

# ORBEL 28

## ”Multi-period vehicle assignment with stochastic load availability”

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# "Multi-period vehicle assignment with stochastic load availability"

## THE PROBLEM

### Vehicle assignment

To maximize profit : select loads to be transported by trucks (FTL-PDP) References : W.B Powell

### Multi-period

Confirmed and projected loads provided over some periods  
Repetitive decision process period per period over an horizon

### Stochastic load availability

Projected loads realize or vanish

Outlines

Multi-period

Vehicle  
Assignment

An example

Stochastic

Bounds

Algorithms

Simulation

Instances and  
results

Robustness

Conclusions

# Outlines

- ▶ Multi-period information and decision framework
- ▶ The Deterministic Vehicle Assignment Problem
- ▶ An example
- ▶ The Stochastic version
- ▶ Bounds : a-priori and a-posteriori information
- ▶ Algorithms : single or multiple scenarios approaches
- ▶ Simulation
- ▶ Instances and Results
- ▶ Robustness Analysis
- ▶ Conclusions

# Multi-period : Rolling horizon

**Decision** : in  $t$  and  $t = 1, 2, \dots, T - H \Rightarrow$  **Policy**



**Parts** : **decision, deterministic, stochastic**

**Case study** ( $Period = day$ ) :

1. Rolling horizon  $H = 4P$
2. **Deterministic**  $RH = 1P$ , **Stochastic**  $3P$

**Dynamism of the system** :

1. Decision and actions in  $t$  (info out)
2. Roll-over 1 period, updates (info in)  $t + 1 \rightarrow t'$ 
  - 2.1 stochastic gets **deterministic**  $t + RH + 1 \rightarrow t' + RH$
  - 2.2 new **stochastic** info in  $t + H + 1 \rightarrow t' + H$
3. Go to 1 with  $t \rightarrow t + 1 = t'$

# Deterministic Vehicle Assignment Problem

- Set of **Cities**  $C_1, \dots, C_N$  and transportation times  $TT_{(C_1, C_2)}$
- Set of **Periods**  $1, \dots, T$
- Set of **Loads**  $j \in J$  ( $DepC_j, ArrC_j, DepP_j, ArrP_j, Gain_j$ )
- Set of **Trucks**  $i \in I$  ( $LocC_i, Un/Loaded_i$  value 0 or  $j$ )

**Actions** : Carry Load $_j$ , Wait in Loc $C_i$ , Unladen to Dep $C_j$

**Objective** : profitable paths i.e. maximize (Gains-Costs)

subject to :

Max 1 Load per Truck, max 1 Truck per Load

Flow conservation constraints

**Network flow structure** : Polynomially Solvable

**Remarks** :

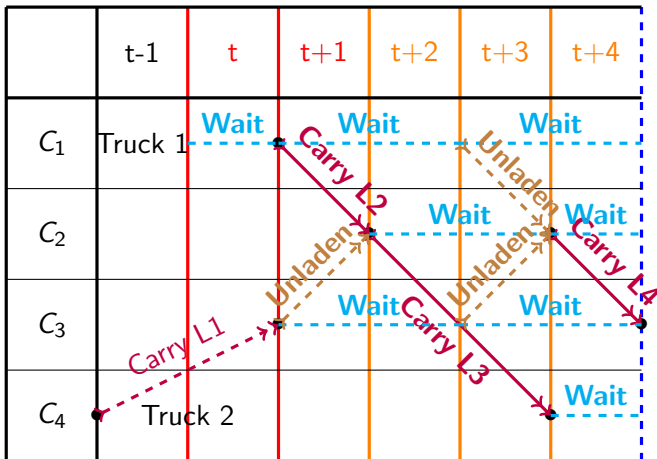
- Full-Truck-Load (FTL), no preemption
- Unloading at the end of  $t = ArrP_j$  if  $un/loaded_i = j$
- $ArrP_j = DepP_j + TT_{(DepC_j, ArrC_j)}$

# Deterministic Vehicle Assignment Problem

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## Decisions for a truck $i$

- ▶ **Carry  $L_j$**  if  $LocC_i = DepC_j$  (Gain)
- ▶ **Wait** in  $LocC_i$  (Cost)
- ▶ Move **Unladen** from  $LocC_i$  to  $DepC_j$  (Cost)



