Modal identification of time-varying systems using Hilbert transform and signal decomposition

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The research focuses on the identification of time-varying systems

$$\boldsymbol{M}(t)\,\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}(t)\,\dot{\boldsymbol{x}}(t) + \boldsymbol{K}(t)\,\boldsymbol{x}(t) = \boldsymbol{f}(t)$$

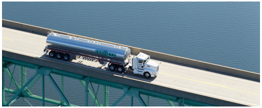
Dynamics of such systems is characterized by :

- Non-stationary time series
- Instantaneous modal properties
  - Frequencies : $\omega_r(t)$
  - Damping ratio's : $\xi_r(t)$
  - Modal deformations : $\boldsymbol{V}_r(t)$

Why time-varying behaviour appears ?

Several possible origins :

- Structural changes



- Operating conditions

- Damages

Outline of the presentation

Introduction to the Hilbert transform and the HVD method

Adaptation of the initial method to overcome some drawbacks

Application to the identification of a test structure

In this work, we use the *Hilbert Transform*

The Hilbert transform $\mathcal{H}$ of a signal $x(t)$ is the convolution product of this signal with the impulse response $h(t) = \frac{1}{\pi\, t}$

$$\begin{aligned}\mathcal{H}(x(t)) &= (h(t) * x(t)) \\ &= \text{p.v.} \int_{-\infty}^{+\infty} x(\tau) h(t-\tau)\, d\tau \\ &= \frac{1}{\pi}\, \text{p.v.} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau}\, d\tau\end{aligned}$$

It is a particular transform that remains in the time domain

It corresponds to a phase shift of $-\frac{\pi}{2}$ of the signal

The Hilbert transform and the *analytic signal*

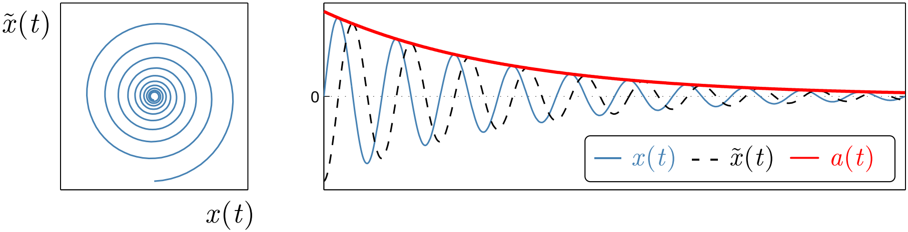
The analytic signal z is built as

$$\begin{aligned} z(t) &= x(t) + i\,\mathcal{H}(x(t)) \\ &= A(t)\, e^{i\phi(t)} \end{aligned}$$

The instantaneous properties of the signal can then be obtained

$$\begin{cases} A(t) = |z(t)| \\ \phi(t) = \angle z(t) \\ \omega(t) = \dfrac{d\phi}{dt} \end{cases}$$

Example of analytic signal construction



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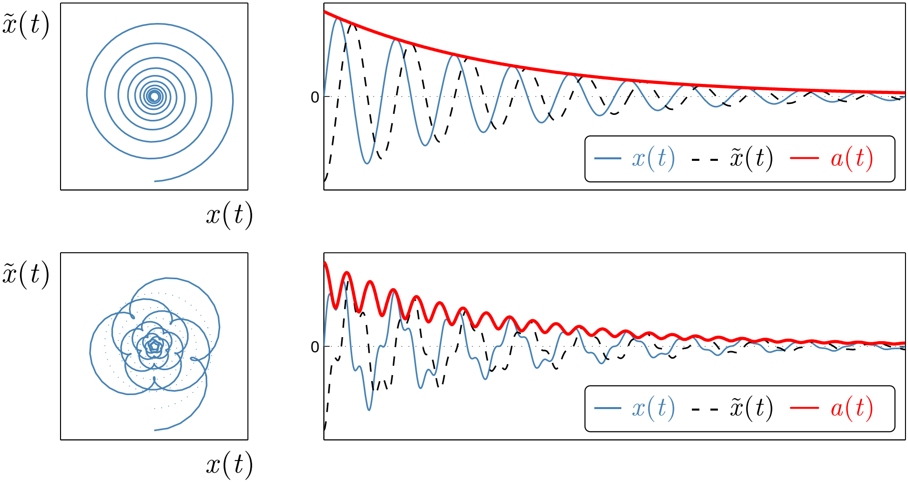
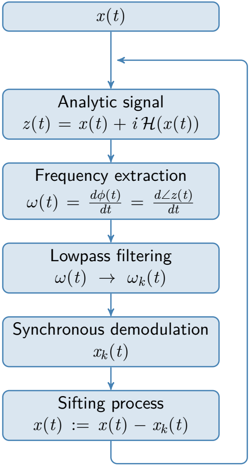


