"Stochastic Optimization in Multi-Periods Problems in Transportation"

HEC-ULg

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1 Outlines

- Multi-periods problems in transportation
- Decision making under uncertainty
- Stochastic Optimization
- A Methodology
  1. Bounds
  2. A picture for manager
  3. Algorithms
  4. Results validation
- 2 cases study
  1. Vehicle Loading
  2. Vehicle-Load Assignment
- Conclusions

2 What it is not !

Classical problems in transportation

- TSP : one truck visit a collection of customers
- VRP : a fleet of trucks visits a collection of customers
- Assumptions : d = T, triangle inequality, max L
- Options :
  1. All or selection of customers (Cost vs Profit)
  2. **Capacity** : demand before or on-road (CVRP-PDP)
  3. **Time** (TSPTW-VRPTW-DARP) usually within a day, finite time, max T

Sometimes it models a reality, sometimes it is a reduction.

Because, time is not finite, actions can be postponed and related (not independent) along time
Drawback : solution of a wrong problem, because of a caricatured model...

TIME PROBLEM => PROCESS

Time in transportation

Options for time

- **Timeless**, distance = time
- **Continuous time** over a finite period (TW)
- **Periodic** (train, bus,...), usually same patterns
- **Dynamic** Parameters ! = Time-dependent (not new customers or demand or arcs)
- Several periods (plan), but one time decision (Harvest)
- **Multi-periods** = fixed periods with rolling horizon over infinite horizon (repeated decisions with interaction among periods)
- **On-line** = decision if new information (In or Out), "brand" new solution ?

From a **solution** to a **policy** for multi-periods, because the truncation effect leads to a sequence of decisions

What we investigate !

[P1]	[P1..Pi]	[Pi+1..RH]	[RH+1..Pn
Decision in P1		**Stochastic**	**Tail**

Frozen-Action [P1,<=Pi ?]

Deterministic [P1,Pi]

In our cases study :

1. action periods=deterministic periods => feasible
2. periods are days
3. rolling horizon = 5 periods =1 week

Dynamism of the system

- Decision in P1
- Actions (info out)
- Roll-over 1 period, updates (in)
  1. stochastic becomes deterministic Pi+1=Pi
  2. new stochastic info RH+1=RH
- Decision in P2=P1

3 What is a solution to a stochastic problem ?

In the future, some elements are stochastic.
Do they follow a distribution law ?
A deterministic world ? Uncertainty principle "Heisenberg"
Model the world and know it at a point of time ?

What is the solution to a stochastic problem ?

- Worst case (oversize solution)
- Chance constrained (95%)
- Robust to variation (Tree)
- Flexible : Easy to recover (Grass)
- **Min or max Expected cost-profit (E*)**

Uncertainty in a rolling horizon

Forecasts => policy also, because of the stochastic part

Stochasticity in transportation

- Customer location, destination, existence
- Demand level
- Transportation time
- **Release dates [TW]**
- **Availability (Y-N)**

Highly discrete distribution laws due to the periods

Probability per period %

4 What can we solve ?

Usual technique "Stochastic programming" (DK)
Model 2 stages :

1. First stage strategic investment : facility location, network design, stock
2. Second stage operational cost : "Demand"

Formulation : **http://en.wikipedia.org/wiki/Stochastic_programming**
Linear, non-linear, continuous or integer variables X, Y
If 2nd stage linear and convexity of recourse function
Technique : **L-Shaped algorithm** (exact)
Introduction of feasibility and optimality constraints

Recourse assumption : continuous or piecewise linear
=> **Discretization :** approximation by a set of scenarios
Scenario generation : gap => correctly wrong N = ?
Few works on Integer + Integer + discrete, so...

Techniques using scenarios

Problem "Noise", randomness in scenario generation
Stability stochastic model not stochastic solution value !

