Structural identifiability analysis of a cardiovascular system model

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Introduction

- To be clinically relevant, mathematical models have to be made *patient-specific*.

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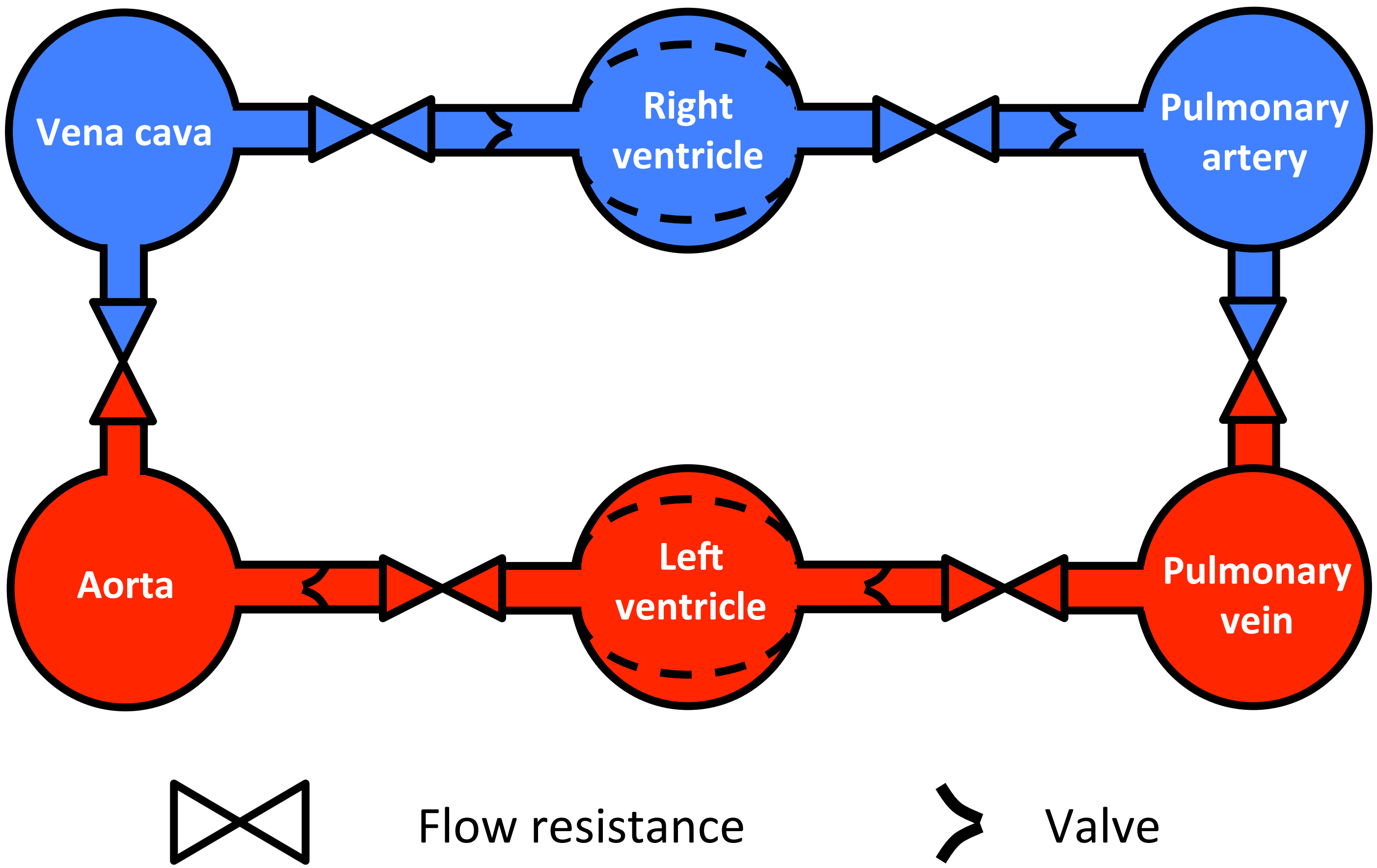
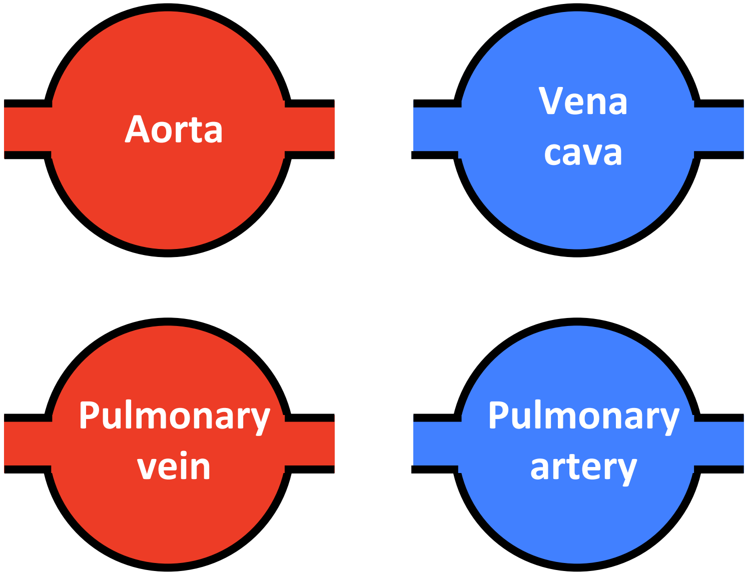
- To be clinically relevant, mathematical models have to be made *patient-specific*.
- *Can we find a measurement set which allows to identify all model parameters?*
- Structural identifiability analysis.
- Goal: investigate structural identifiability of the six-chamber CVS model from a clinically available measurement set.

