehicle routing problem. In: 2011 IEEE/SICE International Symposium on System Integration (SII), pp 750 –755
94. Ruan Q, Zhang Z, Miao L, Shen H (2013) A hybrid approach for the vehicle routing problem with three-dimensional loading constraints. Computers & Operations Research 40(6):1579–1589
95. Strodl J, Doerner K, Tricoire F, Hartl R (2010) On index structures in hybrid metaheuristics for routing problems with hard feasibility checks: An application to the 2-dimensional loading vehicle routing problem. In: Blesa M, Blum C, Raidl G, Roli A, Sampels M (eds) Hybrid Metaheuristics, Lecture Notes in Computer Science, vol 6373, Springer Berlin Heidelberg, pp 160–173
96. Tao Y, Wang F (2013) An effective tabu search approach with improved loading algorithms for the 3l-cvrp. Computers & Operations Research DOI http://dx.doi.org/10.1016/j.cor.2013.10.017
97. Tarantilis C, Zachariadis E, Kiranoudis C (2009) A hybrid metaheuristic algorithm for the integrated vehicle routing and three-dimensional container-loading problem. IEEE Transactions on Intelligent Transportation Systems 10(2):255 –271
98. Toth P, Vigo D (2002) The vehicle routing problem. Monographs on Discrete Mathematics and Applications, Society for Industrial and Applied Mathematics, Philadelphia, USA
99. Tricoire F, Doerner K, Hartl R, Iori M (2011) Heuristic and exact algorithms for the multi-pile vehicle routing problem. OR Spectrum 33(4):931–959
100. Vancroonenburg W, Verstichel J, Tavernier K, Vanden Berghe G (2014) Automatic air cargo selection and weight balancing: a mixed integer programming approach. Transportation Research E, Logistics and Transportation Review 65:70–83
101. Vansteenwegen P, Souffriau W, Van Oudheusden D (2011) The orienteering problem: A survey. European Journal of Operational Research 209(1):1 – 10
102. Vidal T, Crainic T, Gendreau M, Prins C (2013) Heuristics for multi-attribute vehicle routing problems: A survey and synthesis. European Journal of Operational Research 231(1):1 – 21
103. Wäscher G, Hauner H, Schumann H (2007) An improved typology of cutting and packing problems. European Journal of Operational Research 183(3):1109–1130

104. Wei L, Zhang D, Chen Q (2009) A least wasted first heuristic algorithm for the rectangular packing problem. Computers & Operations Research 36(5):1608 – 1614
105. Wisniewski MA, Ritt M, Buriol LS (2012) A tabu search algorithm for the capacitated vehicle routing problem with three-dimensional loading constraints. Tech. rep., Institute of Informatics at UFRGS, URL `http://inf.ufrgs.br/~mrpritt/Publications/P37-sbpo-2011a.pdf`
106. Xu H, Chen ZL, Rajagopal S, Arunapuram S (2003) Solving a practical pickup and delivery problem. Transportation Science 37(3):347–364
107. Zachariadis E, Tarantilis C, Kiranoudis C (2009) A guided tabu search for the vehicle routing problem with two-dimensional loading constraints. European Journal of Operational Research 195(3):729–743
108. Zachariadis E, Tarantilis C, Kiranoudis C (2012) The pallet-packing vehicle routing problem. Transportation Science 46(3):341–358
109. Zachariadis E, Tarantilis C, Kiranoudis C (2013) Designing vehicle routes for a mix of different request types, under time windows and loading constraints. European Journal of Operational Research 229(2):303 – 317
110. Zachariadis E, Tarantilis C, Kiranoudis C (2013) Integrated distribution and loading planning via a compact metaheuristic algorithm. European Journal of Operational Research 228(1):56 – 71
111. Zhu W, Qin H, Lim A, Wang L (2012) A two-stage tabu search algorithm with enhanced packing heuristics for the 3L-CVRP and M3L-CVRP. Computers & Operations Research 39(9):2178–2195