1. "Sample Average Approximation Method" **(SAA)**
   Solve several sets of scenarios
2. "Approximate Dynamic Programming" **(ADP)**
   Go forward, go back on scenarios to approximate decisions values

Our cases study : simulation over scenarios
Integer (NP-hard) + Integer + discrete laws + infinite tail
No convexity, not continuous
Highly discrete distribution => Scenarios OK, $|States|^{|Y|}$
Curse of dimensionality => Intractable ! !
Conclusion on techniques : ***"Hard" to find E****

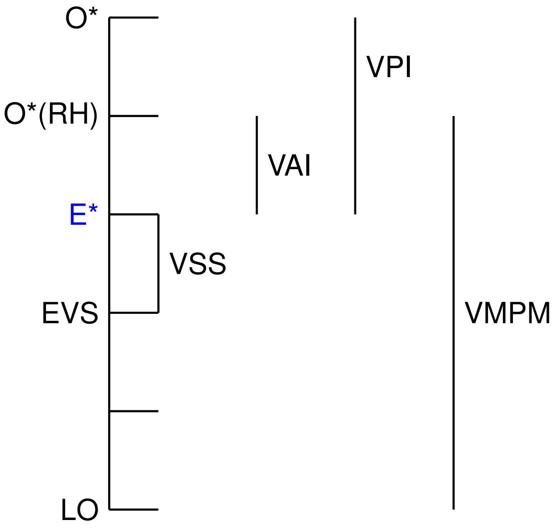
5 Bounds

Oracle : a posteriori **O***
Infinite horizon average value of the deterministic info
Real Bound (Upper or lower) on **E***
VPI : Value of the Perfect Information |O*-E*|>=0
Myopic : Deterministic periods average value **LO**
Bound (Lower or upper) on any policy with forecasts

Usual deterministic approximation : Mean
EVS : Expected Value Solution
VSS : Value of the stochastic solution |E*-EVS|>=0

Multi-period model
Rolling Horizon Oracle **O*(RH)**
VMPM : Value of the multi-period model |O*(RH)-LO|>=0
VAI : Value of the Available Information |O*(RH)-E*|>=0
Practical bound, not strict !

A picture for a manager



"Averages" : EO*, EO*(RH), ELO, EEVS, EVSS, EVAI, EVPI, EVMPM

If I can solve one deterministic scenario ?

Over RH means with t (multi-periods) => Hard !
LO, O*(T), O*(RH), EVS and...others
Algorithms or policies to approximate E*

1. Solve a "good" single scenario
2. **Consensus (Cs) :** Solve "some" scenarios and create a solution with common decisions
3. **Restricted Expectation (RE) :** Solve "some" scenarios and cross-evaluate each solution over other scenarios
4. RE over all scenarios individually (! ! !) 2nd best

No guarantee, just numerical validation !
DK : Characteristic of deterministic solution due to deterministic model
Deterministic solutions are elitist, no option in it
CPU Time : 1, $N + 1$, N^2

If I can solve a subtree of scenarios ?

Full tree : exact solution, but Out of Memory, CPU Time

Approximation by a **Subtree** $(1 * ST \neq ST * 1)$!
Join scenarios, for solution consistency
Non-anticipativity constraints
$X_{1,t} = X_{2,t} = ... = X_{i,t} = X_{ST,t}$
if $(Scenario_1 = Scenario_2 = ...)$ up to period t
But, not all of them, just the common part !

Decision variables are equal until scenarios differ.
Might destroy the nice structure of a model ! ! ! Hard

Multi-periods : non-anticipativity for decision period only !

Results statistical validation

E* remains unknown : Select Best policy
How to compare Policy 1 with Policy 2, E*1 vs E*2 ?
Statistical validation :
"Compare the stochastic solutions from an algorithm sometimes using random calibration scenarios for a random set of scenarios from a random instance"
Solve 30 scenarios by instance over an horizon 20 P
Non Non-Normality check, confidence level, t-student...
Outclassment = significant difference between means
Hypothesis : $\mu_1 \neq \mu_2$, $\mu_1 > \mu_2$?

