"Optimization of Stochastic Multi-Period Problems in Transportation"

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Joint work from

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comex
combinatorial optimization:
metaheuristics & exact methods

0 Message

Transportation models : as decisions making

Set of optimal decisions or optimal sequence of decisions

TSP, VRP, PDP...No past, no future

- Mono-period vs Multi-period (not periodic, not year)
- Independent **and** Subsequent and related
- Deterministic **and** Stochastic Information
- Data **and** Forecasts
- Parameters **and** Distribution laws
- Optimal solutions, Heuristic values **and** Policies
- Instances **and** Scenarios or Futures
- Values **and** Statistical performances
- P solvable, NP-Hard **and** ***"Intractable"***

Contribution : A framework for experimentation

1. Outlines

- Multi-period problems
- Decision making under uncertainty
- Stochastic Optimization Techniques
- A Methodology
  1. Bounds
  2. A picture for manager
  3. Algorithms
  4. Results validation
- Case study (Vehicle-Load Assignment)
- Conclusions

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2. Rolling Horizon

[P1]	[P1..Pi]	[Pi+1..RH]	[RH+1..Pn
Deterministic		**Stochastic**	**Tail**

Decision : in P1
Action : on deterministic part [P1...Pi] ⇒ feasible
Case study :

1. rolling horizon = 5 periods = 5 days = 1 week
2. periods deterministic 2 days, forecasts 3 days
3. action in P1

Dynamism of the system

- Decision in P1
- Actions (info out)
- Roll-over 1 period, updates (in)
  1. stochastic becomes deterministic $Pi + 1 = Pi$
  2. new stochastic info $RH + 1 = RH$
- Decision in P2 = P1

3. Solutions to a stochastic problem ?

- Worst case (oversize solution)
- Chance constrained (95%)
- Robust to variation (Tree)
- Flexible : Easy to recover (Grass)
- **Min or max Expected cost-profit (E*)**

4. Usual techniques

1. "Stochastic Programming" 2-stage, convex, continuous
2. "Markov Chain" states, actions, stability
3. "Approximated Dynamic Programming" ADP
4. "Sample Average Approximation" SAA (continuous)
5. Scenario tree, but rolling horizon

Integer + Discrete distribution laws + Tail

Curse of dimensionality $\Rightarrow$ Intractable !
10 trips per period $2^{10} = 1024$, 3 periods 2^{30}

Conclusion : ***"Hard" to find E****

Methodology :
Simulation and optimization over deterministic scenarios
Solve one, several, some scenarios = "futures"
Literature and algorithms as a brick

Which scenarios ?

5.1 Bounds

Oracle : a posteriori, revealed info **O***
Infinite horizon value with deterministic info
Real Bound (Upper or lower) on **E***

VPI : Value of the Perfect Information |O*-E*|≥0

Myopic : Deterministic periods value **LO**
Bound (Lower or upper) on any policy with forecasts

Deterministic approximation : **Mean**
Mean => EVS : Expected Value Solution

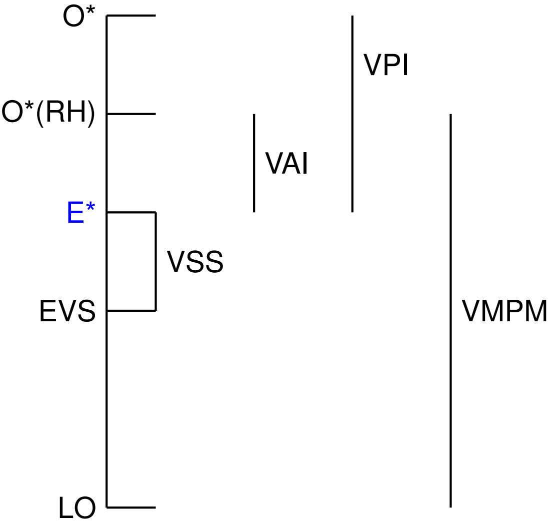
VSS : Value of the stochastic solution |E*-EVS|≥0

Rolling horizon : Finite Oracle **O*(RH)**

VMPM : Value of the multi-period model |O*(RH)-LO|≥0

VAI : Value of the Available Information |O*(RH)-E*|≥0

5.2 A picture for a manager : max



Simulations => *"Expected value of"* : EO*, EO*(RH), ELO, EEVS, EVSS, EVAI, EVPI, EVMPM