Six-chamber CV system model

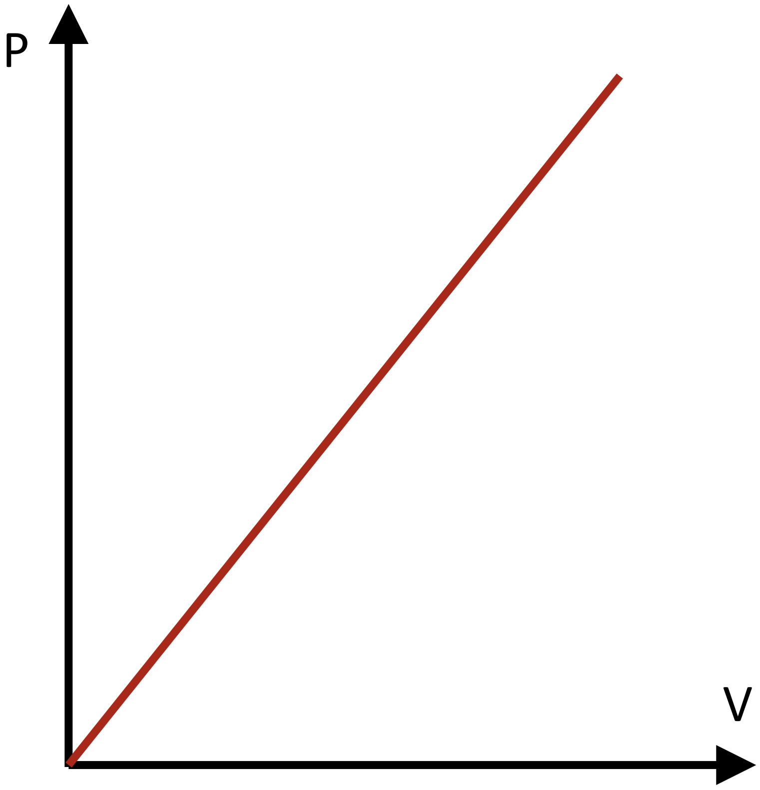


Six-chamber CV system model

Passive chambers:

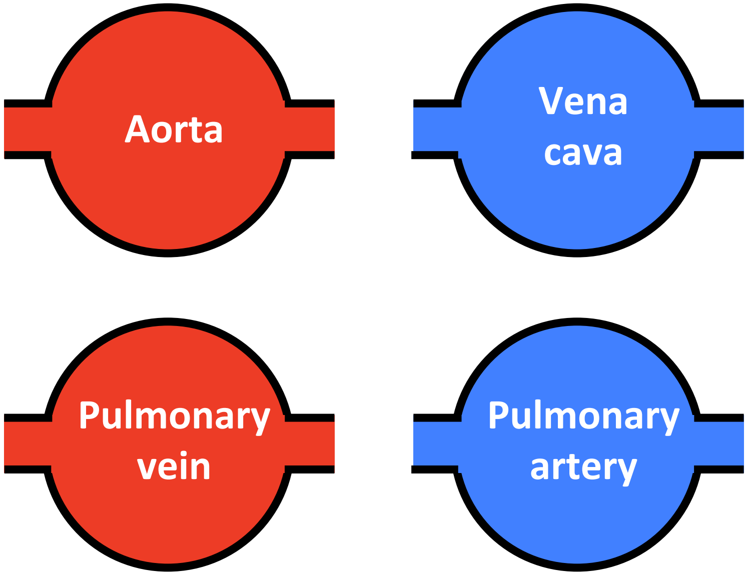


$$P = E \cdot V$$



Six-chamber CV system model

Passive chambers:



$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

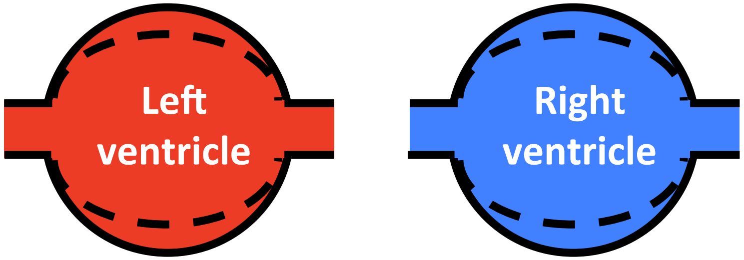
$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$P = E \cdot V$$

Six-chamber CV system model

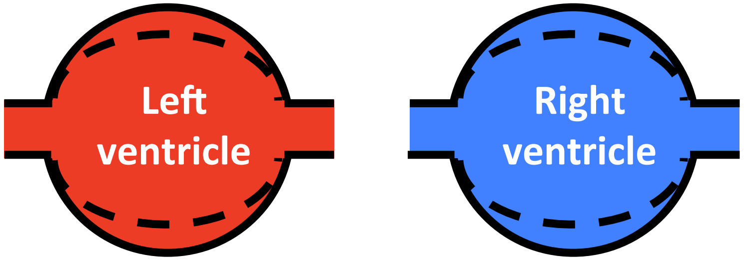
Cardiac chambers:

$$P = E \cdot e \cdot V$$

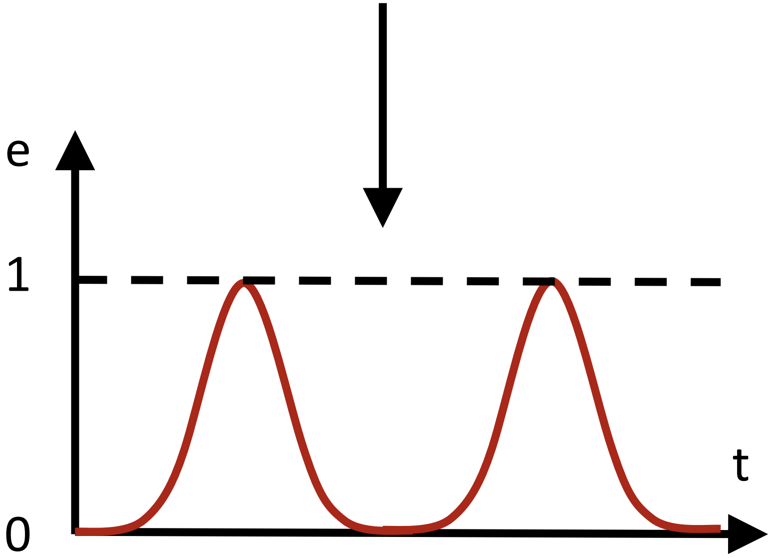


Six-chamber CV system model

Cardiac chambers:



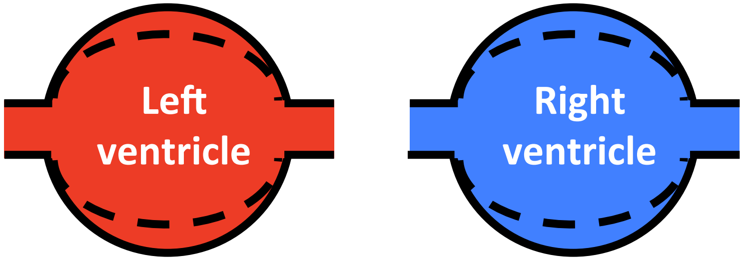
$$P = E \cdot e \cdot V$$



Driver function

Six-chamber CV system model

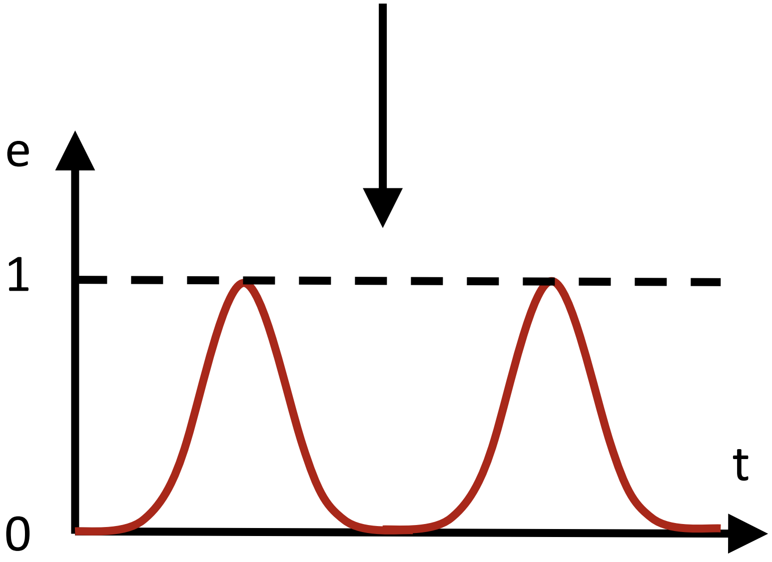
Cardiac chambers:



$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

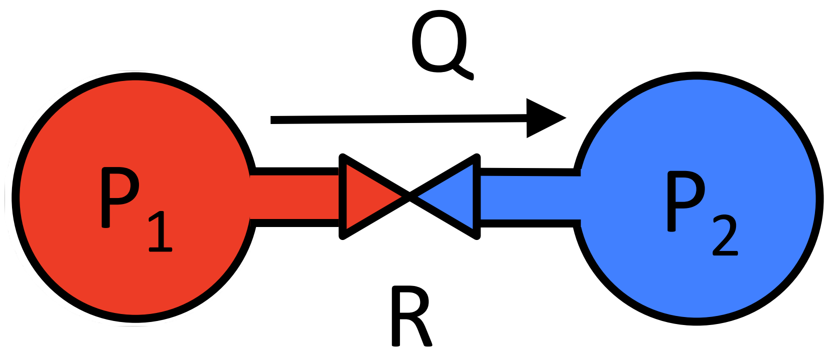
$$P = E \cdot e \cdot V$$



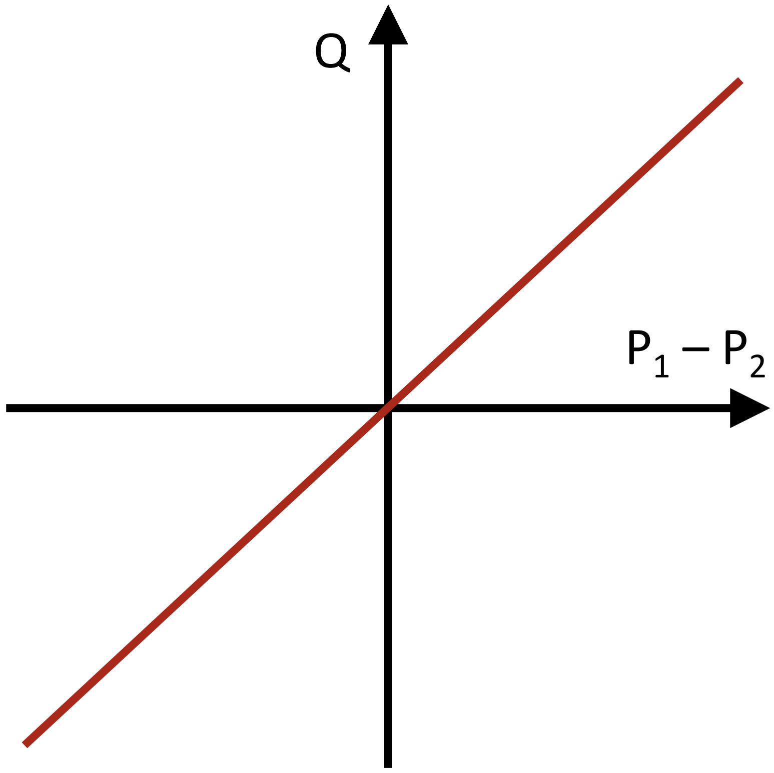
Driver function

Six-chamber CV system model

No valve:

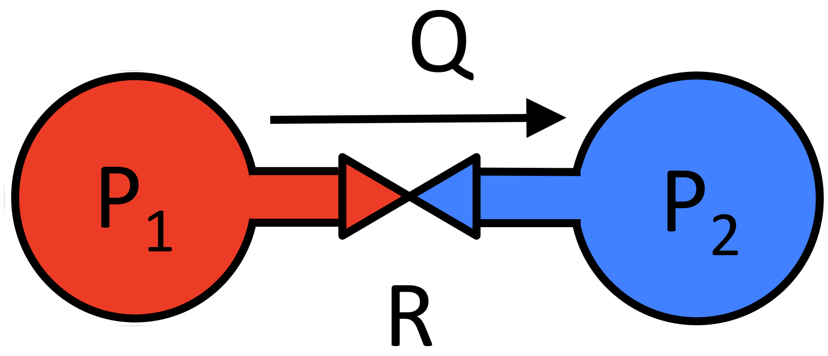


$$Q = \frac{P_1 - P_2}{R}$$



Six-chamber CV system model

No valve:



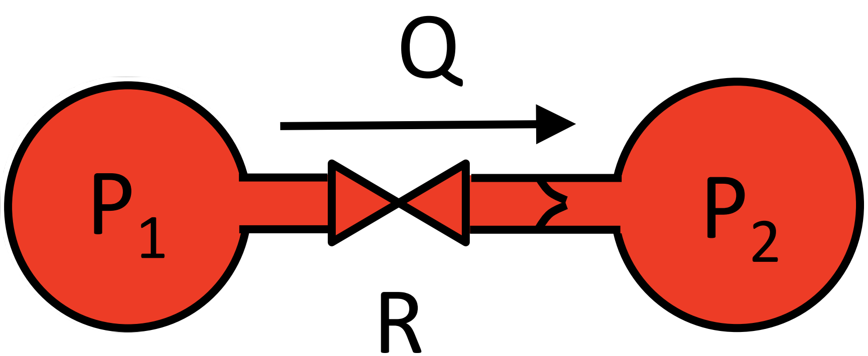
$$Q = \frac{P_1 - P_2}{R}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

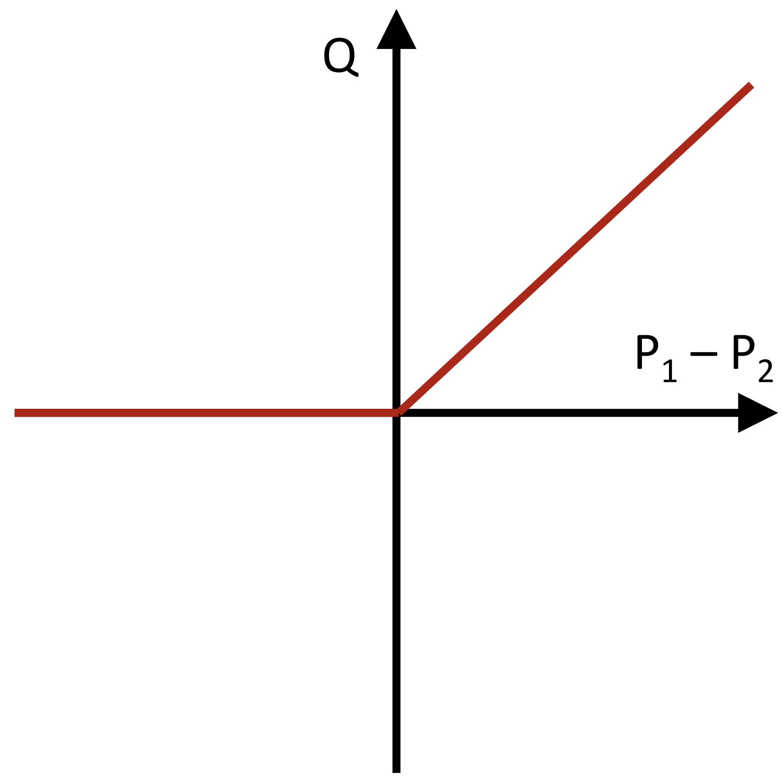
$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

Six-chamber CV system model

Valve:

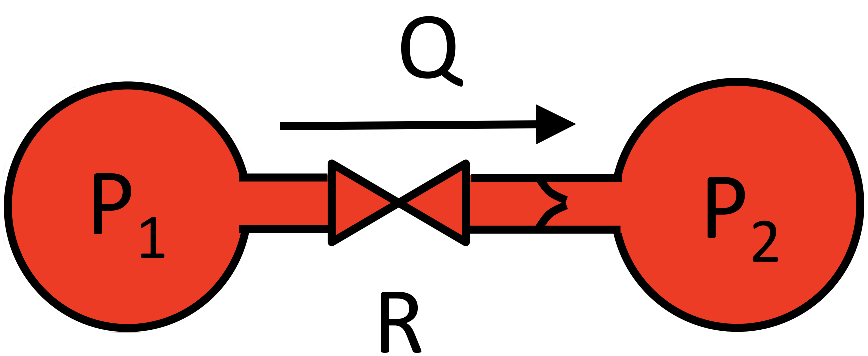


$$Q = \begin{cases} \dfrac{P_1 - P_2}{R} & \text{if } P_1 > P_2 \\ 0 & \text{otherwise} \end{cases}$$



Six-chamber CV system model

Valve:



$$Q = \begin{cases} \dfrac{P_1 - P_2}{R} & \text{if } P_1 > P_2 \\ 0 & \text{otherwise} \end{cases}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

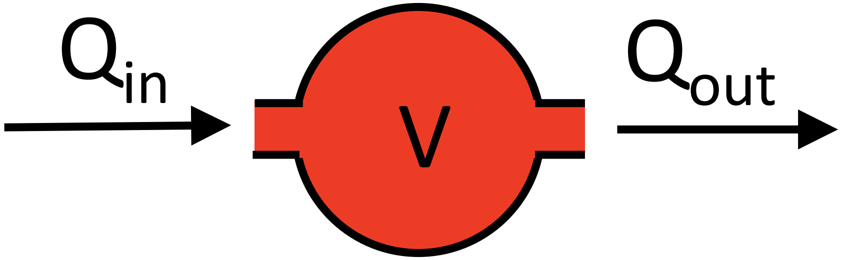
$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

Six-chamber CV system model

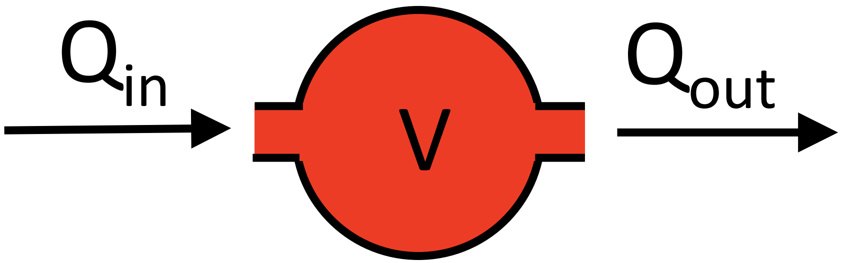
Continuity equation:



$$\dot{V} = Q_{in} - Q_{out}$$

Six-chamber CV system model

Continuity equation:



$$\dot{V} = Q_{in} - Q_{out}$$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$
$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$
$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$
$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$
$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$
$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