Robustness analysis :
Is distribution law known in practice ?
Check performance when the real distribution law differs from the expected one
Calibration scenarios (Cs-RE) differ from test scenarios, other mean EVS

6 Cases study : why 2 cases ?

Differences

Natural class instances vs Theoretical instances
Objective : Min cost vs Max profit
NP-Hard : Set-covering (B&B) vs Network flow (LP $\cong$ IP)
Subtree algorithm : Intractable vs Tractable
1 deterministic period vs 2 deterministic periods
Fleet : Unlimited vs Limited
Capacity : LTL vs FTL
Stochasticity : Release date 4 P TW vs Availability 1 P
The option : Do now/Postpone vs Go/No go
Action periods : P1 vs P1 to P4 if loaded

Similarities

Rolling horizon = 5 periods
Algorithms : LO, O*, O*(5), Cs, RE, EVS
Results statistical validation
Robustness analysis

6.1 Vehicle Loading Problem

Steel industry : Decision coils to be send by trucks
Objective :
Minimize transportation cost (Trucks + Penalties)
Time windows penalties : [Early ; Inv ; Inv ; Late]
Constraint : capacity (weight) and delivery time
Data : stock P1 and forecasts of arrivals from production
Stochasticity : [TW1 % ; TW2 % ; TW3 % ; TW4 %]
4 distribution laws

1. Early [40 ; 30 ; 20 ; 10]
2. Late [10 ; 20 ; 30 ; 40]
3. Uniform [25 ; 25 ; 25 ; 25]
4. Binomial [12.5 ; 37.5 ; 37.5 ; 12.5]

A picture

	Periods				
Coils Weight	**P1**	**P2**	**P3**	**P4**	**P5**
A *0.6*	1	*LAT*			
B *0.3*		*EAR*	*INV*	*INV*	*LAT*
C *0.2*		*EAR*	*INV*	*INV*	*LAT*
D *0.6*	1	*INV*	*INV*	*LAT*	

Results : Early

TABLE: Algorithmic performance – Early distribution

Early	$N = 80$	$N = 120$	$N = 160$	$N = 200$
O^*	100	100	100	100
$O^*(5)$	107.2	105.3	103.8	105.6
LO	193.5	172.5	168.7	159.3
EVS	116.5	113.9	111.1	110.4
Mod	112.2	**108.5**	**107.0**	**107.6**
Cons	122.4	119.5	113.1	117.4
RE	**111.0**	111.7	109.2	111.8
$O^*(5) - O^*$	7.2	5.3	3.8	5.6
VMPM	86.3	67.2	65.0	53.7
VPI	11.0	8.5	7.0	7.6
VAI	3.8	3.2	3.2	2.0
VSS	5.4	5.4	4.1	2.8

Results : Late

TABLE: Algorithmic performance – Late distribution

Late	$N = 80$	$N = 120$	$N = 160$	$N = 200$
O^*	100	100	100	100
$O^*(5)$	102.7	103.0	102.8	103.8
LO	154.3	144.0	142.3	136.8
EVS	119.6	115.0	112.0	113.2
Mod	120.1	117.0	117.9	115.4
Cons	109.7	**110.1**	109.8	111.2
RE	**109.5**	111.0	**109.0**	**109.7**
$O^*(5) - O^*$	2.7	3.0	2.8	3.8
VMPM	51.6	41.1	39.5	33.0
VPI	9.5	10.1	9.0	9.7
VAI	6.8	7.1	6.2	5.9
VSS	10.0	4.9	3.0	3.5

Results : Uniform

TABLE: Algorithmic performance – Uniform distribution

Uniform	$N = 80$	$N = 120$	$N = 160$	$N = 200$
O^*	100	100	100	100
$O^*(5)$	108.1	104.2	102.9	104.9
LO	179.5	159.2	154.6	147.5
EVS	117.7	112.3	**109.6**	109.9
Mod	125.1	118.6	113.9	113.3
Cons	115.7	114.7	111.6	113.3
RE	**112.1**	**112.2**	110.0	**108.7**
$O^*(5) - O^*$	8.1	4.2	2.9	4.9
VMPM	71.4	55.1	51.7	42.6
VPI	12.1	12.2	9.6	8.7
VAI	4.0	8.0	6.7	3.8
VSS	5.6	0.1	**0**	1.2

