Energetical aspects of solar-like oscillations in red giants

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Thesis supervisor: M.A. Dupret
CoRot and Kepler have produced a large variety of power spectra for red giants.

What are the theoretical predictions for linewidths and heights of mixed-modes?

When, during the evolution on the red-giant branch, mixed-modes are detectable?
Solar like Oscillations in red giants

Red giant acoustic structure

\[ \sigma^2 < L^2_l, N^2 \]
\[ \sigma^2 > L^2_l, N^2 \]

Existence of mixed modes → Particular aspects of the power spectra
Mixed modes trapping: typical expectation of a RGB

Energy density of modes trapped
- in the envelope
- in the core

$|dI/d\log T|$ vs $\log T$

Synthetic power spectra

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Mixed modes trapping:
typical expectation of a RGB

Energy density of modes trapped
in the envelope
in the core

frequency
synthetic power spectra

p-type modes

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Mixed modes trapping: typical expectation of a RGB

Energy density of modes trapped
- in the envelope
- in the core

\[ |dI/d\log T| \]

\( \log T \)

\( 8 \quad 7.5 \quad 7 \quad 6.5 \quad 6 \quad 5.5 \quad 5 \quad 4.5 \quad 4 \quad 3.5 \)

\( 1 \quad 0.1 \quad 0.01 \quad 0.001 \quad 1e^{-4} \quad 1e^{-5} \quad 1e^{-6} \quad 1e^{-7} \quad 1e^{-8} \quad 1e^{-9} \)

p-type modes

g-type modes

synthetic power spectra

\( l=0 \)
\( l=1 \)
\( l=2 \)
How to obtain the linewidth and heights of the modes?

Dynamics and energetics of the oscillation

\[ \frac{d^2}{dt^2} z(t) + 2\eta \frac{d}{dt} z(t) + \omega_0^2 z(t) = f(t) \]

Damping
- radiative
- convective

Lorentzian profile (if resolved)

Amplitude \( V(R) \)

Height

The height of a mode in the power spectra depends on its lifetime and on the duration of observation:

Resolved modes
\[ \tau < \frac{T_{obs}}{2} \]
\[ H = V^2(R) \ast \tau \]

Unresolved modes
\[ \tau \geq \frac{T_{obs}}{2} \]
\[ H = V^2(R) \ast \frac{T_{obs}}{2} \]

Dampening e.g. Samadi et al. 2001
Belkacem et al. 2008

Dupret 2002
Grighacene 2005

The height of a mode in the power spectra depends on its lifetime and on the duration of observation.
Damping : Work Integral

\[ \eta = -\frac{\int_V dW}{2\sigma I|\xi_r(R)|^2 M} \]

In the deep radiative zone:

\[-\int_{r_0}^{r_c} \frac{dW}{dr} dr \approx \frac{K(l(l+1))^{3/2}}{2\sigma^3} \int_{r_0}^{r_c} \frac{\nabla_{ad} - \nabla_{ad}N g L}{p r^5} dr\]

Dziembowski 1977 ; Van Hoolst et al. 1998 ; Godart et al. 2009

Mode trapped in the envelope:
- p-type mode
- no damping in the core

Mode trapped in the core:
- g-type mode
- high radiative damping in the core

Bottom of the H-shell
Damping: Work Integral

\[ \eta = -\frac{\int_V dW}{2\sigma l^4 \bar{\xi}_r(R)^2 M} \]

In the deep radiative zone:

\[ -\int_{r_0}^{r_c} \frac{dW}{dr} dr \approx \frac{K(l(l+1))^{3/2}}{2\sigma^3} \int_{r_0}^{r_c} \frac{\nabla_{ad} - \nabla_{ad} N g L}{pr^5} dr \]

Dziembowski 1977; Van Hoolst et al. 1998; Godart et al. 2009

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Bottom of the H-shell
Damping : Work Integral

\[ \eta = -\frac{\int_V dW}{2\sigma I_0 |\xi_r(R)|^2 M} \]

In the deep radiative zone:

\[ -\int_{r_0}^{r_e} \frac{dW}{dr} dr \approx \frac{K(l(l + 1))^{3/2}}{2\sigma^3} \int_{r_0}^{r_e} \nabla_{ad} - \nabla \nabla_{ad} N g L \frac{dr}{pr^5} \]

Dziembowski 1977; Van Hoolst et al. 1998; Godart et al. 2009

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Dziembowski 1977; Van Hoolst et al. 1998; Godart et al. 2009

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Bottom of the H-shell

**Solar like Oscillations in red giants : Energetical aspects**

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Damping : convective contribution

Interaction between convection and oscillation is difficult to model (Gabriel 1996, Grighacene et al. 2005; Gough 1977; Xiong 1997)

- A perturbative approach of the mixing-length theory
- Involves a free parameter $\beta$ in the closure term of the perturbed energy equation

Results are sensible to the $\beta$ parameter so we have to constrain it.