Six-chamber CV system model

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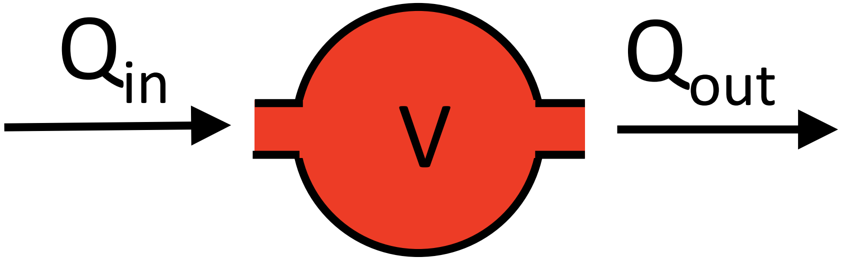

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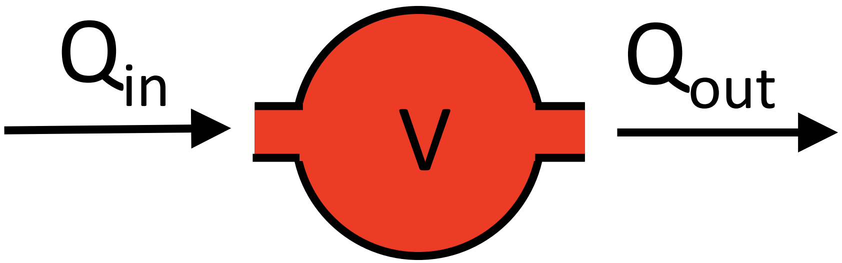
$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$
$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$
$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$\dot{V}_{lv} + \dot{V}_{ao} + \dot{V}_{vc} + \dot{V}_{rv} + \dot{V}_{pa} + \dot{V}_{pu} = 0$$

Six-chamber CV system model

Continuity equation:



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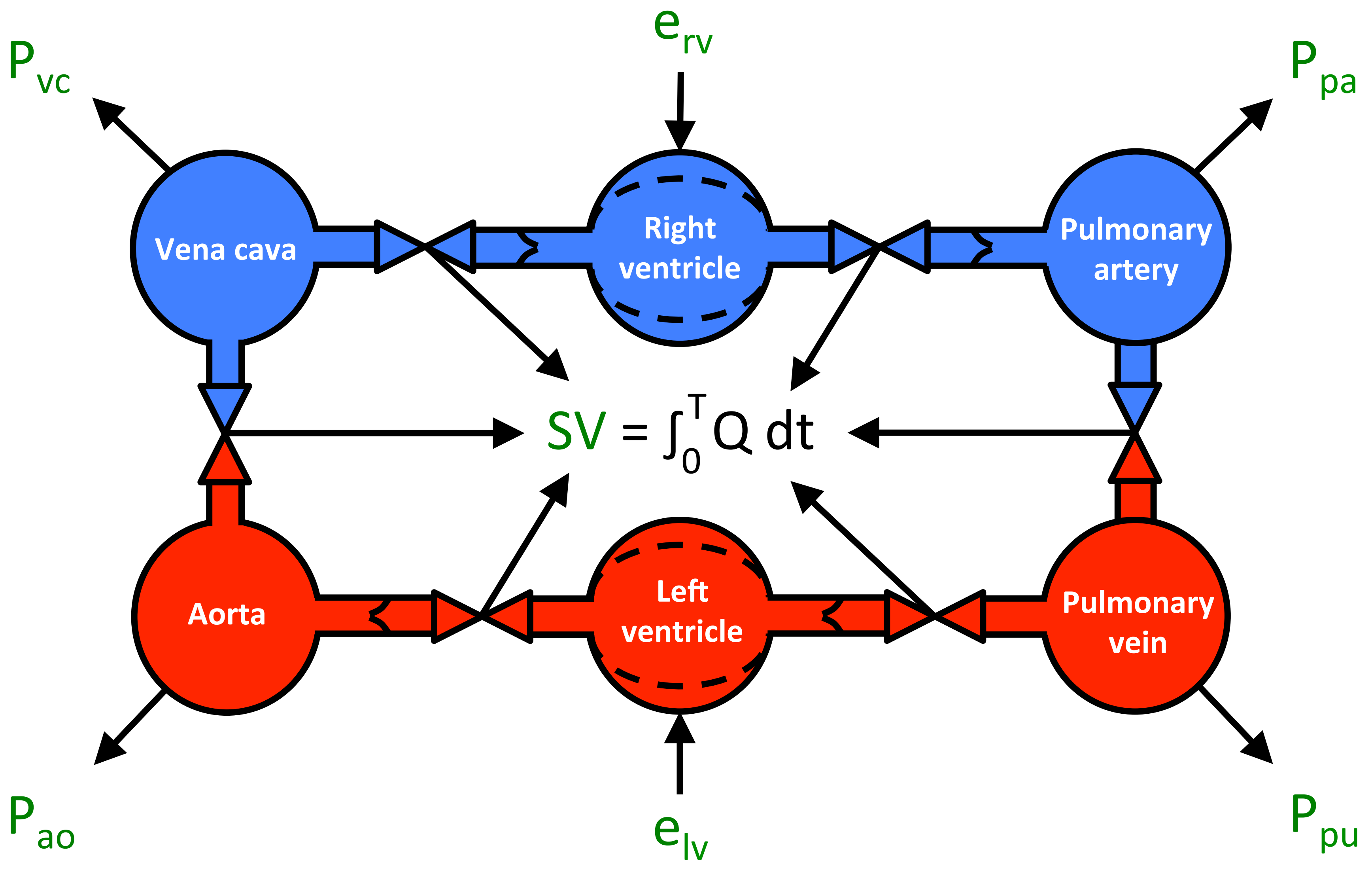
$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$
$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$\dot{V}_{lv} + \dot{V}_{ao} + \dot{V}_{vc} + \dot{V}_{rv} + \dot{V}_{pa} + \dot{V}_{pu} = 0$$

$$V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu} = SBV$$

Identifiability of the CV system model



During the whole cardiac cycle

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During the whole cardiac cycle