Results : Binomial

TABLE: Algorithmic performance – Binomial distribution

Binomial	$N = 80$	$N = 120$	$N = 160$	$N = 200$
O^*	100	100	100	100
$O^*(5)$	107.2	105.8	104.4	106.1
LO	184.8	179.0	160.9	157.3
EVS	123.4	117.2	114.7	116.7
Mod	123.4	**109.9**	112.4	115.0
Cons	114.7	114.0	113.5	115.3
RE	**113.0**	111.6	**112.1**	**111.9**
$O^*(5) - O^*$	7.2	5.8	4.4	6.1
VMPM	77.6	73.2	56.5	51.2
VPI	13.0	9.9	12.1	11.9
VAI	5.8	4.1	7.7	5.8
VSS	10.5	7.3	2.6	4.8

Statistical validation : RE is the best ?

TABLE: Comparison of means for *RE* vs. alternative algorithms

	$\mathcal{A} = EVS$		$\mathcal{A} = Mod$		$\mathcal{A} = Cs$	
Reject H_0 vs. H_1	Yes	No	Yes	No	Yes	No
$H_1 : \mu_{RE} \neq \mu_{\mathcal{A}}$?	12	4	13	3	7	9
$H_1 : \mu_{RE} < \mu_{\mathcal{A}}$?	12	0	10	0	7	0
$H_1 : \mu_{\mathcal{A}} < \mu_{RE}$?	0	0	**3**	0	0	0

Results with Optimist

TABLE: Algorithmic performance – Early distribution

Real $\mathcal{R}$ = Early	$N = 80$	$N = 120$	$N = 160$	$N = 200$
O^*	100	100	100	100
$O^*(5)$	107.2	105.3	103.8	105.6
RE_{Early}	**111.1**	111.7	109.2	111.8
Optimist	112.2	**108.5**	**107.0**	**107.6**

TABLE: Algorithmic performance – Late distribution

Real $\mathcal{R}$ = Late	$N = 80$	$N = 120$	$N = 160$	$N = 200$
O^*	100	100	100	100
$O^*(5)$	102.7	103.0	102.8	103.8
RE	**109.5**	111.0	109.0	109.7
RE_{Early}	111.7	**108.6**	**106.9**	108.8
Optimist	110.1	109.2	107.3	**108.3**

Results with Optimist

TABLE: Algorithmic performance – Uniform distribution

Real $\mathcal{R}$ = **Uniform**	$N = 80$	$N = 120$	$N = 160$	$N = 200$
O^*	100	100	100	100
$O^*(5)$	108.1	104.2	102.9	104.9
RE	**112.1**	112.2	110.0	108.7
RE_{Early}	113.7	111.4	107.8	109.5
Optimist	113.6	**109.7**	**107.3**	**108.4**

TABLE: Algorithmic performance – Binomial distribution

Real $\mathcal{R}$ = **Binomial**	$N = 80$	$N = 120$	$N = 160$	$N = 200$
O^*	100	100	100	100
$O^*(5)$	107.2	105.8	104.4	106.1
RE	**112.9**	111.6	112.1	111.9
RE_{Early}	117.3	113.5	112.1	111.7
Optimist	116.0	**109.0**	**109.5**	**110.3**

Statistical validation : Optimist is the best ?

TABLE: Comparison of means for *Optimist* vs. alternative algorithms

	$\mathcal{A} = RE_{Early}$		$\mathcal{A} = RE$ (L-U-B)	
Reject H_0 vs. H_1	Yes	No	Yes	No
$\mu_{Optimist} \neq \mu_{\mathcal{A}}$	6	10	6	6
$\mu_{Optimist} < \mu_{\mathcal{A}}$	6	0	6	0
$\mu_{\mathcal{A}} < \mu_{Optimist}$	0	0	0	0

Conclusions Case Study 1

- **VMPM** high
- **VSS** relevant
- **VAI, VPI** relevant, but IS or process problems
- Subtree infeasible
- Optimist : single scenario heuristic : fast, easy and
- **Robust** : independent from the distribution law !

