"Multi-period vehicle assignment with stochastic load availability”

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### Vehicle assignment
To maximize profit: select loads to be transported by trucks (FTL-PDP) References: W.B. Powell

### Multi-period
Confirmed and projected loads provided over some periods
Repetitive decision process period per period over an horizon

### Stochastic load availability
Projected loads realize or vanish
Outlines

- Multi-period information and decision framework
- The Deterministic Vehicle Assignment Problem
- Bounds
- Algorithms
- Instances and Results
- Robustness Analysis
- Conclusions
Multi-period : Rolling horizon

**Decision**: in \( t \) and \( t = 1, 2, \ldots, T - H \) \( \Rightarrow \) Policy

- \( t \)
- \( t+1, \ldots, t+RH \)
- \( t+RH+1, \ldots, t+H \)
- \( t+H+1, \ldots, T \)

**Deterministic** \quad **Stochastic** \quad **Tail**

**Parts**: decision, deterministic, stochastic

1. Rolling horizon \( H = 4P = 4 \) days
2. Deterministic \( RH = 1P \), Stochastic \( 3P \)

**Dynamism of the system**:

1. Decision and actions in \( t \) (info out)
2. Roll-over 1 period, updates (info in) \( t \rightarrow t + 1 \)
   1. stochastic gets deterministic \( t + RH + 1 \rightarrow t' + RH \)
   2. new stochastic info in \( t + H + 1 \rightarrow t' + H \)
3. Go to 1 with \( t \rightarrow t + 1 \)
Vehicle Assignment Problem: Description

**Full truckload selection**

**Data**: Cities, Distances, Periods, Loads, Trucks  
**Actions**: Carry, Wait, Move unladen  
**Objective function**: maximize Profit (Gains-Costs)  
**Constraints**: Space, Time, Max 1 Load per Truck

**Stochastic data**: Stochastic Load Availability in one period  
Discrete and finite Bernoulli distribution for load $L_j$

$$P(q_j = x) = \begin{cases} 
p_j & \text{if } x = 1 \\
1 - p_j & \text{if } x = 0 \end{cases}$$
**Single scenario model**

**Deterministic formulation**: Network flow structure

Polynomially solvable
Feasible links: time and space aggregated

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<th>t</th>
<th>t+1</th>
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<td>Wait</td>
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<td><strong>C₂</strong></td>
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<td>Unladen</td>
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<td>Carry L₃</td>
<td>Wait</td>
<td></td>
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Specific Scenarios => Bounds

**Bounds**: fully revealed information scenarios

1. Myopic or a-priori policy over RH: \( O_{RH}^* \)
2. Oracle or a-posteriori policy over H: \( O_H^* \)
3. Oracle or a-posteriori solution over T: \( O_T^* \)

**Stochastic problem**

Expected Value Scenario => Expected Value 'Solution' \( EVS \)

Optimal policy for the stochastic problem: \( E^* \)

Maximization: \( O_T^* \geq O_H^* \geq E^* \geq EVS \geq O_{RH}^* \)

**Value of information**:

\( VPI : \) Value of the Perfect Information \( O_T^* - E^* \geq 0 \)
Problem: Found $E^*$ the optimal policy
Approximate models and algorithms

**Bounds**: Fully revealed information

\[ O^*, O_H^* \text{ (UB=100%)}, O_{RH}^* \text{ (LB=0%)} \]

**Mono scenario approximation**

**EVS** expected reward, Modal and **Optimist** (all loads)

**Multiple scenario approaches**: 10 to 30 scenarios

- **Consensus**: Aggregate per action in \( t \) and per city
  \[ \Rightarrow \text{Allocate action per truck decreasingly} \]

- **Restricted Expectation**: Cross-evaluation of decisions in \( t \) inserted in other scenarios, highest cumulated gain

- **Subtree**: Non-anticipativity constraints in \( t \), Tractable
## Instances and Results

**Instances**: 10 Trucks, 10-15-20-25 Cities, 150-200 Loads, 20 P
Probability of availability ($p_j$) linked to distance or city sizes

<table>
<thead>
<tr>
<th>Info</th>
<th>Inst./Alg.</th>
<th>$O^*_T$</th>
<th>LB</th>
<th>EVS</th>
<th>Cs</th>
<th>ST</th>
<th>UB</th>
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<td><strong>56.6</strong></td>
<td><strong>46.7</strong></td>
<td><strong>67.6</strong></td>
<td><strong>100</strong></td>
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Results analysis

Observations:

- High value of **EVMPM**
- Graphs or distributions do not seem to influence the results
- **EVTI, EVPI** are high on average (e.g. 78.4%, 110.7%)
- **ST is mostly** $\mu_{\pi^*}$ rarely Cs or **EVS**
- **ST** never under-performs and closes 2/3 of the gap
  
  $O_{RH}^{*} - O_{H}^{*}$

- **EVS** performs "well" (e.g. $\text{EVSS} = +/\text{-11\%}$)
Robustness analysis

**Robustness :**

*forecast availabilities based on a probability $p$ in algorithm $ST^p$ compared with real availabilities $p'$*

<table>
<thead>
<tr>
<th>Reality/Forecast</th>
<th>EVS</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
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<td>$ST^{30}$</td>
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<td>Alg.</td>
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<td>Reality Medium 50%</td>
<td>36.4</td>
<td>31.9</td>
<td>55.1</td>
<td>30.2</td>
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</table>

**Aim :**

to be independent from distribution
Conclusions

1. Importance of stochastic multi-period models
2. VPI, VMPM, VSS are relevant information values
3. ST is the best algo and others under-perform
4. ST^{50} (calibrated with a 50% availability) is robust
5. ST solvable by a LP solver
6. e.g Independent of graph shape, size or distribution laws

Perspectives:
1. Repositioning strategy
2. Investigate the VTI