ROADEF 2014

”Multi-period vehicle assignment with stochastic load availability”

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"Multi-period vehicle assignment with stochastic load availability"

THE PROBLEM

Vehicle assignment

To maximize profit : select loads to be transported by trucks (FTL-PDP) References : W.B. Powell

Multi-period

Confirmed and projected loads provided over some periods
Repetitive decision process period per period over an horizon

Stochastic load availability

Projected loads realize or vanish

Outlines

- Multi-period information and decision framework
- The Deterministic Vehicle Assignment Problem
- The Stochastic version
- Bounds
- Algorithms
- Instances and Results
- Robustness Analysis
- Conclusions

Multi-period : Rolling horizon

Decision : in t and $t = 1, 2, ..., T - H$ => **Policy**

Parts : decision, deterministic, stochastic

Case study ($Period = day$) :

1. Rolling horizon $H = 4P$
2. Deterministic $RH = 1P$, Stochastic $3P$

Dynamism of the system :

1. Decision and actions in t (info out)
2. Roll-over 1 period, updates (info in) $t + 1 \rightarrow t'$
   - 2.1 stochastic gets deterministic $t + RH + 1 \rightarrow t' + RH$
   - 2.2 new stochastic info in $t + H + 1 \rightarrow t' + H$
3. Go to 1 with $t \rightarrow t + 1 = t'$

Deterministic Vehicle Assignment Problem

- Set of **Cities** $C_1, \ldots, C_N$ and transportation times $TT_{(C_1,\ C_2)}$
- Set of **Periods** $1, \ldots, T$
- Set of **Loads** $j \in J$ $(DepC_j,\ ArrC_j,\ DepP_j,\ ArrP_j,\ Gain_j)$
- Set of **Trucks** $i \in I$ $(LocC_i,\ Un/Loaded_i$ value 0 or $j)$

Actions : Carry $Load_j$, **Wait in** $LocC_i$, **Unladen to** $DepC_j$

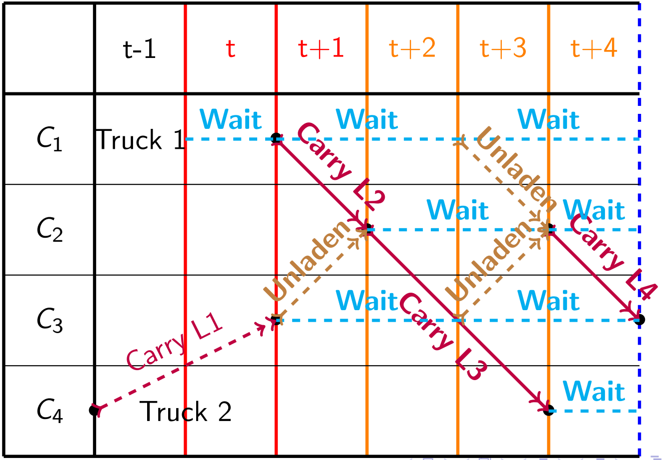
Objective : profitable paths i.e. maximize (Gains-Costs)
subject to :
Max 1 Load per Truck, max 1 Truck per Load
Flow conservation constraints

Network flow structure : Polynomially Solvable

Deterministic Vehicle Assignment Problem

Decisions for a truck *i*

- **Carry Lj** if $LocC_i = DepC_j$ (Gain)
- **Wait** in $LocC_i$ (Cost)
- Move **Unladen** from $LocC_i$ to $DepC_j$ (Cost)



Stochastic Vehicle Assignment Problem

Stochastic framework : **Stochastic Load Availability**
If $DepP_j \in t + RH + 1, .., t + H$, (*)
the stochastic availability of load j is represented by a discrete distribution law :

$$P(q_j = x) = \begin{cases} p_j & if \;\; x = 1 \\ 1 - p_j & if \;\; x = 0 \end{cases} \tag{1}$$

Projection q_j materializes (1) or not (0)
when $t + RH + 1 \rightarrow t' + RH$

Scenario : specific outcome of q_j $\forall j \in J$ if (*).

Specific Scenarios => Bounds

t | t+1,...,t+RH | t+RH+1,...,t+H | t+H+1,...,T

Deterministic **Stochastic** **Tail**

Bounds : fully revealed information scenarios

1. Myopic or a-priori policy over RH : O^*_{RH}
2. Oracle or a-posteriori policy over H : O^*_H
3. Oracle or a-posteriori solution over T : O^*_T