This work is based on the idea of the *Hilbert Vibration Decomposition* (HVD) method



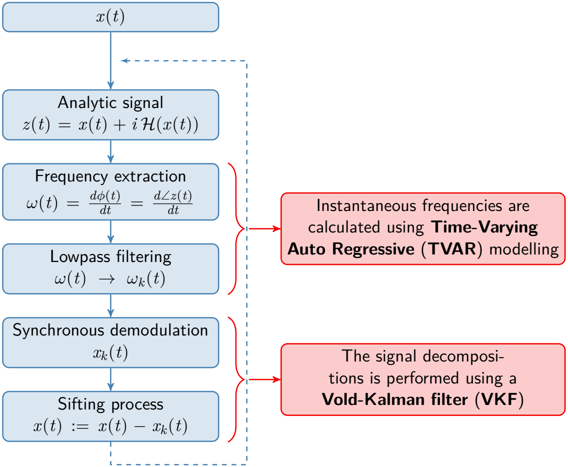
It is an iterative process:

- Detection of the instantaneous frequency of the dominent component
- Demodulation of its related monocomponent
- Extraction of the monocomponent from the signal

The sifting of the signal extracts monocomponents from higher to lower instantaneous amplitude

It is applicable to single channel measurement and crossing monocomponents may be a problem

The identification of the instantaneous frequencies and the signal decomposition are modified



Time–Varying Auto Regressive model is chosen for the identification of the instantaneous frequencies

The response is assumed to be a linear combination of its past values

In time-varying systems, the regression coefficients are time–dependant

$$x(t) = \sum_{i=1}^{p} a_i(t)\, x(t-i) + e(t),$$

leading to the time-varying transfer function

$$H(\omega, t) = \frac{1}{1 - \sum_{i=1}^{p} a_i(t)\, e^{-j\,\omega\, i}}$$

The roots of the denominator give the instantaneous poles of the system

The basis function approach is adopted to manage the time dependency of the regressive coefficients

It is assumed that the time–variation of the regressive coefficients can be represented as a linear combination of a set of known functions

$$a_i(t) = \sum_{k=0}^{q} a_{i,k} \, u_k(t)$$

In that way, only the $a_{i,k}$ coefficients are estimated, in a time invariant problem

$$\boldsymbol{G} \, \boldsymbol{\alpha} = \boldsymbol{h}$$

in which $\boldsymbol{\alpha}$ gathers the regression coefficients and $\boldsymbol{G}$ and $\boldsymbol{h}$ are functions of x and u_k

A global estimation is possible

Because the poles of the system are global properties, the regressive coefficients should be the same if identified from any sensor

A least squares estimate of the regression coefficients is possible by gathering all the sensors

$$\begin{bmatrix} \boldsymbol{G}_1 \\ \boldsymbol{G}_2 \\ \vdots \\ \boldsymbol{G}_n \end{bmatrix} \boldsymbol{\alpha} = \begin{bmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \\ \vdots \\ \boldsymbol{h}_n \end{bmatrix} \begin{matrix} \rightarrow \text{sensor } 1 \\ \rightarrow \text{sensor } 2 \\ \vdots \\ \rightarrow \text{sensor } n \end{matrix}$$

Response signals are decomposed using a Vold–Kalman filter

This method allows to retrieve signal subcomponents based on their phase

$$x(t) = \sum_k \underbrace{a_k(t)\, e^{i \int_0^t \omega_k(t) \mathrm{d}t}}_{x_{(k)}(t)} + \delta(t)$$

The complex amplitudes of the components minimise the data equation

$$x(t) - \sum_k a_k(t)\, e^{i\,\phi_k(t)} = \delta(t)$$

and the structural equations

$$a_k(t-1) - 2a_k(t) + a_k(t+1) = \varepsilon_k(t)$$

Monocomponents and complex amplitudes are extracted

The Vold-Kalman model and the modal expansion are very similar.

The extracted complex amplitudes are then considered as unscaled mode shapes

Vold-Kalman filter: $\boldsymbol{x}(t) = \sum_k \quad \boldsymbol{a_k}(t) \quad e^{i\,\phi_k(t)}$

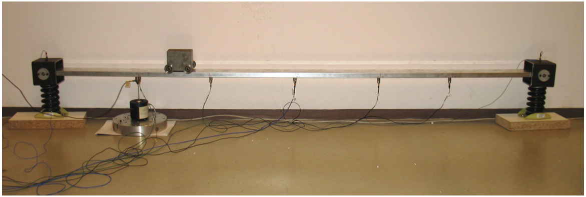
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad \updownarrow \qquad\quad \updownarrow$

Modal expansion: $\boldsymbol{x}(t) = \sum_k \quad \boldsymbol{V_k}(t) \quad \eta_k(t)$