The $\beta$ complex parameter is adjusted so that the depression of the damping rates occurs at $v_{\text{max}}$ predicted by scaling relations (Belkacem et al. 2012)
Evolution of a 1.5M star

Following the evolution of power spectra with the evolution of the star

1.5 $M_\odot$

ATON (Ventura et al. 2005)
Theoretical power spectrum: Ascending the RGB

Lifetimes

$\eta = - \frac{\int_V dW}{2\sigma l |\xi_r(R)|^2 M}$

$T_{\text{obs}} = 1 \text{ year}$
Theoretical power spectrum: Ascending the RGB

Lifetimes

\[ \eta = -\frac{\int_V dW}{2\sigma I|\xi_r(R)|^2M} \]

\( T_{\text{obs}} = 1 \text{ year} \)

1.5 \( M_\odot \)

\( l=0 \quad l=1 \quad l=2 \)

A B C D

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Theoretical power spectrum: Ascending the RGB

Lifetimes

\[ \eta = - \frac{\int_V dW}{2\sigma I |\xi_r(R)|^2 M} \]

- \( l = 0 \)
- \( l = 1 \)
- \( l = 2 \)

Tobs = 1 year

1.5 M☉
Theoretical power spectrum: Ascending the RGB

Lifetimes

1.5 M☉

<table>
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<tr>
<th>l=0</th>
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<th>l=2</th>
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<tr>
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<td>green</td>
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Tobs = 1 year

\[ \eta = -\frac{\int_V dW}{2\sigma I|\xi_r(R)|^2 M} \]
Theoretical power spectrum: Ascending the RGB

Lifetimes

1.5 M☉

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<td>blue</td>
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<tr>
<td>2</td>
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Tobs = 1 year

\[ \eta = - \frac{\int_V dW}{2\sigma I |\xi_r(R)|^2 M} \]
Theoretical power spectrum: Ascending the RGB

Lifetimes

\[ \eta = - \frac{\int_V dW}{2\sigma^2 |\xi_r(R)|^2 M} \]

1.5 \( M_\odot \)

<table>
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<th>( \text{Lifetime (days)} )</th>
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</tr>
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<td>2</td>
<td>90</td>
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Tobs = 1 year

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Theoretical power spectrum: Ascending the RGB

Lifetimes

\[ \eta = -\frac{\int_V dW}{2\sigma l |\xi_r(R)|^2 M} \]

1.5 M☉

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Tobs = 1 year
Theoretical power spectrum: Ascending the RGB

Lifetimes

1.5 M☉

\( \eta = - \frac{\int_V dW}{2 \sigma I |\xi_r(R)|^2 M} \)

Tobs = 1 year
For 1.5 $\text{M}_\odot$ star ascending the RGB (and $T_{\text{obs}} = 1$ year) Detectable g-dominated mixed-modes for stars with $v_{\text{max}} \geq 50$ µHz and $\Delta v \geq 4.9$ µHz
power spectrum aspect along the red-giant branch

Theoretical power spectrum: Ascending the RGB

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What is the iso-detectability criterium of mixed-modes?

We selected models with the same number of mixed modes in a large separation.
Extension to other masses

$T_{\text{obs}} = 1 \text{ year}$

$1M_\odot$

$\nu_{\text{max}} = 84 \mu Hz$

$\Delta \nu = 8.5 \mu Hz$

$1.7M_\odot$

$\nu_{\text{max}} = 90 \mu Hz$

$\Delta \nu = 7.7 \mu Hz$

$2.1M_\odot$

$\nu_{\text{max}} = 62 \mu Hz$

$\Delta \nu = 5.7 \mu Hz$
Effect of Tobs

Tobs = 100 days
Effect of Tobs

Tobs = 360 days
Effect of $T_{obs}$

$T_{obs} = 1500$ days

adimensional frequency
Effect of $T_{obs}$

$T_{obs} = 10000$ days

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Conclusion

- Theoretical detectability limit on the RGB for $1.5 \, M_{\odot}$ and $T_{\text{obs}} = 1 \, \text{year}$, $g$-dominated mixed modes are detectable for stars with $\nu_{\text{max}} \geq 50 \, \mu\text{Hz}$ and $\Delta \nu \geq 4.9 \, \mu\text{Hz}$

- Models with the same number of mixed modes in a large separation presents similar power spectra

$$n_g \approx \frac{\Delta \nu}{\Delta P_{\nu_{\text{max}}}} \propto \left[ \int \frac{N}{r} \, dr \right] M^{3/2} R^{5/2} T_{\text{eff}} \propto \sqrt{\frac{\langle \rho_c \rangle}{\langle \rho \rangle}} \frac{R}{M} T_{\text{eff}}$$

- We extend the detectability limit to other masses

Perspectives:

- Impact of chemical composition, metallicity, overshoot, ...

- Quantitative comparison to observations (measure of individual linewidth and heights Benomar et al. 2013)