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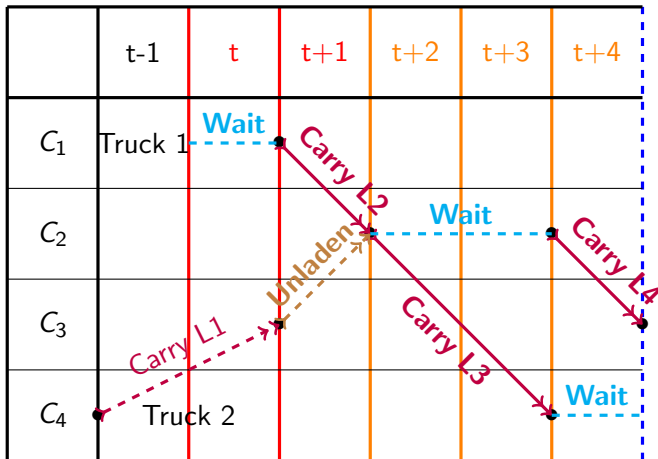
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# Deterministic Vehicle Assignment Problem

An optimal solution for a period  $t$



**Multi-period policy** : cumulated value of actions in  $t$

# Stochastic Vehicle Assignment Problem

Stochastic framework : **Stochastic Load Availability**

If  $DepP_j \in t + RH + 1, \dots, t + H, (*)$

the stochastic availability of load  $j$  is represented by a discrete distribution law :

$$P(q_j = x) = \begin{cases} p_j & \text{if } x = 1 \\ 1 - p_j & \text{if } x = 0 \end{cases} \quad (1)$$

Projection  $q_j$  materializes (1) or not (0)

when  $t + RH + 1 \rightarrow t' + RH$

**Scenario** : specific outcome of  $q_j \forall j \in J$  if (\*).

**Simulation** : Stochastic becomes Deterministic

1 scenario technique

Multiple scenarios : Separately or Simultaneously



# Specific Scenarios $\Rightarrow$ Bounds

Optimal policy for the stochastic problem :  $E^*$

## Bounds from deterministic scenarios :

1. Myopic or a-priori policy over  $RH$  :  $O_{RH}^*$
2. Oracle or a-posteriori policy over  $H$  :  $O_H^*$
3. Oracle or a-posteriori solution over  $T$  :  $O_T^*$

Expected Value Scenario  $\Rightarrow$  Expected Value 'Solution'  $EVS$

Maximization :  $O_T^* \geq O_H^* \geq E^* \geq EVS \geq O_{RH}^*$

**VPI** : Value of the Perfect Information  $O_T^* - E^* \geq 0$

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## A picture : maximization

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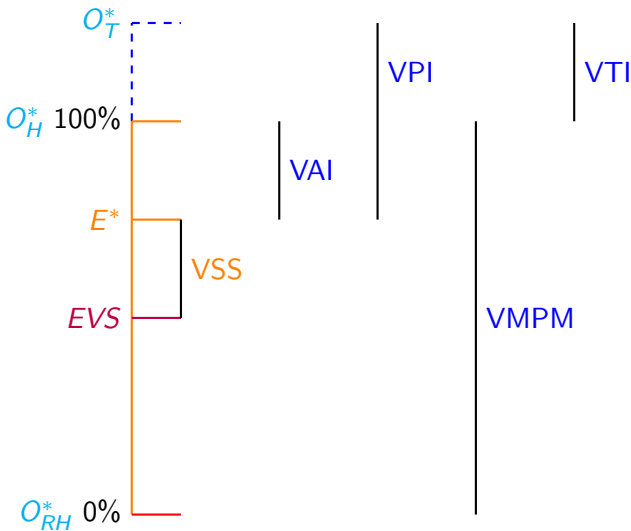
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**Problem :** Found  $E^*$  the optimal policy

# Algorithms : single or multiple scenarios approaches

Approximations of  $E^*$

**Single scenarios** (Mean, Modal, "Optimist", Dedicated)

**Multiple Scenarios Approaches (MSA)**

▶ **Consensus (Cs) :**

1. Solve  $N$  scenarios
2. Create a new solution with frequent decisions

▶ **Restricted Expectation (RE) :** Solve scenarios  $i, j$  and cross-evaluate action  $i$  over scenarios  $j$

1. Insert actions of solution  $i$  in scenario  $j$
2. Scenarios  $i \neq j$  ( $i, j \in N$ )  $\Rightarrow$  Solutions ( $i$  in  $j$ )
3. Cumulated value of Solutions ( $i$  in  $j$ )
4. Select the best action  $i$