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$SV = \int_0^T Q \, dt$$

During the whole cardiac cycle

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$\downarrow$$

$$SV = \int_0^T Q\, dt$$

$$\int_0^T Q_{sys}\, dt = \frac{\int_0^T (P_{ao} - P_{vc})\, dt}{R_{sys}}$$

During the whole cardiac cycle

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$SV = \int_0^T Q \, dt \longrightarrow SV = \frac{\int_0^T (P_{ao} - P_{vc}) \, dt}{R_{sys}}$$

During the whole cardiac cycle

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$SV = \int_0^T Q \, dt$$

$$R_{sys} = \frac{\int_0^T (P_{ao} - P_{vc}) \, dt}{SV}$$

During the whole cardiac cycle

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During the whole cardiac cycle

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During the whole cardiac cycle

$$SV = \int_0^T Q \, dt$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During the whole cardiac cycle

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During the whole cardiac cycle

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

At the beginning of systole: $P_{lv,BS} = P_{ao,BS}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$
$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$
$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$
$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$
$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$
$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$
$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$
$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$
$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$
$$P_{ao} = E_{ao} \cdot V_{ao}$$
$$P_{vc} = E_{vc} \cdot V_{vc}$$
$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$
$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$
$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$
$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$
$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$
$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$
$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

At the beginning of systole: $P_{lv,BS} = P_{ao,BS}$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

At the beginning of systole: $P_{lv,BS} = P_{ao,BS}$

$$P_{lv,BS} = E_{lv} \cdot e_{lv,BS} \cdot V_{lv,BS}$$

At the beginning of systole: $P_{lv,BS} = P_{ao,BS}$

$$P_{ao,BS} = E_{lv} \cdot e_{lv,BS} \cdot V_{lv,BS}$$

At the beginning of systole: $P_{lv,BS} = P_{ao,BS}$

$$V_{lv,BS} = \frac{P_{ao,BS}}{E_{lv} \cdot e_{lv,BS}}$$

At the end of systole: $P_{lv,ES} = P_{ao,ES}$

$$V_{lv,BS} = \frac{P_{ao,BS}}{E_{lv} \cdot e_{lv,BS}} \qquad V_{lv,ES} = \frac{P_{ao,ES}}{E_{lv} \cdot e_{lv,ES}}$$

At the end of systole: $P_{lv,ES} = P_{ao,ES}$

$$V_{lv,BS} = \frac{P_{ao,BS}}{E_{lv} \cdot e_{lv,BS}} \qquad V_{lv,ES} = \frac{P_{ao,ES}}{E_{lv} \cdot e_{lv,ES}}$$

$$SV = V_{lv,BS} - V_{lv,ES}$$

At the end of systole: $P_{lv,ES} = P_{ao,ES}$

$$V_{lv,BS} = \frac{P_{ao,BS}}{E_{lv} \cdot e_{lv,BS}} \qquad V_{lv,ES} = \frac{P_{ao,ES}}{E_{lv} \cdot e_{lv,ES}}$$

$$SV = \frac{P_{ao,ES}}{E_{lv} \cdot e_{lv,ES}} - \frac{P_{ao,BS}}{E_{lv} \cdot e_{lv,BS}}$$

At the end of systole: $P_{lv,ES} = P_{ao,ES}$

$$V_{lv,BS} = \frac{P_{ao,BS}}{E_{lv} \cdot e_{lv,BS}} \qquad V_{lv,ES} = \frac{P_{ao,ES}}{E_{lv} \cdot e_{lv,ES}}$$

$$E_{lv} = \frac{P_{ao,ES}}{SV \cdot e_{lv,ES}} - \frac{P_{ao,BS}}{SV \cdot e_{lv,BS}}$$

At the end of systole: $P_{lv,ES} = P_{ao,ES}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

At the end of systole: $P_{lv,ES} = P_{ao,ES}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$
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$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$
$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$
$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$
$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$
$$P_{ao} = E_{ao} \cdot V_{ao}$$
$$P_{vc} = E_{vc} \cdot V_{vc}$$
$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$
$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$
$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$
$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$
$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$
$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$
$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

At the end of systole: $P_{rv,ES} = P_{pa,ES}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$
$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$
$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$
$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$
$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$
$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$
$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$
$$SV = \int_0^T Q\,dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$
$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$
$$P_{ao} = E_{ao} \cdot V_{ao}$$
$$P_{vc} = E_{vc} \cdot V_{vc}$$
$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$
$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$
$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$
$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$
$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$
$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$
$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac ejection: $P_{rv} > P_{vc}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac ejection: $P_{rv} > P_{vc}$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

During cardiac ejection: $P_{rv} > P_{vc}$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = 0$$

During cardiac ejection: $P_{rv} > P_{vc}$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$\dot{P}_{vc} = E_{vc} \cdot \dot{V}_{vc}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = 0$$

During cardiac ejection: $P_{rv} > P_{vc}$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = 0$$

$$\dot{P}_{vc} = E_{vc} \cdot (Q_{sys} - Q_{tc})$$

During cardiac ejection: $P_{rv} > P_{vc}$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = 0$$

$$\dot{P}_{vc} = E_{vc} \cdot \frac{P_{ao} - P_{vc}}{R_{sys}}$$

During cardiac ejection: $P_{rv} > P_{vc}$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = 0$$

$$\frac{P_{vc} \cdot R_{sys}}{P_{ao} - P_{vc}} = E_{vc}$$

During cardiac ejection: $P_{rv} > P_{vc}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac ejection: $P_{rv} > P_{vc}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$
$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$
$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$
$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$
$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$
$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$
$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$
$$SV = \int_0^T Q\,dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$
$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$
$$P_{ao} = E_{ao} \cdot V_{ao}$$
$$P_{vc} = E_{vc} \cdot V_{vc}$$
$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$
$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$
$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$
$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$
$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$
$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$
$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac ejection: $P_{lv} > P_{pu}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q\,dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac filling: $P_{ao} > P_{lv}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac filling: $P_{ao} > P_{lv}$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