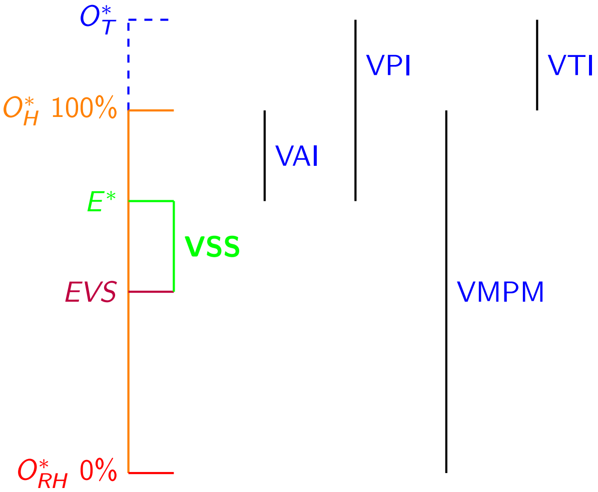
Expected Value Scenario => Expected Value 'Solution' EVS

Optimal policy for the stochastic problem : E^*

Maximization : $O^*_T \geq O^*_H \geq E^* \geq EVS \geq O^*_{RH}$

VPI : Value of the Perfect Information O^*_T - $E^* \geq 0$

A picture : maximization



O_T^*

O_H^* 100%

E^*

EVS

O_{RH}^* 0%

VSS

VAI

VPI

VTI

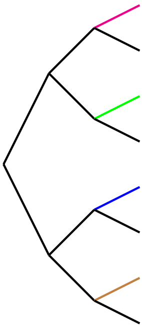
VMPM

Problem : Found E^* the optimal policy

Algorithms : Single scenario approach

Approximations of E^* Truck 1 Carry 🔴 Truck 2 Wait 🔵

Single scenarios (Mean, Modal, "Optimist", Dedicated)



Solution 1 Truck 1 🔵 Truck 2 🔴
Solution 2 Truck 1 🔵 Truck 2 🔵
Solution 3 Truck 1 🔴 Truck 2 🔵
Solution 4 Truck 1 🔵 Truck 2 🔴
Solution 5 Truck 1 🔴 Truck 2 🔴
Solution 6 Truck 1 🔴 Truck 2 🔴
Solution 7 Truck 1 🔵 Truck 2 🔵
Solution 8 Truck 1 🔵 Truck 2 🔴

Found a single scenario such as Solution 3=E^* ?

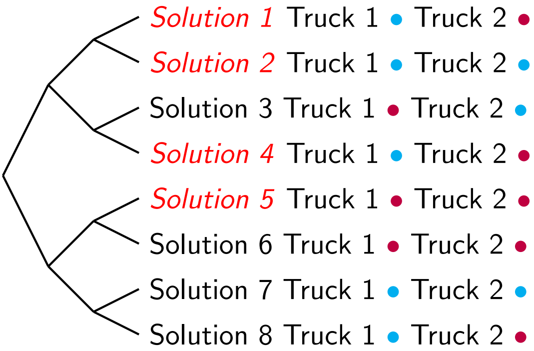
Algorithms : Multiple Scenario Approaches

Approximations of E^* Truck 1 Carry 🔴 Truck 2 Wait 🔵

MSA : Consensus (Cs) algorithm

1. Solve *N scenarios*
2. Create a new solution with frequent decisions

Solution found : Truck 1 Wait 🔵 Truck 2 Carry 🔴



Solution 1 Truck 1 🔵 Truck 2 🔴
Solution 2 Truck 1 🔵 Truck 2 🔵
Solution 3 Truck 1 🔴 Truck 2 🔵
Solution 4 Truck 1 🔵 Truck 2 🔴
Solution 5 Truck 1 🔴 Truck 2 🔴
Solution 6 Truck 1 🔴 Truck 2 🔴
Solution 7 Truck 1 🔵 Truck 2 🔵
Solution 8 Truck 1 🔵 Truck 2 🔴

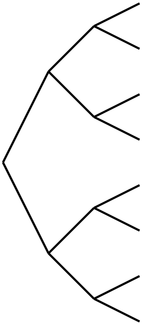
Algorithms : Multiple Scenario Approaches

Approximations of E^* Truck 1 Carry 🔴 Truck 2 Wait 🔵

MSA : Restricted Expectation (RE) algorithm

1. Solve *N scenarios*
2. Insert actions of i in scenario j $i \neq j \in N$ and solve
3. Cumulated value of solution of scenarios j with action i
4. Select the best action i after cross evaluation

Solution found : Truck 1 Wait 🔵 Truck 2 Carry 🔴



Solution 1 Truck 1 🔵 Truck 2 🔴

Solution 2 Truck 1 🔵 Truck 2 🔵

Solution 3 Truck 1 🔴 Truck 2 🔵

Solution 4 Truck 1 🔵 Truck 2 🔴

Solution 5 Truck 1 🔴 Truck 2 🔴

Solution 6 Truck 1 🔴 Truck 2 🔴

Solution 7 Truck 1 🔵 Truck 2 🔵

Solution 8 Truck 1 🔵 Truck 2 🔴

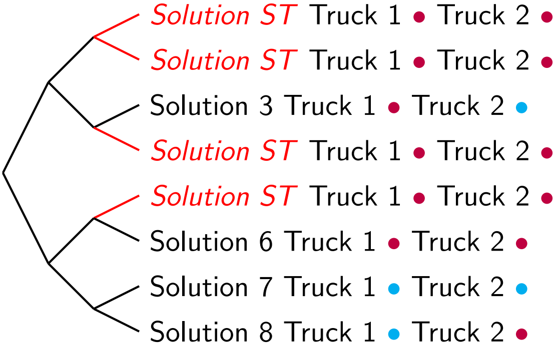
Algorithms : Multiple Scenario Approaches