▶ **Subtree :** Solve  $ST$  scenarios and add non-anticipativity constraints (Linear Relaxation = Optimal in practice)

**Statistical validation :**

How to compare Policy 1 with Policy 2 values ? ( $\mu_1, \mu_2$ )

**Outclassment** = significant statistical difference of means

**"Paired sample comparison"**

Hypothesis :  $\mu_1 \neq \mu_2, \mu_1 > \mu_2 ?$

Solve 30 scenarios by instance

Normality check, confidence level, Z-test

**Warm-up and End of horizon biases :**

Warm-up : remove  $H$  periods

End of horizon :  $T$  long, unit per period

## Instances and Results

10 Trucks, 10-15-20-25 Cities, **150-200** Loads, 20P ( $RH = 1$ ,  $H = 4$ ) stochasticity linked to city sizes, Subtree (30 scenarios)

Info		LB	EVS			UB
Inst./Alg.	$O_T^*$	$O_{RH}^*$	EVS	$C_s$	$ST_{30}$	$O_H^*$
5-15-25 A	222.0	0	73.6	<b>80.0</b>	79.2	100
6-15-25 A	156.1	0	78.6	<b>90.8</b>	89.7	100
7-15-25 A	171.0	0	57.2	68.0	<b>70.7</b>	100
8-15-25 A	187.3	0	<b>54.3</b>	13.8	53.4	100
5-15-25 B	153.1	0	57.7	61.2	<b>81.6</b>	100
6-15-25 B	165.7	0	55.8	42.8	<b>60.3</b>	100
7-15-25 B	194.7	0	56.5	60.4	<b>61.0</b>	100
8-15-25 B	201.4	0	86.7	60.8	<b>100.0</b>	100
5-15-25 C	192.4	0	64.1	53.8	<b>78.8</b>	100
6-15-25 C	125.9	0	62.7	78.3	<b>88.0</b>	100
7-15-25 C	179.2	0	63.9	49.6	<b>70.4</b>	100
8-15-25 C	192.0	0	47.0	20.0	<b>63.5</b>	100
5-20-25 A	195.1	0	63.9	45.2	<b>65.9</b>	100
6-20-25 A	153.8	0	52.1	54.4	<b>74.3</b>	100
7-20-25 A	253.9	0	38.6	32.1	<b>44.5</b>	100
8-20-25 A	225.7	0	7.3	-36.5	<b>21.9</b>	100
5-20-25 B	141.9	0	62.9	33.2	<b>68.4</b>	100
6-20-25 B	147.4	0	62.7	53.4	<b>74.2</b>	100
7-20-25 B	176.7	0	52.1	52.7	<b>66.1</b>	100
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8-20-25 C	150.3	0	67.3	41.7	<b>71.3</b>	100
<b>Average</b>	<b>178.4</b>	<b>0</b>	<b>56.6</b>	46.7	<b>67.6</b>	<b>100</b>

# Results analysis

Neither the graphs, nor the laws influence the results  
 The **VPI**, **VTI** are high on average 110.8%, 78.4%

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## Results analysis

$ST_{30}$  best values (average 2/3 of the gap) **except twice** for Cs and **once** for *EVS*.  $ST_{30}$  never under-performs

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## Results analysis

*EVS* performs "well", only 11% behind  $ST_{30}$

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# Robustness analysis

**Robustness** : forecast availabilities based on a probability  $p$  in algorithm  $ST^p$  compared with real availabilities  $p'$

Table: Robustness of distribution law parameter

Forecast	EVS	Low	Medium	High
Alg.	$EVS_{50}$	$ST^{30}$	$ST^{50}$	$ST^{70}$
Reality Low 20%	23.8	55.0	48.1	20.1
Reality High 80%	60.4	67.0	84.9	87.6
Alg.	$EVS_{50}$	$ST^{20}$	$ST^{50}$	$ST^{80}$
Reality Medium 50%	36.4	31.9	55.1	30.2

1. Importance of stochastic multi-period models
2. **VPI, VMPPM, VSS** are relevant information values
3. **ST** is the best algo and others under-perform
4. **ST<sup>50</sup>** (calibrated with a 50% availability) is robust
5. **ST** solvable by a LP solver
6. e.g Independent of graph shape, size or distribution laws

## Perspectives :

1. Repositioning strategy
2. Investigate the **VTI**
3. Compare with ADP