A wavelet-based analysis of surface air temperature variability

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Hölder exponent

Definition

Let f be a signal and x_0 a real number. Then f belongs to the Hölder space $C^{\alpha}(x_0)$ if there exists a polynomial $P_{x_0,\alpha}$ of degree at most α satisfying

$$|f(x) - P_{x_0,\alpha}(x)| \sim |x - x_0|^{\alpha}$$

for all x close to x_0.

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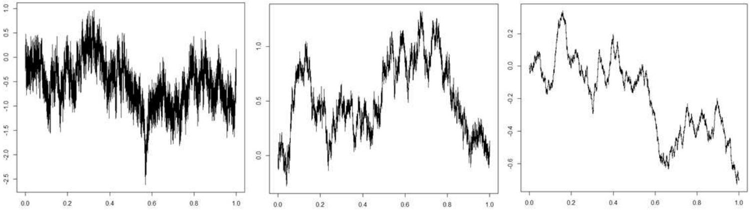
Definition

The Hölder exponent $h(x_0)$ of f at x_0 is defined as the supremum of the exponents α such that f belongs to $C^\alpha(x_0)$:

$$h(x_0) = \sup\{\alpha : f \in C^\alpha(x_0)\}.$$

Monofractality

- Hölder exponent changes from point to point : f multifractal
- Constant Hölder exponent : f monofractal, i.e. f is regularly irregular
- Example of a monofractal function : fractional Brownian motion



Fractional Brownian motions with Hölder exponents 0.2, 0.4, 0.6 almost surely.

How to check the monofractality of a signal and get its Hölder exponent ?

Wavelet leaders method (WLM)

1) Wavelet decomposition of the signal :

$$f(x) = \sum_{j,k \in \mathbb{Z}} c_{j,k} \psi(2^j x - k) = \sum_{\lambda \in \Lambda} c_\lambda \psi_\lambda$$

where ψ is a wavelet and $c_{j,k}$ is the wavelet coefficient associated to the dyadic interval λ at scale j and position k :

$$\lambda = \lambda_{j,k} = [2^{-j}k, 2^{-j}(k+1)[$$

and

$$c_{j,k} = 2^j \int_{\mathbb{R}} f(x)\psi(2^j x - k)dx.$$

2) For each λ, compute the wavelet leaders

$$d_\lambda = \sup_{\lambda' \subset \lambda} |c_{\lambda'}|$$

Wavelet leaders method (WLM)

3) Remove the null wavelet leaders and compute

$$S(q,j) = \frac{1}{2^j} \sum_{\lambda \in \Lambda_j} d_\lambda^q,$$

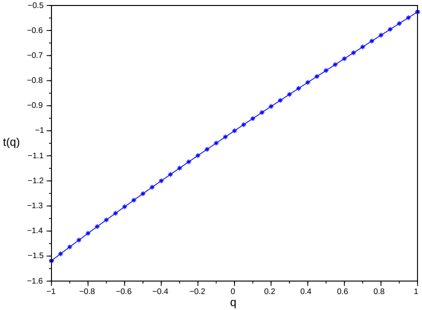
where Λ_j is the set of dyadic intervals at scale j.

4) Compute the function τ defined as

$$\tau(q) = \underline{\lim}_{j \to +\infty} \frac{\log(S(q,j))}{\log 2^{-j}},$$

which is numerically obtained through the slopes of linear regressions at small scales of $\log(S(q,j))$ seen as a function of j.

Wavelet leaders method (WLM)



τ function associated to a FBM with Hölder exponent 0.5. Linear regression gives a slope of 0.494021.

5) Remark : If τ is a straight line, then f is monofractal, in which case the Hölder exponent of f is the slope of τ.

Wavelet leaders method (WLM)

- Remark : if τ is a straight line, then f is monofractal, in which case the Hölder exponent of f is the slope of τ.
- If f is a monofractal signal with Hölder exponent H, then f belongs to the uniform Hölder space C^H, and a norm in this space is defined by

$$\|f\|_{C^H} = \sup_{j,k}\{|c_{j,k}|/2^{jH}\} := N$$

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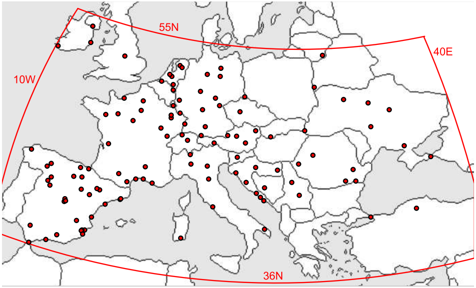
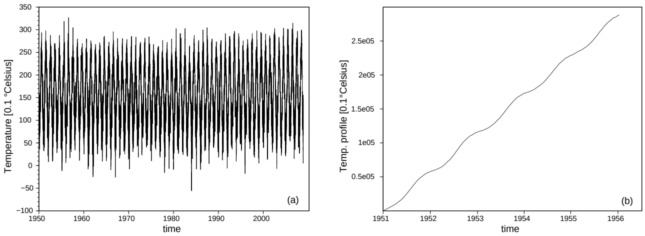
Analyzed data



