"Multi-period vehicle assignment with stochastic load availability"

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QuantOM

comex combinatorial optimization: metaheuristics & exact methods

"Multi-period vehicle assignment with stochastic load availability"

Vehicle assignment

To maximize profit : select loads to be transported by trucks (FTL-PDP) References : W.B. Powell

Multi-period

Confirmed and projected loads provided over some periods
Repetitive decision process period per period over an horizon

Stochastic load availability

Projected loads realize or vanish

Outlines

- **Multi-period information and decision framework**
- **The Deterministic Vehicle Assignment Problem**
- **Bounds**
- **Algorithms**
- **Instances and Results**
- **Robustness Analysis**
- **Conclusions**

Multi-period : Rolling horizon

Decision : in t and $t = 1, 2, ..., T - H$ => Policy

t	t+1,...,t+RH	t+RH+1,...,t+H	t+H+1,...,T
	Deterministic	Stochastic	Tail

Parts : decision, deterministic, stochastic

1. Rolling horizon $H = 4P = 4$ days
2. Deterministic $RH = 1P$, Stochastic $3P$

Dynamism of the system :

1. Decision and actions in t (info out)
2. Roll-over 1 period, updates (info in) $t \rightarrow t + 1$
   1. stochastic gets deterministic $t + RH + 1 \rightarrow t' + RH$
   2. new stochastic info in $t + H + 1 \rightarrow t' + H$
3. Go to 1 with $t \rightarrow t + 1$

Vehicle Assignment Problem : Description

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Full truckload selection

Data : Cities, Distances, Periods, Loads, Trucks
Actions : Carry, **Wait**, **Move unladen**
Objective function : maximize Profit (Gains-Costs)
Constraints : Space, Time, Max 1 Load per Truck

Stochastic data : Stochastic Load Availability in one period

Discrete and finite Bernoulli distribution for load L_j

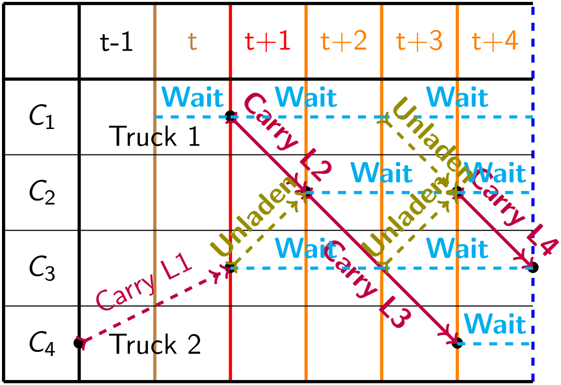
$$P(q_j = x) = \begin{cases} p_j & if \ \ x = 1 \\ 1 - p_j & if \ \ x = 0 \end{cases}$$

Single scenario model

Deterministic formulation : Network flow structure

Polynomially solvable

Feasible links : time and space aggregated



Specific Scenarios => Bounds

t	t+1,...,t+RH	t+RH+1,...,t+H	t+H+1,...,T
	Deterministic	**Stochastic**	**Tail**

Bounds : fully revealed information scenarios

1. Myopic or a-priori policy over RH : O^*_{RH}
2. Oracle or a-posteriori policy over H : O^*_H
3. Oracle or a-posteriori solution over T : O^*_T

Stochastic problem

Expected Value Scenario => Expected Value 'Solution' EVS

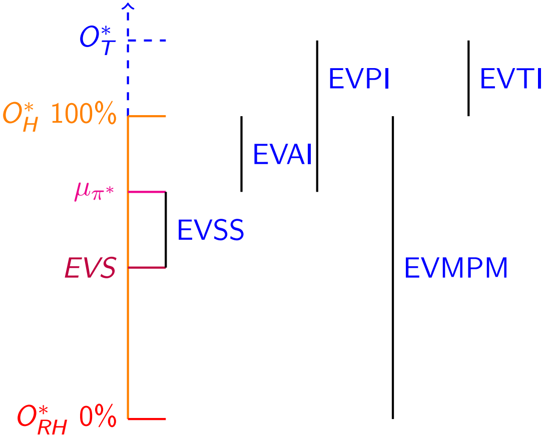
Optimal policy for the stochastic problem : E^*

Maximization : $O^*_T \geq O^*_H \geq E^* \geq EVS \geq O^*_{RH}$

Value of information :

