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Shuttlecock dynamics

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Abstract

We study experimentally the dynamics of shuttlecocks. We show that their trajectory is completely different from classical parabola : for a same launch, the flight of the shuttlecock quickly curves downwards and almost reaches a vertical asymptote. We solve the equation of motion with gravity and drag at high Reynolds number and find an analytical expression for the range. At high velocity, this reach does not depend on the velocity anymore. This phenomenon, that we call the "aerodynamic wall", is highly observable in badminton. Then we study how the shuttlecock shape influences the badminton game. The shuttlecock always flies the nose forehead, which means after the impact it has to flip. Actually it returns, oscillates and then stabilizes. We understand these damping oscillations by distinguishing the mass and aerodynamic center of a shuttlecock.

Keywords: Badminton; shuttlecock

Nomenclature

m	Mass of the shuttlecock
R	Cross section radius of the shuttlecock
S	Cross section of a shuttlecock
C_D	Shuttlecock drag coefficient
U	Shuttlecock velocity
U_∞	Shuttlecock terminal velocity for a free fall

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U_0	Shuttlecock initial velocity
θ_0	Shuttlecock initial angle with the horizontal
ρ	Air density
L	Aerodynamic length

1. Introduction

Since the 18th century, badminton is played with a shuttlecock. It is a conical object of mass m = 5 g which geometrical characteristics are presented on figure 1. It is made up of a cork (3 g) and a feather skirt (2 g) with a cross section $S = \pi R^2 = 30\ cm^2$ (cf. figure 1.a). During the game this object can move at a very high velocity, up to 100 m/s. The 117 m/s speed achieved by Malaysian Tan Boon Heong has been officially entered into the Guinness Book of Word Records as the fastest smash in history. These huge velocities make badminton one of the fastest sport among all. Nevertheless one can notice that badminton court is generally smaller than other sports fields, tennis one for example. These observations highlight the importance of aerodynamic effects in badminton. Those effects can be summarized with a drag force proportional to the square velocity of the moving object. Considering this drag force we investigate shuttlecocks trajectories in section 2 and we get analytically an expression for their range depending on initial velocity and angle and a unique parameter *L*. This parameter, called the aerodynamic length, depends only on shuttlecock and fluid characteristics. We study the dependency of this parameter *L* with shuttlecock characteristics (mass, cross section and drag coefficient). This study allows us to understand the usual distinction made by badminton players between a feather and plastic shuttlecock.

Another property of a badminton shuttlecock is to break spherical symmetry unlike other sports balls. This unique shape allows the shuttlecock to flip after each impact with the racket and to fly the cork ahead. We inspect in section 3 the characteristic times of shuttlecock flip. The order of magnitude and impact condition dependency could be understood considering the special geometry of the shuttlecock. We compare these characteristic times with the time of exchange. This comparison allows us to understand how a player can use shuttlecock flip to lure the opponent.

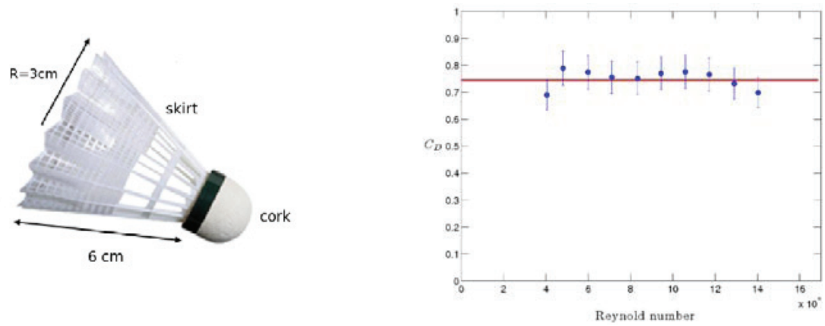

Fig. 1. (a) Shuttlecock characteristics; (b) Shuttlecock drag coefficient depending on Reynold number

2. Shuttlecock ballistic

2.1. Shuttlecock experimental trajectory

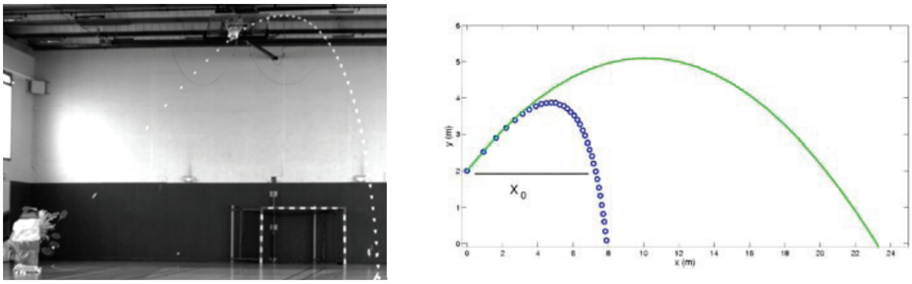

Fig. 2. (a) High clear chronophotography. Each frame are separated by 20 ms; (b) High clear trajectory (blue cirles) compared with the parabola (green line)