During cardiac filling: $P_{ao} > P_{lv}$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$Q_{av} = 0$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

During cardiac filling: $P_{ao} > P_{lv}$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$Q_{av} = 0$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$\dot{P}_{ao} = E_{ao} \cdot \dot{V}_{ao}$$

During cardiac filling: $P_{ao} > P_{lv}$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$Q_{av} = 0$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$\dot{P}_{ao} = E_{ao} \cdot (Q_{av} - Q_{sys})$$

During cardiac filling: $P_{ao} > P_{lv}$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$Q_{av} = 0$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$\dot{P}_{ao} = -E_{ao} \cdot \frac{P_{ao} - P_{vc}}{R_{sys}}$$

During cardiac filling: $P_{ao} > P_{lv}$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$Q_{av} = 0$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$-\frac{\dot{P}_{ao} . R_{sys}}{P_{ao} - P_{vc}} = E_{ao}$$

During cardiac filling: $P_{ao} > P_{lv}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac filling: $P_{ao} > P_{lv}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac filling: $P_{vc} > P_{rv}$ and $P_{rv} < P_{pa}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac filling: $P_{vc} > P_{rv}$ and $P_{rv} < P_{pa}$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

During cardiac filling: $P_{vc} > P_{rv}$ and $P_{rv} < P_{pa}$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{P_{vc} - P_{rv}}{R_{tc}}$$

$$Q_{pv} = 0$$

During cardiac filling: $P_{vc} > P_{rv}$ and $P_{rv} < P_{pa}$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{P_{vc} - P_{rv}}{R_{tc}}$$

$$\dot{P}_{vc} = E_{vc} \cdot \dot{V}_{vc}$$

$$Q_{pv} = 0$$

During cardiac filling: $P_{vc} > P_{rv}$ and $P_{rv} < P_{pa}$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{P_{vc} - P_{rv}}{R_{tc}}$$

$$\dot{P}_{vc} = E_{vc} \cdot (Q_{sys} - Q_{tc})$$

$$Q_{pv} = 0$$

During cardiac filling: $P_{vc} > P_{rv}$ and $P_{rv} < P_{pa}$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{P_{vc} - P_{rv}}{R_{tc}}$$

$$\dot{P}_{vc} = E_{vc} \cdot \left(\frac{P_{ao} - P_{vc}}{R_{sys}} - \frac{P_{vc} - P_{rv}}{R_{tc}} \right)$$

$$Q_{pv} = 0$$

During cardiac filling: $P_{vc} > P_{rv}$ and $P_{rv} < P_{pa}$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{P_{vc} - P_{rv}}{R_{tc}}$$

$$\dot{P}_{vc} = E_{vc} \cdot \left(\frac{P_{ao} - P_{vc}}{R_{sys}} - \frac{P_{vc} - P_{rv}}{R_{tc}} \right)$$

$$Q_{pv} = 0$$

$$\ddot{P}_{vc} = E_{vc} \cdot \left(\frac{\dot{P}_{ao} - \dot{P}_{vc}}{R_{sys}} - \frac{\dot{P}_{vc} - \dot{P}_{rv}}{R_{tc}} \right)$$

During cardiac filling: $P_{vc} > P_{rv}$ and $P_{rv} < P_{pa}$

$$\dot{P}_{rv} = E_{rv} \cdot \dot{e}_{rv} \cdot V_{rv} + E_{rv} \cdot e_{rv} \cdot \dot{V}_{rv}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{P_{vc} - P_{rv}}{R_{tc}}$$

$$Q_{pv} = 0$$

$$\dot{P}_{vc} = E_{vc} \cdot \left(\frac{P_{ao} - P_{vc}}{R_{sys}} - \frac{P_{vc} - P_{rv}}{R_{tc}} \right)$$

$$\ddot{P}_{vc} = E_{vc} \cdot \left(\frac{\dot{P}_{ao} - \dot{P}_{vc}}{R_{sys}} - \frac{\dot{P}_{vc} - \dot{P}_{rv}}{R_{tc}} \right)$$

During cardiac filling: $P_{vc} > P_{rv}$ and $P_{rv} < P_{pa}$

$$\dot{P}_{rv} = E_{rv} \cdot \dot{e}_{rv} \cdot V_{rv} + E_{rv} \cdot e_{rv} \cdot \dot{V}_{rv}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = \frac{P_{vc} - P_{rv}}{R_{tc}}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{P_{vc} - P_{rv}}{R_{tc}}$$

$$Q_{pv} = 0$$

$$\dot{P}_{vc} = E_{vc} \cdot \left(\frac{P_{ao} - P_{vc}}{R_{sys}} - \frac{P_{vc} - P_{rv}}{R_{tc}} \right)$$

$$\ddot{P}_{vc} = E_{vc} \cdot \left(\frac{\dot{P}_{ao} - \dot{P}_{vc}}{R_{sys}} - \frac{\dot{P}_{vc} - \dot{P}_{rv}}{R_{tc}} \right)$$

During cardiac filling: $P_{vc} > P_{rv}$ and $P_{rv} < P_{pa}$

$$\dot{P}_{rv} = E_{rv} \cdot \dot{e}_{rv} \cdot V_{rv} + E_{rv} \cdot e_{rv} \cdot \dot{V}_{rv}$$

$$\dot{V}_{rv} = \frac{P_{vc} - P_{rv}}{R_{tc}}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$\dot{P}_{vc} = E_{vc} \cdot \left(\frac{P_{ao} - P_{vc}}{R_{sys}} - \frac{P_{vc} - P_{rv}}{R_{tc}} \right)$$

$$\ddot{P}_{vc} = E_{vc} \cdot \left(\frac{\dot{P}_{ao} - \dot{P}_{vc}}{R_{sys}} - \frac{\dot{P}_{vc} - \dot{P}_{rv}}{R_{tc}} \right)$$

