Advanced optimization methods for power systems

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Overview

Context: An increasing complexity

Integration of Renewable energies

Intermittent power: less predictable, less observable, less controllable
Best location faraway from load centers: off shore wind or dispersed in distribution grids

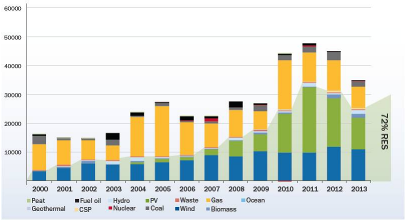


Figure: Generation capacity in Europe (MW): installed per year

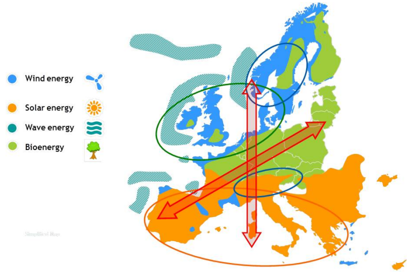


Figure: Best location of RES in Europe

Context: An increasing complexity

Low public acceptance of new infrastructures

- Not In My Back Yard (NIMBY)
- Fear of hypothetical impacts of EMF on health
- ☞ Complex and costly solutions

Aging of grid assets

- Significant part of the grids' assets is more than 50 years old
- ☞ Challenges in asset management and maintenance: Large number of assets approaching simultaneously the end of their life times.

Supra national electricity markets

- Global optimizer, Optimal utilization of assets $\Longrightarrow$ Operation closer to the grid's limits
- ☞ Electrical phenomena don't stop at administrative borders $\Longrightarrow$ Large system

Context: New "complex" solutions

Hardware solutions:

- New conductors (HTLS) for existing overhead power lines.
- Special devices: Phase Shifting Transformers, Static Var Compensators
- Long distance HVAC underground cables with reactive compensators.
- HVDC underground cables in // with AC grid controlled of AC/DC converters.
- Ultimately, HVDC grids: offshore wind farms and cheap interconnections.

Software solutions:

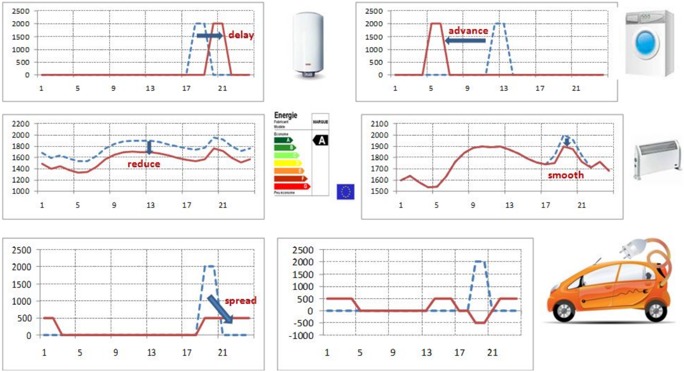
- Dynamic Ratings: Lines, Cables, Transformers.
- Advanced measurement system: PMUs, non conventional PT/CT.
- Advanced controls and protection schemes using new ICT capabilities.

Context: Demand Response or Dispersed Storage

A paradigm shift ?

- Business models and costs are still questionable.

☞ Rethinking of operating practices: loads could be not anymore purely uncontrollable stochastic variables.



Grid operators manage extremely complex decision making processes

To ensure the reliability and quality of supply at minimal cost over different time horizons:

- Long term (10-20 years): planning stage
  - Where to build new power lines? Which technology? Which capacity?
- Mid term (2-5 years):
  - Installation of control devices: substation design, var/reactive support, PSTs, replacement of conductors, SPS/RAS design.
  - Asset management and maintenance: which equipment to upgrade, to replace, to repair and when?
- Short term (monthly-weekly):
  - Outage management, must-run generators, preparation of corrective actions, definition of required margins.
- Real Time (two days ahead to real time):
  - Interaction with energy markets: definition of grid capacities;
  - Selection of substation's topology, settings of SPS/RAS, adjustment of generating units.