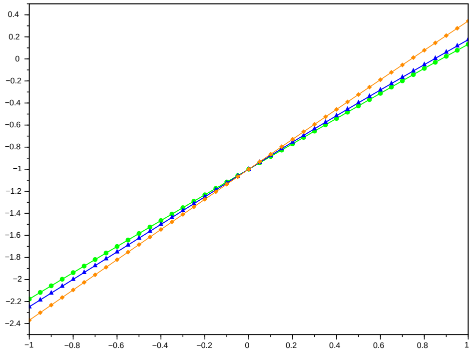
- Daily mean temperature data from 1951 to 2003
- Weather stations located below 1000 meters of altitude
- 115 stations selected
- Missing data up to 7%, less than 1% for 97 stations
- Temp. profile considered for more stable numerical results (i.e. x_n replaced by $\sum_{j=1}^{n} x_j$.)

Analyzed data



Temperature signal of Rome (1951-2003) and corresponding temperature profile (1951-1956).

Monofractal nature of the signals



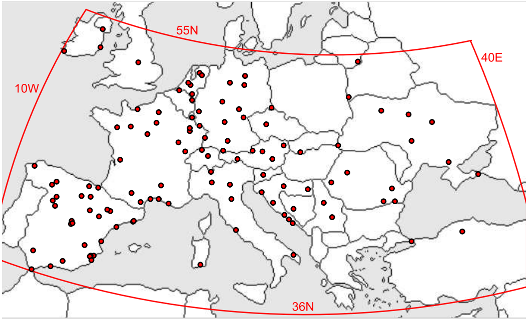
τ functions associated to Aachen (green), Шепетивка (blue) and Rome (orange), with respective slopes 1.156, 1.218, 1.358.

Hölder exponents and norms

- τ linear $\Longrightarrow$ signals are monofractal
- Mean coefficient of determination : $R^2 = 0.9975 \pm 0.0028$
- Indication of uniformity in temperature variability
- Hölder exponents ranging from 1.093 to 1.43
- Norms ranging from 8.23 to 30.45

- First idea : temperature variability is linked to standard deviation of pressure anomalies.

Relation with pressure anomalies



- Normalize the Hölder exponents between 0 and 1, as well as the standard deviation of pressure anomalies.
- Consider these values as matrices representing Europe (Mh, Mp).

Relation with pressure anomalies

- Normalize the Hölder exponents from WLM between 0 and 1, as well as the standard deviation of pressure anomalies.
- Consider these values as matrices representing Europe (Mh, Mp).
- Compute the Frobenius distance between these matrices :

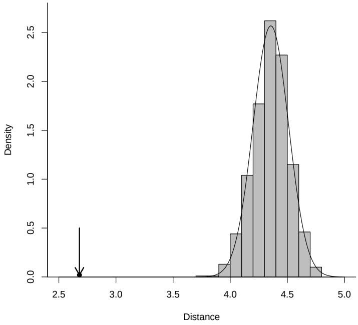
$$d(M,N) = \sqrt{\sum_{i,j}(M_{i,j} - N_{i,j})^2}.$$

We get $d(Mh, Mp) = 2.68$.

- Is this distance significant ? Confirmation that *Mh* and *Mp* are correlated ?

Relation with pressure anomalies

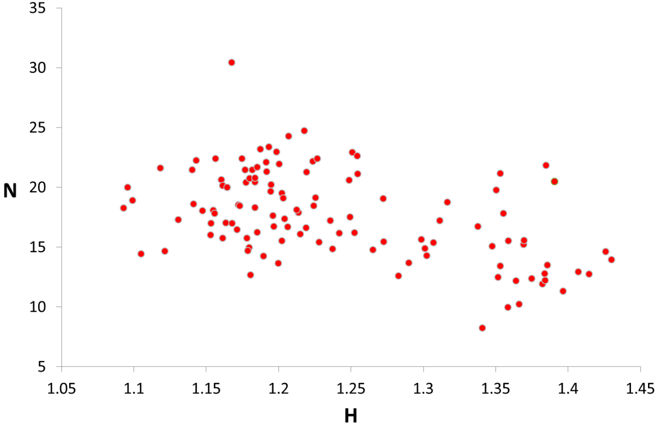
Mh shuffled 1000 times and distance with *Mp* measured.



Indication of a correlation between *Mh* and *Mp*. Hölder exponents seem to be linked to pressure anomalies.