We study experimentally shuttlecocks trajectories with a high-speed camera. A similar work has been previously done by A. Cooke [1] [2]. One of these trajectories is reported on figure 2.a. Initially the trajectory is straight. One can notice the high deceleration of the shuttlecock due to air friction at high Reynolds number. When the speed is low enough, the gravity curves the trajectory. Finally weight becomes dominant and the shuttlecock falls nearly vertically. Figure 2.b also compares the experimental trajectory with the expected one in the pure gravitational limit, that is to say without air friction. The observed trajectory is highly asymmetric, unlike parabola, and its range is considerably lower. This shape of trajectory had been early drawn by Tartaglia when he looked at cannonballs trajectories [3].

2.2. Theoretical study of these trajectories

As the air exerts no lift on a shuttlecock and its drag coefficient is constant over typical game Reynolds number (cf. fig. 1.b), the equation of dynamics for this object could be expressed the following way [4] :

$$m\frac{d\vec{U}}{dt} = m\vec{g} - \frac{1}{2}\rho S C_D U\vec{U} \qquad (1)$$

This equation contains three different terms: inertia, gravity and drag. At the beginning of the trajectory, the initial velocity is high as the drag force. In this regime the gravity could be neglected. So the shuttlecock follows a straight line and decelerates. At one point, the velocity reduces and the drag becomes comparable with weight. In this regime, gravity curves the trajectory. Finally the shuttlecock tends toward a steady state where the weight counterbalances the drag. In this final regime the velocity is collinear to the gravity and its value is $U_\infty = \sqrt{gL}$ with $L = 2m/\rho SC_D$ the aerodynamic length.

Clanet and al. [5] solve analytically this equation for projectile in air. They found a good approximation of the range of these projectiles depending on initial velocity, angle and the aerodynamic length.

$$X_0 = \frac{L\cos\theta_0}{2} \ln\left(1 + 4\left(\frac{U_0}{U_\infty}\right)^2 \sin\theta_0\right) \qquad (2)$$

2.3. Comparison between theory and experiments

From different shuttlecock shoots (different initial velocity and angle) we measure the horizontal distance over which the projectile is again at the initial altitude, in other words the range. Figure 3 reports our experiments results. We represent the non-dimensional range $2\,X_0\cos\theta_0\,/L$ depending on the non-dimensional square velocity $(U_0/U_\infty)^2 \sin\theta_0$. The aerodynamic length of this shuttlecock was determined by measuring the terminal velocity of this object after a long free fall. For the shuttlecock used in these experiments the aerodynamic length was $L = 6,5\,m$.

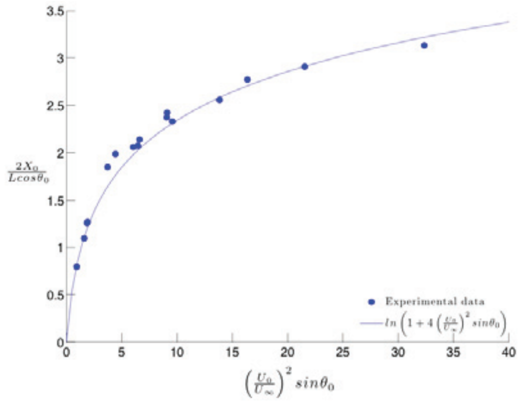

Fig. 3. Non-dimensional range for a shuttlecock depending on non-dimensional square velocity

As predicted by the theoretical expression (2), we observe a saturation of the range for high velocity. This observation corresponds to the game reality where a high increase in initial velocities does not provide a high increase of the range. In this regime of high initial velocities, the shuttlecock range scales as the aerodynamic length. Finally, measurements of this length allow us to predict the behavior of a shuttlecock in the game.

2.4. Distinction between plastic and feather shuttlecock

Badminton players make a distinction between plastic and feather shuttlecocks. They prefer feather shuttlecocks because of their shorter and more curved trajectories compared with plastic ones. Figure 4 reports two experimental trajectories of a feather and a plastic shuttlecock for same initial conditions.