Approximations of E^* Truck 1 Carry ● Truck 2 Wait ●

MSA : Subtree (ST) algorithm

1. Add non-anticipativity constraints among *N scenarios*
2. Solve *N scenarios* at once, linear relaxation is optimal

Solution found : Truck 1 Carry ● Truck 2 Carry ●



Solution ST Truck 1 ● Truck 2 ●
Solution ST Truck 1 ● Truck 2 ●
Solution 3 Truck 1 ● Truck 2 ●
Solution ST Truck 1 ● Truck 2 ●
Solution ST Truck 1 ● Truck 2 ●
Solution 6 Truck 1 ● Truck 2 ●
Solution 7 Truck 1 ● Truck 2 ●
Solution 8 Truck 1 ● Truck 2 ●

Instances and Results

10 Trucks, 10-**15**-**20**-25 Cities, **150**-200 Loads, 20P ($RH = 1$, $H = 4$) availability linked to city sizes, Subtree (30 scenarios)

Info		**LB**	**EVS**			**UB**
Inst./Alg.	O^*_T	O^*_{RH}	*EVS*	*Cs*	*ST*	O^*_H
5-15-25 A	222.0	0	73.6	**80.0**	79.2	100
6-15-25 A	156.1	0	78.6	**90.8**	89.7	100
7-15-25 A	171.0	0	57.2	68.0	**70.7**	100
8-15-25 A	187.3	0	**54.3**	13.8	53.4	100
5-15-25 B	153.1	0	57.7	61.2	**81.6**	100
6-15-25 B	165.7	0	55.8	42.8	**60.3**	100
7-15-25 B	194.7	0	56.5	60.4	**61.0**	100
8-15-25 B	201.4	0	86.7	60.8	**100.0**	100
5-15-25 C	192.4	0	64.1	53.8	**78.8**	100
6-15-25 C	125.9	0	62.7	78.3	**88.0**	100
7-15-25 C	179.2	0	63.9	49.6	**70.4**	100
8-15-25 C	192.0	0	47.0	20.0	**63.5**	100
5-20-25 A	195.1	0	63.9	45.2	**65.9**	100
6-20-25 A	153.8	0	52.1	54.4	**74.3**	100
7-20-25 A	253.9	0	38.6	32.1	**44.5**	100
8-20-25 A	225.7	0	7.3	-36.5	**21.9**	100
5-20-25 B	141.9	0	62.9	33.2	**68.4**	100
6-20-25 B	147.4	0	62.7	53.4	**74.2**	100
7-20-25 B	176.7	0	52.1	52.7	**66.1**	100
8-20-25 B	165.1	0	49.8	25.6	**54.2**	100
5-20-25 C	171.7	0	51.4	61.2	**67.7**	100
6-20-25 C	215.3	0	39.1	23.6	**56.1**	100
7-20-25 C	142.9	0	53.6	54.0	**61.3**	100
8-20-25 C	150.3	0	67.3	41.7	**71.3**	100
Average	**178.4**	0	**56.6**	46.7	**67.6**	100

Results analysis

Neither the graphs, nor the laws influence the results

The VPI, VTI are high on average 110.8%, 78.4%

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Results analysis

ST best values (average 2/3 of the gap) except twice for *Cs* and once for *EVS*. *ST* never under-performs

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Results analysis

EVS performs "well", only 11% behind *ST*

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Robustness analysis

Robustness : forecast availabilities based on a probability p in algorithm ST^p compared with real availabilities p'

Table: Robustness of distribution law parameter

Reality/**Forecast**	**EVS**	Low	Medium	High
Alg.	EVS^{50}	ST^{30}	ST^{50}	ST^{70}
Reality Low 20%	23.8	**55.0**	48.1	20.1
Reality High 80%	60.4	67.0	84.9	**87.6**
Alg.	EVS^{50}	ST^{20}	ST^{50}	ST^{80}
Reality Medium 50%	36.4	31.9	**55.1**	30.2

Aim : to be independent from distribution law

Conclusions

1. Importance of stochastic multi-period models
2. VPI, VMPM, VSS are relevant information values
3. ST is the best algo and others under-perform
4. ST^{50} (calibrated with a 50% availability) is robust
5. ST solvable by a LP solver
6. e.g Independent of graph shape, size or distribution laws

Perspectives :

1. Repositioning strategy
2. Investigate the VTI
3. Compare with ADP