VPI : Value of the Perfect Information O^*_T - $E^* \geq 0$

A picture : maximization



Problem : Found E^* the optimal policy

Approximate models and algorithms

Bounds : Fully revealed information

O^*, O^*_H (UB=100%), O^*_{RH} (LB=0%)

Mono scenario approximation

EVS expected reward, Modal and **Opt**imist (all loads)

Multiple scenario approaches : 10 to 30 scenarios

- **Consensus** : Aggregate per action in t and per city => Allocate action per truck decreasingly
- **Restricted Expectation :** Cross-evaluation of decisions in t inserted in other scenarios, highest cumulated gain
- **Subtree :** Non-anticipativity constraints in t, Tractable

Instances and Results

Instances : 10 Trucks, 10-15-20-25 Cities, 150-200 Loads, 20 P
Probability of availability (p_j) linked to distance or city sizes

Info		LB	EVS			UB
Inst./Alg.	O_T^*	O_{RH}^*	EVS	Cs	ST	O_H^*
5-15-25 A	222.0	0	73.6	**80.0**	79.2	100
6-15-25 A	156.1	0	78.6	**90.8**	89.7	100
7-15-25 A	171.0	0	57.2	68.0	**70.7**	100
8-15-25 A	187.3	0	**54.3**	13.8	53.4	100
5-15-25 B	153.1	0	57.7	61.2	**81.6**	100
6-15-25 B	165.7	0	55.8	42.8	**60.3**	100
7-15-25 B	194.7	0	56.5	60.4	**61.0**	100
8-15-25 B	201.4	0	86.7	60.8	**100.0**	100
5-15-25 C	192.4	0	64.1	53.8	**78.8**	100
6-15-25 C	125.9	0	62.7	78.3	**88.0**	100
7-15-25 C	179.2	0	63.9	49.6	**70.4**	100
8-15-25 C	192.0	0	47.0	20.0	**63.5**	100
5-20-25 A	195.1	0	63.9	45.2	**65.9**	100
6-20-25 A	153.8	0	52.1	54.4	**74.3**	100
7-20-25 A	253.9	0	38.6	32.1	**44.5**	100
8-20-25 A	225.7	0	7.3	-36.5	**21.9**	100
5-20-25 B	141.9	0	62.9	33.2	**68.4**	100
6-20-25 B	147.4	0	62.7	53.4	**74.2**	100
7-20-25 B	176.7	0	52.1	52.7	**66.1**	100
8-20-25 B	165.1	0	49.8	25.6	**54.2**	100
5-20-25 C	171.7	0	51.4	61.2	**67.7**	100
6-20-25 C	215.3	0	39.1	23.6	**56.1**	100
7-20-25 C	142.9	0	53.6	54.0	**61.3**	100
8-20-25 C	150.3	0	67.3	41.7	**71.3**	100
Average	**178.4**	0	**56.6**	46.7	**67.6**	100

Results analysis

Observations :

- High value of **EVMPM**
- Graphs or distributions do not seem to influence the results
- **EVTI**, **EVPI** are high on average (e.g. 78.4%, 110.7%)
- ST **is mostly** μ_{π^*} rarely Cs or EVS
- ST never under-performs and closes 2/3 of the gap O^*_{RH}-O^*_{H}
- EVS performs "well" (e.g **EVSS**=+/-11%)

Robustness analysis

Robustness :

forecast availabilities based on a probability p in algorithm ST^p compared with real availabilities p'

Reality/Forecast	EVS	Low	Medium	High
Alg.	EVS^{50}	ST^{30}	ST^{50}	ST^{70}
Reality Low 20%	23.8	**55.0**	48.1	20.1
Reality High 80%	60.4	67.0	84.9	**87.6**
Alg.	EVS^{50}	ST^{20}	ST^{50}	ST^{80}
Reality Medium 50%	36.4	31.9	**55.1**	30.2

Aim :

to be independent from distribution

Conclusions

Conclusions

1. Importance of stochastic multi-period models
2. VPI, VMPM, VSS are relevant information values
3. ST is the best algo and others under-perform
4. ST^{50} (calibrated with a 50% availability) is robust
5. ST solvable by a LP solver
6. e.g Independent of graph shape, size or distribution laws

Perspectives :

1. Repositioning strategy
2. Investigate the VTI