5.3 Approximations of E*

- Solve a "good" **single scenario** (Mean, Mod)
- **Consensus (Cs) :**
  1. Solve N scenarios
  2. Create a new solution with common decisions
- **Restricted Expectation (RE) :** Solve scenarios i,j and cross-evaluate action i over scenarios j
  1. Scenarios i,j ($\in N$) $\Rightarrow$ Solutions i,j $\Rightarrow$ Actions i,j
  2. Evaluate value of Action i on Scenario j
  3. Cumulated value of Actions i, j
  4. Select the best action

Questions

- Reduced actions and state techniques ?
- Scenarios generation ?
- Stochastic solution from deterministic model ?
- Deterministic solutions are elitist, no option in it
- CPU Time : 1, $N + 1$, N^2

5.3 Approximations of E*

Full tree : Deterministic equivalent
$\Rightarrow$ One common action for all futures
Links : **Non-anticipativity constraints**
Action variables are equal in each scenario

In practice : Out of Memory, CPU Time, B & B

Approximation by a **Subtree** ($1 * ST \neq ST * 1$)

Subtree formulation often harder than a single scenario

5.4 Statistical validation and Robustness

Statistical validation :
E* = Best policy we can found
How to compare Policy 1 with Policy 2, E*1 vs E*2 ?

Outclassment = significant difference between means

"Paired sample comparison"
Hypothesis : $\mu_1 \neq \mu_2$, $\mu_1 > \mu_2$?
Solve 30 scenarios by instance over an horizon 20 P
Non Non-Normality check, confidence level, t-student...

Robustness analysis :
Assumption : Distribution law is known in practice
Test : Calibration law $\neq$ Real law (Cs, RE, Mean)

6.1 Vehicle-Load Assignment Problem

Problem Assign trip to truck FTL (PDP with selection)

Decisions : Wait, Move Empty, Load

Objective :
Maximize Profit (Load-Empty Moves-Waiting)

Constraint : loading if at place on time, no preemption

Data : [1,2] and forecasts on available loads [3,4,5]

Stochasticity : Availability [...%] of a trip from A to B, start in [3,4,5]

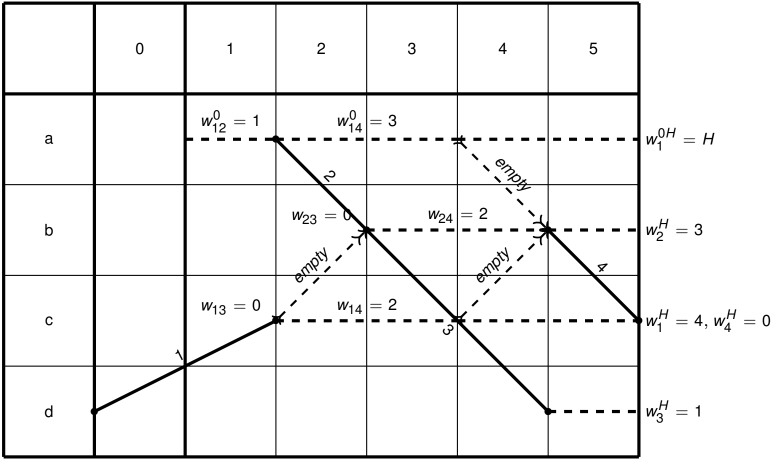
Distribution laws [...%] linked to :

1. Traveled distance (1, 2, 3, 4)
2. City size (B, M, S)

Solution : Network flow problem

6.2 A Picture for a scenario

A representation of the time-space (Periods, Cities)



6.3 Results : one example 150 loads 20 P

TABLE: Distribution laws linked to distance

Info		**LB**			**EVS**					**UB**
Alg	O^*	O^*2	*Opt*	*Mod*	*EG*	*Cs*	RE^*	TR_{10}	TR_{30}	O^*5
1-10	120.4	0	22.8	17.3	37.3	31.2	12.4	48.6	**58.2**	100
1-15-25A	153.0	0	12.9	38.8	38.4	51.4	43.1	65.7	**70.2**	100
1-15-25B	153.8	0	13.7	44.7	49.2	45.5	26.7	66.5	**75.5**	100
1-15-25C	176.0	0	32.8	43.5	67.1	45.2	36.9	72.8	**85.2**	100
1-20	135.0	0	14.8	41.3	52.5	38.9	46.5	69.8	**71.0**	100
1-20-25A	167.8	0	6.8	32.5	62.4	21.3	44.9	73.1	**78.1**	100
1-20-25B	149.6	0	23.3	41.0	46.2	42.0	31.8	**70.6**	60.0	100
1-20-25C	199.8	0	-22.1	30.1	24.1	27.0	-24.7	61.5	**67.9**	100
1-25	164.9	0	-83.6	6.5	7.9	12.6	-32.1	**54.9**	50.6	100
2-10	163.7	0	18.4	38.9	44.7	37.7	26.8	67.4	**74.3**	100
2-15-25A	221.3	0	69.2	70.2	65.8	70.9	63.7	**77.2**	76.4	100
2-15-25B	186.1	0	65.1	66.3	70.3	51.0	62.4	83.2	**87.4**	100
2-15-25C	136.6	0	36.7	60.4	67.5	73.1	42.5	78.3	**82.4**	100
2-20	204.6	0	59.6	**74.5**	57.7	53.0	39.3	71.6	70.1	100
2-20-25A	190.1	0	51.6	71.1	81.1	69.4	60.2	82.7	**83.3**	100
2-20-25B	150.9	0	30.4	40.0	54.5	57.4	53.2	**77.5**	74.2	100
2-20-25C	180.9	0	65.2	86.5	87.1	79.6	62.0	86.3	**89.2**	100
2-25	167.3	0	11.4	50.0	65.0	64.2	42.6	**69.8**	61.0	100
...	...	0	...	...	...	...	...	...	...	100
Aver.	**168.4**	0	15.2	35.6	**46.2**	43.2	25.8	65.6	**69.3**	100