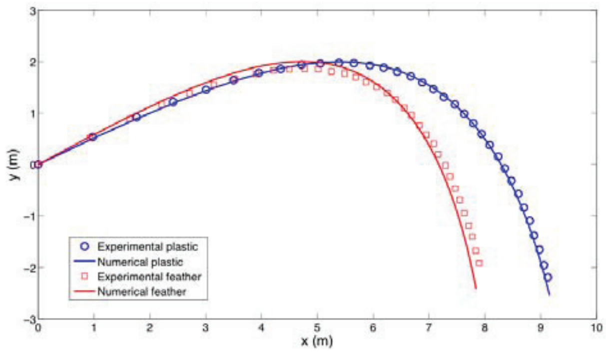

Fig. 4. Feather and plastic shuttlecock trajectory for same initial conditions

We can observe on figure 4 that these trajectories have a similar shape but a different range. The feather shuttlecock has a lower range than the plastic one. This observation is in good agreement with our measurements of aerodynamic lengths made for these both projectiles. Actually, we found a shorter aerodynamic length for the feather shuttlecock, $L_f = 6,5\,m$ than for plastic shuttlecock, $L_p = 7,0\,m$. As the range scales with the aerodynamic length in this regime, it explains players' feeling in the game. It is interesting to notice that this difference of aerodynamic lengths values mainly comes from a difference in masses. Indeed, plastic and feather shuttlecock have nearly the same cross section and drag coefficient but it is hard to manufacture a synthetic shuttlecock as rigid and light as feather. Our conclusion is true for a large variety of plastic and feather shuttlecocks.

Then we solve numerically equation (1) with initial conditions of the previous experiment and measured aerodynamic lengths. These solutions are represented with continuous line on figure 4 and they are in good agreement with experimental trajectories. This result leads to the conclusion of no difference in shape between a plastic and feather shuttlecock trajectory but only in the aerodynamic length so in their range.

3. Shuttlecock flip

3.1. Experimental flip observation

Shuttlecock is a specific sport ball because of the lack of spherical symmetry. During game the player hits rather the cork than the skirt to control better the shuttlecock. He uses the fact that a shuttlecock always flies the cork ahead. Hence the shuttlecock has to flip after each exchange. So we visualize the flipping behavior of a shuttlecock after it impacts on a racket. Our observation is reported on figure 5 where we superimpose all frames of an high-speed movie.

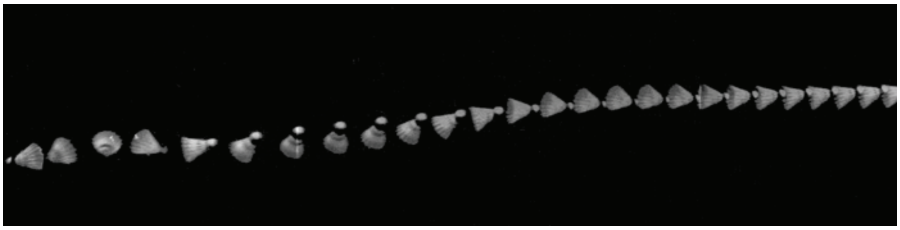

Fig. 5. Chronophotography of a flipping shuttlecock after an impact with the racket. Here the racket comes from the left of the image. Each frame is separated by 5 ms

Figure 5 allows us to characterize the flip of a shuttlecock. After racket impact the shuttlecock reverses of half a turn in about 20 ms. Afterwards the shuttlecock symmetry axis oscillates compared with its velocity direction. Period of these oscillations are about 80 ms. Finally the shuttlecock direction stabilizes along its velocity direction with a stabilizing time equal to about 200 ms.

3.2. Flip explanation

The shuttlecock flip is possible because this object has distinguished centre of mass and center of pressure [6]. Actually, the shuttlecock cork is denser than its skirt so the center of mass is close to the cork for those objects. Meanwhile the aerodynamic center, where the drag is exerted, is close to the center of the volume that is to say close to the center of the skirt.

When a shuttlecock is not aligned with its velocity direction the drag force, which is exerted on the aerodynamic center, submits a stabilizing torque to the shuttlecock. This stabilizing torque, reported on figure 6, explains why the shuttlecock flies the cork ahead and flips after racket impact.

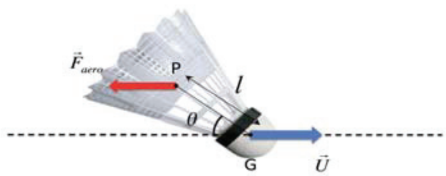

Fig. 6. Aerodynamic torque applied on a shuttlecock, as it is not aligned with its velocity direction

Writing torque equilibrium on the shuttlecock provides a prediction for reversing, oscillating and stabilizing time.

3.3. During the game

For a high clear, the time of an exchange is typically 2 s. In this case the stabilizing time (about 0,2 s) is short compared with this time of exchange. We can neglect the flip in the dynamic of the shuttlecock and the approach of the second section, which assumes a constant cross section, is validated. However, near the let, the time of exchange could be comparable with the stabilizing time. In this case, players try to give a high spin to the shuttlecock in order to make impossible for the opponent to hit the cork and return the ball.

4. Conclusion

We inspect the effect of the aerodynamic drag on a shuttlecock dynamics. This force has a huge impact on the shape and the range of shuttlecock trajectory. These trajectories have nothing in common with the usual parabola. Beside, aerodynamic force applies a stabilizing torque as the shuttlecock is not aligned with its velocity direction. This torque provides the shuttlecock flip ability and explains why it always flies the cork ahead.

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