"Optimal" decisions over these different time horizons

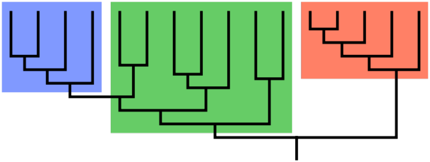
- Some are not formalized as optimization problems but based on knowledge of experts.
- Complexity increases $\Longrightarrow$ Decision support tools mandatory to help experts.
- Challenge: Ensuring consistency
  - Multistage decision making processes considering all the different time horizons.
  - Decisions at planning stage requires modeling/simulation of asset management and operation and the same between asset management and operation.
  - Approximations are required and relevant "proxies" must be found.

☞ Valuable to explicitly formulate them as optimization problems, even if they are hard to solve exactly.

☞ Significant progress recently both in computational and in mathematical respects

Taxonomy of optimization problems

- Modeling the optimization problem from a formal viewpoint.



- Modeling the physics of the power system

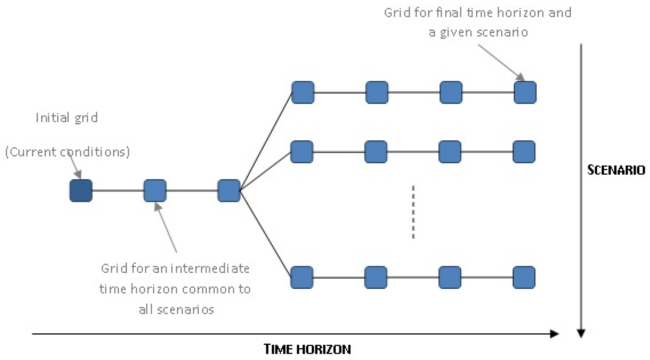
Modeling the optimization problem from a formal viewpoint

Multistage decision making problem under uncertainty.

We can divide these decisions in three classes (analogy with IT systems):

1. Decisions changing the structure of the system (developing the hardware)
2. Decisions changing policies or control/protection schemes (developing the software)
3. Decisions modifying the operating points of the system (selecting input data to run the software on the hardware)

A very complex optimization problem



Expansion Planning: Formulation

Formulation as stochastic dynamic programming problem.

Scenario-based approaches $\Longrightarrow$ some level of robustness as proposed by Rockafellar and Wets in 1991 [1].

Key questions: reliability criteria and how to implement them.

- Monetization of "Energy Not Served" or "Loss of Load"
- Expected value, no cap on the maximum risk

☞ Chance constrained or robust optimization could offer more relevant solutions

☞ Dramatic changes in case of generalization of Demand Response as proposed by Schweppe in 1978 [2]

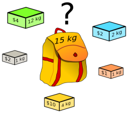
Expansion Planning: Complexity

We can identify three different dimensions: spatial, temporal and stochastic:

- Spatial: from large transmission grid to active distribution grids
- Temporal: from decades to milliseconds
- Stochastic: uncontrollable loads and renewable energies, contingencies, failures ....

☞ Difficult to deal simultaneously with these 3 dimensions

Selection of relevant technological options $\Longrightarrow$ combinatorial optimization: a "knapsack" problem.



Modeling of uncertainties

- Key factor to find optimal decisions.
- Neglecting spatial, temporal correlations $\Longrightarrow$ Very sub optimal decisions.

☞ Probabilistic methods and risk based approaches:

When probabilistic properties partially known $\Longrightarrow$ generalized semi-infinite programming:

$$\min_{x \in X} f(x)$$
$$\text{subject to: } \forall \delta \in \Delta : g(x,\delta) \leq 0,$$

x: decision variables and δ: uncertainties.

Modeling of expectations and behaviors of grid users

- Long term planning: "perfect" market.
  - Minimization of the total cost: Capex and Opex.

- Shorter term: actual behaviors and imperfect market design $\Longrightarrow$ Agent based approach

☞ Very complex problem: finding "Nash Equilibrium".