Why ? Optimist postpones more and captures the option !
To appear in :
"EURO Journal on Transportation and Logistics"

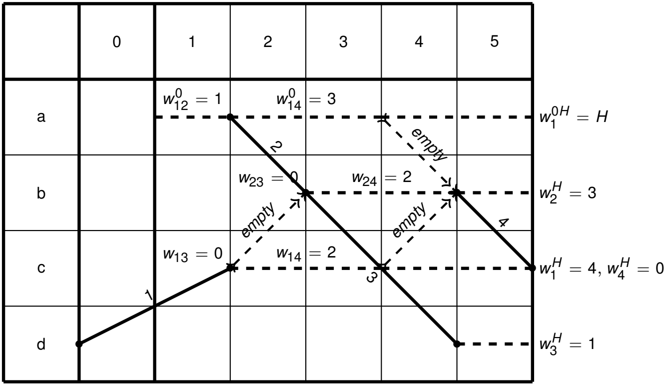
6.2 Vehicle-Load Assignment Problem

Transportation industry : Decision travel to truck FTL (PDP with selection)
Decisions : Wait, Move Empty, Load
Objective :
Maximize Profit (Load-Empty Moves-Waiting)
Constraint : loading if at place on time, no preemption
Data : [P1, P2] and forecasts on available travels [3,4,5]
Stochasticity :
Availability [...%] for a travel from A to B in period 3<=Pi<=5
Distribution laws [...%] linked to :

1. traveled distance (1, 2, 3, 4)
2. city size (B, M, S)

A Picture

A representation of the time-space (Periods, Cities)



Explain loads (city to city and periods)
Explain trucks (loaded-empty)
Explain actions (Wait, Move empty, Load)

Results : one example 150 loads

TABLE: Distribution laws linked to distance

Info	VPI	LB			EVS					UB
Alg	*O^**	*O^*2*	*Opt*	*Mod*	*EG*	*Cs*	*RE^**	TR_{10}	TR_{30}	O^*5
1-10	120.4	0	22.8	17.3	37.3	31.2	12.4	48.6	**58.2**	100
1-15-25A	153.0	0	12.9	38.8	38.4	51.4	43.1	65.7	**70.2**	100
1-15-25B	153.8	0	13.7	44.7	49.2	45.5	26.7	66.5	**75.5**	100
1-15-25C	176.0	0	32.8	43.5	67.1	45.2	36.9	72.8	**85.2**	100
1-20	135.0	0	14.8	41.3	52.5	38.9	46.5	69.8	**71.0**	100
1-20-25A	167.8	0	6.8	32.5	62.4	21.3	44.9	73.1	**78.1**	100
1-20-25B	149.6	0	23.3	41.0	46.2	42.0	31.8	**70.6**	60.0	100
1-20-25C	199.8	0	-22.1	30.1	24.1	27.0	-24.7	61.5	**67.9**	100
1-25	164.9	0	-83.6	6.5	7.9	12.6	-32.1	**54.9**	50.6	100
2-10	163.7	0	18.4	38.9	44.7	37.7	26.8	67.4	**74.3**	100
2-15-25A	221.3	0	69.2	70.2	65.8	70.9	63.7	**77.2**	76.4	100
2-15-25B	186.1	0	65.1	66.3	70.3	51.0	62.4	83.2	**87.4**	100
2-15-25C	136.6	0	36.7	60.4	67.5	73.1	42.5	78.3	**82.4**	100
2-20	204.6	0	59.6	**74.5**	57.7	53.0	39.3	71.6	70.1	100
2-20-25A	190.1	0	51.6	71.1	81.1	69.4	60.2	82.7	**83.3**	100
2-20-25B	150.9	0	30.4	40.0	54.5	57.4	53.2	**77.5**	74.2	100
2-20-25C	180.9	0	65.2	86.5	87.1	79.6	62.0	86.3	**89.2**	100
2-25	167.3	0	11.4	50.0	65.0	64.2	42.6	**69.8**	61.0	100
...	...	0	...	...	...	...	...	...	...	100
Aver.	**168.4**	0	15.2	35.6	**46.2**	43.2	25.8	65.6	**69.3**	100