Second idea : Is there a link with climate types ?

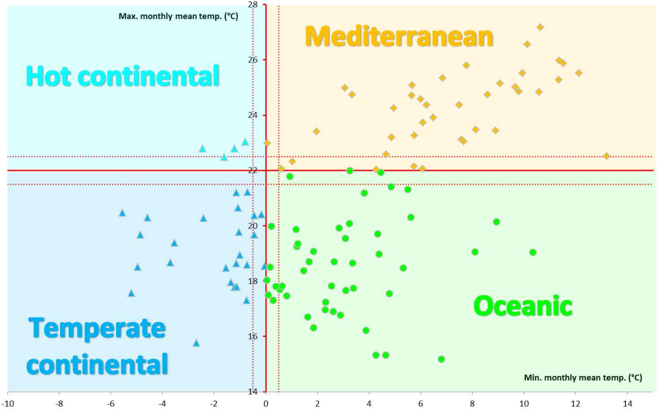
Distribution of the exponents and norms



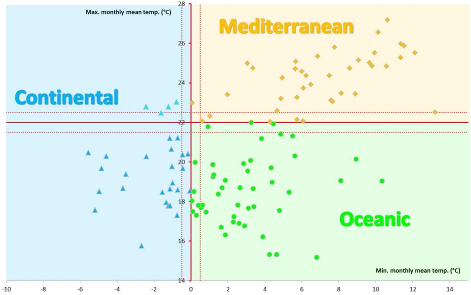
Link with climate types ?

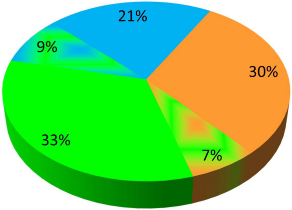
Köppen-Geiger climate classification

Classification based on maximum and minimum monthly mean temperatures (references fixed at 22°C and 0°C). Stations close to 0.5°C of another type of climate were also associated to this second category. Here, precipitations were not taken into account.

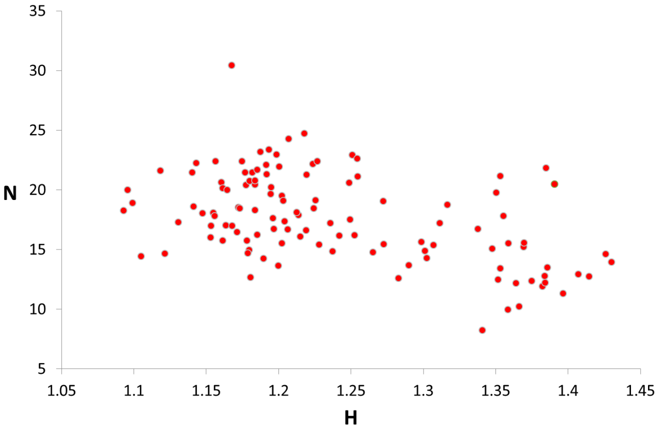


Climate distribution





Distribution of the exponents and norms



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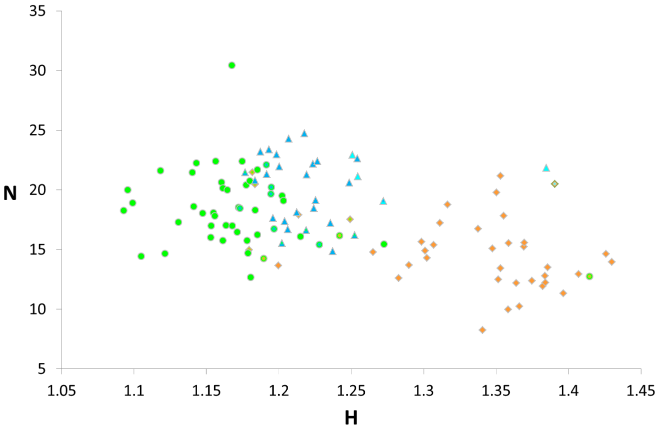

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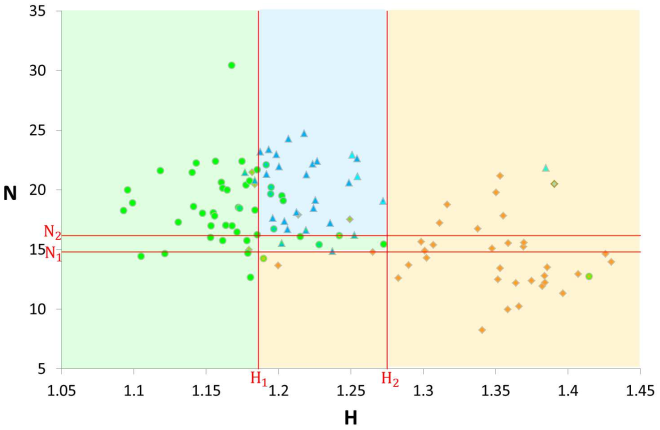