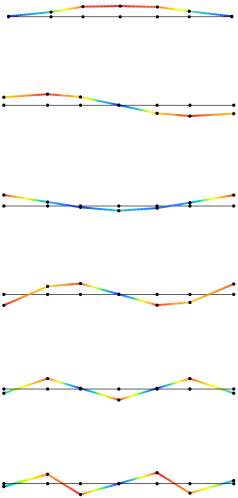
The experimental set-up

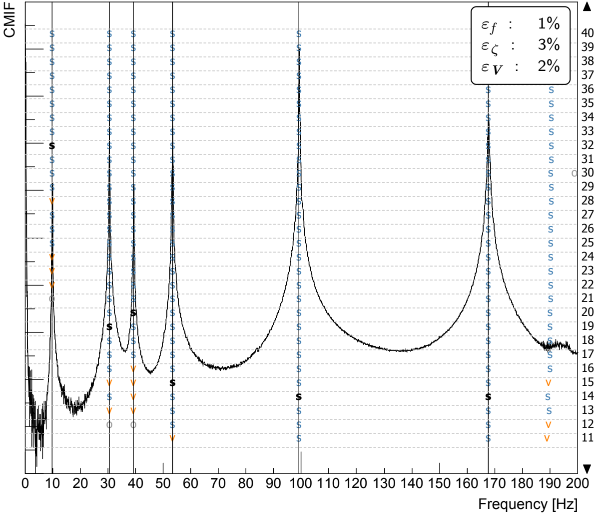
2.1 meter aluminum beam
Steel block ($\approx$ 3.5 kg, 38.6%)

1 shaker
7 accelerometers
LMS SCADAS & LMS Test.Lab system



Time invariant modal identification of the beam subsystem



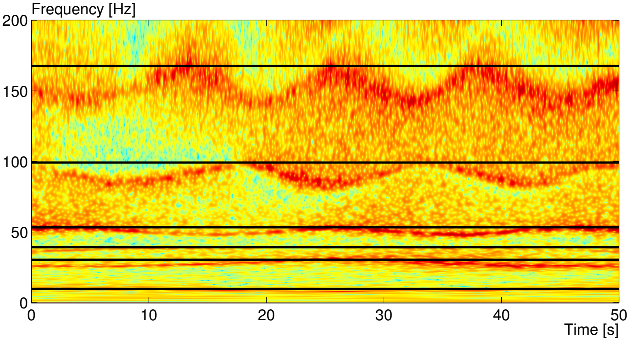


A record is performed in time–varying conditions

The shaker excites the structure with a random force

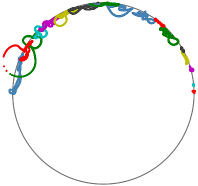
The mass is pulled along the beam

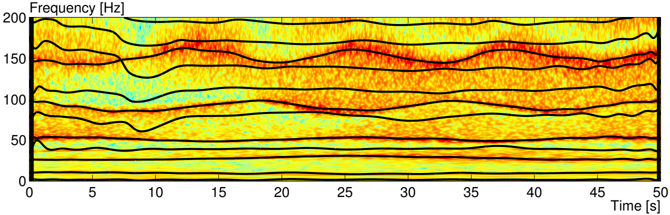
The accelerations of the beam are recorded



The time–varying identification is now performed

- Legendre polynomials are chosen as basis functions
- The model order and size of the basis are chosen ($p = 17,\ q = 32$)





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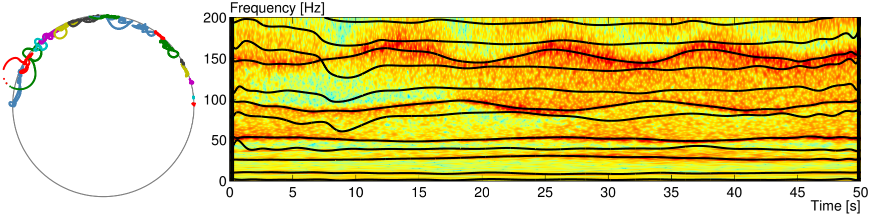

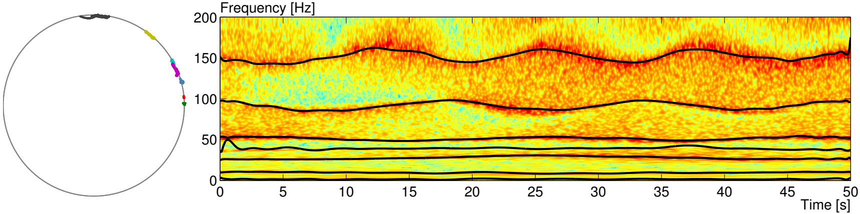

The moving mass has an influence on the mode shapes of the structure

The mode shapes are the most disrupted when the mass lies on an anti-node of vibration

Conversely, when the mass lies on a node of a mode, the latter is no more perturbed

Thank you for your attention