Preliminary conclusions

Maximization problem

1. VPI is high
2. Results do not depends on graph type, distribution laws...
3. Subtree algorithm is usually the best
4. Subtree 30 often better than Subtree 10
5. Subtree never under-performs
6. EVS is the second best, but behind

Subsequent tests :

Algorithmic parameter : calibration scenarios number ?
Subtree 50 (mean increases, variance reduces)
No statistical outclassment Subtree 30, once Subtree 10
CPU time increases "linearly", LP solution $\cong$ IP

Robustness

TABLE: Robustness of distribution law parameter

Info	VPI	LB	EVS				UB
Inst./Alg.	O^*	O^*2	EG_{50}	TR_{30}^{30}	TR_{30}^{50}	TR_{30}^{70}	O^*5
d-20-15-25 A	361.4	0	25.2	40.2	**65.0**	27.3	100
w-20-15-25 A	283.7	0	34.5	**82.9**	72.5	15.1	100
d-20-20-25 A	229.0	0	31.9	**63.6**	45.3	35.6	100
w-20-20-25 A	298.4	0	3.7	**33.0**	9.7	2.6	100
Average 20	293.1	0	23.8	**55.0**	48.1	20.1	100
d-80-15-25 A	152.6	0	91.0	86.0	**111.2**	**111.2**	100
w-80-15-25 A	217.0	0	44.4	55.7	**87.1**	86.0	100
d-80-20-25 A	129.7	0	85.3	71.0	96.1	**103.4**	100
w-80-20-25 A	184.4	0	20.8	**55.2**	45.4	49.8	100
Average 80	170.9	0	60.4	67.0	84.9	**87.6**	100
Inst./Alg.	O^*	O^*2	EG_{50}	TR_{30}^{20}	TR_{30}^{50}	TR_{30}^{80}	O^*5
d-50-15-25 A	201.6	0	49.1	48.0	79.1	59.2	100
w-50-15-25 A	187.3	0	54.3	21.9	53.4	35.8	100
d-50-20-25 A	145.1	0	34.8	47.5	66.0	43.3	100
w-50-20-25 A	225.7	0	7.3	10.3	21.9	**-17.6**	100
Average 50	189.9	0	36.4	31.9	**55.1**	30.2	100

Conclusions Case Study 2

1. VPI is usually high
2. VMPM is relevant
3. Independent of graph shape, size or distribution laws
4. Subtree is the best algo and others under-perform
5. Subtree30 for simulation, Subtree50 in practice
6. By default, calibrate subtree for 50% availability (2nd best/3 and outclasses if reality is 50%)
7. **Robustness :** better to stick to distribution and approximate by the center
8. Less uncertainty on information closes the gap and reduces the VPI

7 Conclusions

- Importance of stochastic multi-periods models
- Tool to measure the values of informations
- Understandable bounds for managers
- A toolbox of algorithms to tackle those problems
- A statistical validation of algorithms, outclassment
- A robust single scenario heuristic for case 1
- A subtree solvable by a LP Solver for case 2
- Both formulations without t to solve 1 scenario

Perspectives

- Metaheuristics (many statistical issues)
- Subtree generation
- Exact : Column generation in subtree
- Improve Cs and RE algorithms
- Improve calibration scenarios generation
- Repositioning strategy, LTL (PDP)... in Case 2
- Investigate the gap between VPI and VAI
- Compare with ADP
- Strategy to find what is the option
- Answer your questions, comments, remarks...