Hölder spaces-based climate classification and results

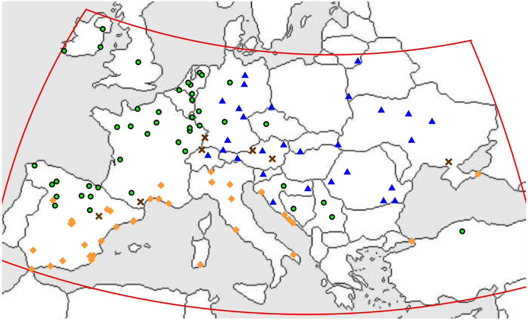
Maximum matching with Köppen-Geiger classification if

$$\begin{aligned} H_1 &= 1.186 \\ H_2 &= 1.275 \\ N_1 &= 14.81 \\ N_2 &= 16.18 \end{aligned}$$

Result : 93.9% correctly associated

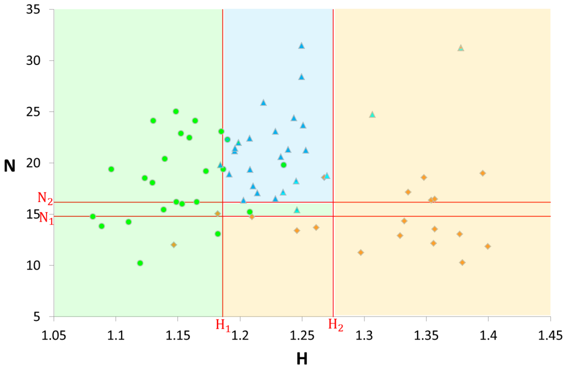
Remark : without the norm, 89.6% correctly associated.

Results on the map



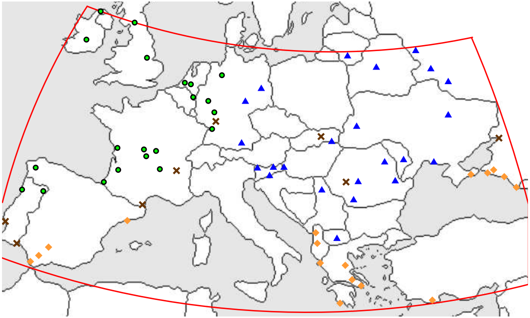
Stations marked with a brown cross are the ones whose type of climate was erroneously predicted. The others were correctly predicted ; green discs stand for Oceanic climate, blue triangles correspond to continental stations and orange orange diamonds are the Mediterranean ones.

Blind test



- 69 other stations
- 40 years of data between 1951 and 2003

Blind test

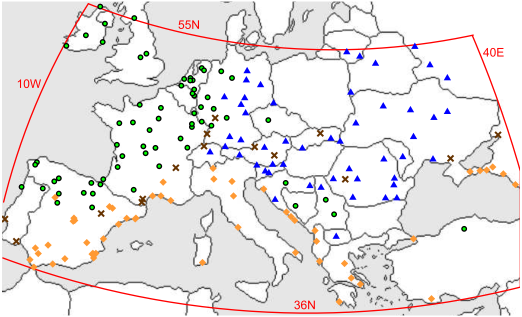


Result : 88.4% correctly associated

Remark : without the norm, 84.1% correctly associated.

All the stations

Stations of reference and blind test.



Result : 91.8% correctly associated

Discussion of the results

Results

Oceanic	$\longleftrightarrow$	Lowest Hölder exp.	$\longleftrightarrow$	Largest std of pres. ano.
Continental	$\longleftrightarrow$	Intermediate Hölder exp.	$\longleftrightarrow$	Intermediate std of pres. ano.
Mediterranean	$\longleftrightarrow$	Largest Hölder exp.	$\longleftrightarrow$	Lowest std of pres. ano.

Discussion

- On a daily basis, Oceanic climate is more irregular than the Continental weather, which is less regular than Mediterranean climate.
- Explanation could be the North Atlantic Oscillation (NAO), anticyclonic conditions in Southern Europe (linked to low std of pressure anomalies),...

Conclusions and future work

Conclusions

- WLM shows surface air temperatures signals are monofractal signals
- This monofractal nature indicates uniformity in climate variability
- Hölder exponents seem linked to standard deviations of pressure anomalies
- Hölder exponents reflect the temperature-based Köppen-Geiger climate type
- Algorithm and results confirmed through blind tests

Future work

- Checking of the validity of current climatic models
- Analysis of other climate indices (pressure, precipitation,...)
- Generalization to global temperatures

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