During cardiac filling: $P_{vc} > P_{rv}$ and $P_{rv} < P_{pa}$

$$R_{tc} = \frac{\dot{P}_{vc} \cdot e_{rv} \cdot E_{vc} \cdot R_{sys} - \dot{e}_{rv} \cdot E_{vc} \cdot P_{vc} \cdot R_{sys} + e_{rv}^2 \cdot E_{rv} \cdot (E_{vc} \cdot (P_{vc} - P_{ao}) + \ldots}{\dot{P}_{ao} \cdot e_{rv} \cdot E_{vc} + \dot{e}_{rv} \cdot E_{vc} \cdot (P_{vc} - P_{ao}) - \ddot{P}_{vc} \cdot e_{rv} \cdot R_{sys} + \dot{P}_{vc} \cdot (\dot{e}_{rv} \cdot R_{sys} \ldots}$$

$$P_{rv} = \frac{e_{rv} \cdot (e_{rv} \cdot E_{rv} \cdot (E_{vc} \cdot (P_{vc} - P_{ao}) + \dot{P}_{vc} \cdot R_{sys})^2 + E_{vc} \cdot R_{sys} \cdot (P_{ao} \cdot E_{vc} \cdot \ldots}{E_{vc} \cdot R_{sys} \cdot (\dot{P}_{ao} \cdot e_{rv} \cdot E_{vc} + \dot{e}_{rv} \cdot E_{vc} \cdot (P_{vc} - P_{ao}) - \ddot{P}_{vc} \cdot e_{rv} \cdot R_{sys} + P_{v} \ldots}$$

$$V_{rv} = \frac{e_{rv} \cdot E_{rv} \cdot (E_{vc} \cdot (P_{vc} - P_{ao}) + \dot{P}_{vc} \cdot R_{sys})^2 + E_{vc} \cdot R_{sys} \cdot (\dot{P}_{ao} \cdot E_{vc} \cdot P_{vc} \ldots}{E_{rv} \cdot E_{vc} \cdot R_{sys} \cdot (\dot{P}_{ao} \cdot e_{rv} \cdot E_{vc} + \dot{e}_{rv} \cdot E_{vc} \cdot (P_{vc} - P_{ao}) - \ddot{P}_{vc} \cdot e_{rv} \cdot R_{sys} \ldots}$$

$$\dot{P}_{rv} = \frac{(\dot{P}_{ao} - \dot{P}_{vc}) \cdot e_{rv}^2 \cdot E_{rv} \cdot E_{vc}^2 \cdot (P_{ao} - P_{vc}) + E_{vc} \cdot (\dot{P}_{vc}^2 \cdot e_{rv}^2 \cdot E_{rv} - \dot{e}_{rv} \cdot \dot{P}_{v} \ldots}{E_{vc} \cdot R_{sys} \cdot (\dot{P}_{ao} \cdot e_{rv} \cdot E_{vc} + \dot{e}_{rv} \cdot E_{vc} \cdot (P_{vc} - P_{ao}) - \ddot{P}_{vc} \cdot e_{rv} \cdot R_{sys} + \dot{P}_{v} \ldots}$$

$$\dot{V}_{rv} = \frac{P_{ao} - P_{vc}}{R_{sys}} - \frac{\dot{P}_{vc}}{E_{vc}}$$

During cardiac filling: $P_{vc} > P_{rv}$ and $P_{rv} < P_{pa}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac filling: $P_{vc} > P_{rv}$ and $P_{rv} < P_{pa}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q\, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac filling: $P_{pu} > P_{lv}$ and $P_{lv} < P_{ao}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q\,dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac ejection: $P_{pu} < P_{lv}$ and $P_{lv} > P_{ao}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac ejection: $P_{vc} < P_{rv}$ and $P_{rv} > P_{pa}$

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

During cardiac filling

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

During cardiac filling

$$P_{pu} = E_{pu} \cdot V_{pu}$$

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$$P_{pu} = E_{pu} \cdot V_{pu}$$

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During cardiac filling

$$\dot{V}_{lv} = Q_{mt} - Q_{av}$$

$$\dot{V}_{ao} = Q_{av} - Q_{sys}$$

$$\dot{V}_{vc} = Q_{sys} - Q_{tc}$$

$$\dot{V}_{rv} = Q_{tc} - Q_{pv}$$

$$\dot{V}_{pa} = Q_{pv} - Q_{pul}$$

$$\dot{V}_{pu} = Q_{pul} - Q_{mt}$$

$$SBV = V_{lv} + V_{ao} + V_{vc} + V_{rv} + V_{pa} + V_{pu}$$

$$SV = \int_0^T Q \, dt$$

$$P_{pu} = E_{pu} \cdot V_{pu}$$

$$P_{lv} = E_{lv} \cdot e_{lv} \cdot V_{lv}$$

$$P_{ao} = E_{ao} \cdot V_{ao}$$

$$P_{vc} = E_{vc} \cdot V_{vc}$$

$$P_{rv} = E_{rv} \cdot e_{rv} \cdot V_{rv}$$

$$P_{pa} = E_{pa} \cdot V_{pa}$$

$$Q_{mt} = \frac{r(P_{pu} - P_{lv})}{R_{mt}}$$

$$Q_{av} = \frac{r(P_{lv} - P_{ao})}{R_{av}}$$

$$Q_{sys} = \frac{P_{ao} - P_{vc}}{R_{sys}}$$

$$Q_{tc} = \frac{r(P_{vc} - P_{rv})}{R_{tc}}$$

$$Q_{pv} = \frac{r(P_{rv} - P_{pa})}{R_{pv}}$$

$$Q_{pul} = \frac{P_{pa} - P_{pu}}{R_{pul}}$$

Results

The 13 model parameters can be uniquely retrieved from (perfect) measurements of

- aortic pressure,
- pulmonary artery pressure,
- vena cava pressure,
- pulmonary vein pressure,
- stroke volume

and knowledge of the two driver functions.

Practical considerations

The 13 model parameters can be uniquely retrieved from (perfect) measurements of

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Conclusions

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- The model parameters computed from this dataset are theoretically unique.
- Hence, the parameter values are fully suitable to be used for diagnosis.

Thanks for your attention!