N players with payoff function f_p and x_p strategy for each player p

$$\exists\ x_p^*, \forall\ x_p : f_p(x_p^*, x_{-p}^*) \geq f_p(x_p, x_{-p}^*)$$

x_{-p}: strategies of all other players except p

☞ Stochastic Behaviors: Potential links with Mean field Game Theory
  - Study of strategic decision making in very large populations of small interacting individuals

Multi-objective optimization problems

e.g: Minimize looses while keeping reactive reserves, minimize CO_2 emissions and generation costs ...

- Currently, $\min(w_1.f_1(x) + w_2.f_2(x) + ... + w_n.f_n(x))$
- But finding value for w_i: difficult and questionable

☞ $\min(f_1(x), f_2(x), ..., f_n(x)) \Longrightarrow$ Pareto optimal solutions

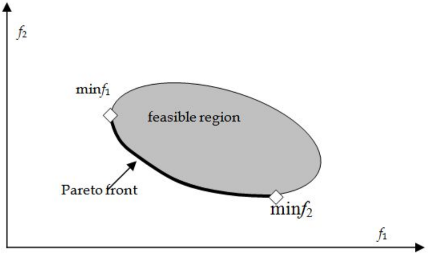


Figure: Complex optimization ⇒ Meta Heuristics Methods

Modeling the physics of the power system

Solving optimization problems based on not realistic enough modeling is useless ⇒ Right balance between realism and complexity.

- Currently, static and deterministic modeling using linearizations ⇒ review required
- A significant number of controls are discrete:
  - switch on/off of breakers, switch on/off capacitor or reactor banks, tap changers on transformers, generating units producing with non zero minimal active power ...
  - ☞ Naive relaxation (round off strategy) ⇒ Difficult to find feasible solutions.
- Some local controls and protection schemes are not event-based but measurement based.
  - ☞ Conditional corrective actions ⇒ Binary variables as proposed in 1999 by Bemborad and Morari. [3]
- Some constraints are related to stability and dynamic behaviors
  - ☞ Ultimate solution: Modeling using DAEs ⇒ Intractable problem
  - Modeling via "proxies" = rules learned off line as proposed in 1994 by Wehenkel and al. [4]

Conclusion: Taxonomy of optimization problem

- We could see that power system management could lead to a large diversity of optimization problems.

☞ The proper formulation of each problem has to be well thought out before searching for computational solutions.

Integer Variables

- Treatment of integer variables in optimization problems is a difficult task.

- Industrial and efficient solutions for large linear problems (MILP):
  - Very efficient presolvers: reducing dramatically the number of constraints
  - Parallel implementation but efficiency is problem depend and requires tuning
  - Hot start capabilities speed up sequence of optimization problems

☞ For non linear problems (MINLP), no very good industrial solutions

⚠ In our case: non linear but moreover non convex when integer variables are transformed in continuous variables

Modeling of local controls in optimization problems

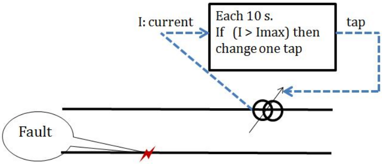
$\min \sum_{g=1}^{n_g} |p_g - p_g^0|$

subject to:
$F(P,\theta,\phi_0) = 0$
$C(\theta,\phi_0) \le L_0$
$P_{min} \le P \le P_{max}$
$-\phi_m \le \phi_0 \le \phi_m$

for each contingency: $k \in S_c$
$F_k(P,\theta_k,\phi_0) = 0$
$C_k(\theta_k,\phi_0) \le L_1$
$\boldsymbol{if\ (I_{pst}(\theta_k,\phi_0) \le I_{max})\ then}\ \{\boldsymbol{\phi_k = \phi_0}\}$
$-\phi_m \le \phi_k \le \phi_m$
$F_k(P,\theta_k^c,\phi_k) = 0$
$C_k(\theta_k^c,\phi_k) \le L_2$

F, F_k, C, C_k and I_{pst}: linear functions.