6.4 Preliminary conclusions

1. Dynamism is important : VMPM
2. VPI is high (68.4% + 30.7%)
3. No influence of graphs, distribution laws...
4. Subtree algorithm is usually the best
5. Subtree 30 often better than Subtree 10
6. Subtree never under-performs
7. EVS is the second best after subtree
8. the VSS is important 23.1%

Subsequent tests for the subtree :
Algorithmic parameter : calibration scenarios 10-30-50...
$\Rightarrow$ Subtree 50 (mean increases, variance reduces)
No statistical outclassment 50 vs 30, once 50 vs 10
CPU time increases "linearly", LP solution $\cong$ IP

6.5 Robustness

TABLE: Robustness of distribution law parameter

Info		LB	EVS				UB
Inst./Alg.	O^*	O^*2	EG^{50}	TR^{30}_{30}	TR^{50}_{30}	TR^{70}_{30}	O^*5
d-20-15-25 A	361.4	0	25.2	40.2	**65.0**	27.3	100
w-20-15-25 A	283.7	0	34.5	**82.9**	72.5	15.1	100
d-20-20-25 A	229.0	0	31.9	**63.6**	45.3	35.6	100
w-20-20-25 A	298.4	0	3.7	**33.0**	9.7	2.6	100
Average 20	293.1	0	23.8	**55.0**	48.1	20.1	100
d-80-15-25 A	152.6	0	91.0	86.0	**111.2**	**111.2**	100
w-80-15-25 A	217.0	0	44.4	55.7	**87.1**	86.0	100
d-80-20-25 A	129.7	0	85.3	71.0	96.1	**103.4**	100
w-80-20-25 A	184.4	0	20.8	**55.2**	45.4	49.8	100
Average 80	170.9	0	60.4	67.0	84.9	**87.6**	100
Inst./Alg.	O^*	O^*2	EG^{50}	TR^{20}_{30}	TR^{50}_{30}	TR^{80}_{30}	O^*5
d-50-15-25 A	201.6	0	49.1	48.0	79.1	59.2	100
w-50-15-25 A	187.3	0	54.3	21.9	53.4	35.8	100
d-50-20-25 A	145.1	0	34.8	47.5	66.0	43.3	100
w-50-20-25 A	225.7	0	7.3	10.3	21.9	**-17.6**	100
Average 50	189.9	0	36.4	31.9	**55.1**	30.2	100

6.6 Conclusions Case Study

1. VPI, VMPM, VSS are relevant
2. Independent of graph shape, size or distribution laws
3. Subtree is the best algo and others under-perform
4. Subtree with 30 or 50 scenarios is enough
5. By default, calibrate subtree for 50% availability (2nd best/3 and outclasses if reality is 50%)
6. Robustness : better to stick to distribution and approximate by the center
7. Less uncertainty on information closes the gap and reduces the VPI

7.1 Conclusions-contribution

- Importance of stochastic multi-period models
- Tool to measure the values of informations
- Understandable bounds for managers
- A toolbox of algorithms to tackle those problems
- A statistical validation of algorithms, outclassment
- A subtree solvable by a LP Solver in the case study

7.2 Perspectives

- Metaheuristics (many statistical issues)
- Subtree generation
- Exact : Column generation in subtree if hard
- Improve Cs and RE algorithms
- Improve calibration scenarios generation
- Repositioning strategy, LTL (PDP)
- Investigate the gap between VPI and VAI
- Compare with ADP
- Answer your questions, comments, remarks...