Illustrative example: Phase Shifter Transformer control



Conditional action modeled using a binary variable

$\boldsymbol{\delta.I_{max} \le I_{pst}(\theta_k,\phi_0) \le \delta.M_I + I_{max}}$
$\boldsymbol{-\delta.M_\phi + \phi_0 \le \phi_k \le \delta.M_\phi + \phi_0}$

δ: a binary variable, M_I and M_ϕ: two big positive constants

Powerful general framework for modeling of logical and integer relations

"Control of systems integrating logic, dynamics, and constraints" by Bemborad and Morari [3]

☞ But case by case analysis required; finding not too big ”big M" ....

Another important application: Expansion planning (disjunctive model as proposed in 2001 by Bahiense and al. [5])

- Product between difference of phase angles and added incremental admittance ⇒ Linear Equations

Mixed Integer Non Linear Programming (MINLP)

☞ MINLP remains challenging: Leyffer and al. [6] give a general diagnostic on practical problems

- State of the art: Sequential MILPs or round-off methods $\Rightarrow$ tuning required, finding a feasible solution could be difficult.
- Interesting alternative: Mathematical Programming with Equilibrium Constraints (MPEC) $\Rightarrow$ ensure the feasibility
  - x: binary variable, $x \in \{0,1\} \iff x \perp (x-1) = 0$
  - Implementations using Interior point methods via relaxation or penalization
  - Useful to manage very large size problems as shown in two European Projects: PEGASE [7], iTesla [1]

  ☞ Only sub-optimal solutions for non-convex MINLP problems.

Conclusion: Need for convexification

- ACOPF are at the core of all our optimization problems

⚠ Large amount of dedicated tunings and heuristics required to solve each specific practical problem.

- Impossible to ensure that an AC feasible solution could be obtained using an iterative linear method.

☞ Convexification of ACOPF is a promising generic method to avoid most of these tunings and heuristics.

- In the following, we present two recent promising convexification methods.

PART 2: What's new in solution approaches for ACOPF ?

- Two promizing ACOPF convexification approaches
- Novel results in handling of uncertainties in optimization
- Other IT and CS progresses beyond our wishes

On the essence of the general class of ACOPF problems

- Ingredients:
  - A graph defined by a set of n buses and by $b \leq \frac{1}{2}n(n-1)$ branches
  - State is defined by vector of complex voltages over the set of buses
- Nature of optimization variables, objective function, feasible set:
  - Optimization variables: injections & branch parameters are inputs; voltages and currents are 'outputs'
  - Objective function: smooth in terms of optimization variables (e.g. a polynomial of a certain degree).
  - Physical constraints: KLaws, device physics.
  - Technical constraints: bounds on injections, discrete nature of parameter choices (e.g. topology, steps of transformers, on-off status of generators, load-shedding steps, etc.)
- The main (actually only) source of non-convexity is the nature of the feasible domain:
  - Power flow equations are quadratic equality constraints, and integrality constraints are by essence non convex.

Lifting and relaxation to build novel ACOPF algorithms

- The underlying rationale is based on the YOGA[1] iteration:
  - Let's relax: enlargen original non convex feasible set by a convex one, as small as possible (i.e. something close to a convex hull).
  - Let's work: solve the 'relaxed' problem with existing solvers.
  - Let's contemplate the result:
    - We get a lower bound on the value of the original problem.
    - If we are lucky, we get a feasible and hence (globally) optimal solution to the original problem: BINGO

- Most YOGA iterations are based on two successive steps:
  - Lifting: map the problem into a higher dimensional space, where additional constraints are added so as to make this a 1-to-1 map.
  - Relaxation: in the lifted formulation, choose a set of constraints to be removed, so as to remain close to convex hull, at least in the area where optimal solutions can be located.

[1] *YOGA*: acronym for *Your Optimization with a Gentle Attitude*

A tiny naive illustrative example: in equations

- An originally non-convex problem in dimension 1:

$$\min_{x\in\mathbb{R}} \{x^2 - x\} \text{ s.t. } [x^2 - 4 \geq 0]$$

- is first **lifted** into an equivalent 2-dimensional problem:

$$\min_{(y_1,y_2)\in\mathbb{R}^2} \{y_2 - y_1\} \text{ s.t. } [y_2 - 4 \geq 0 \text{ and } y_2 = (y_1)^2]$$

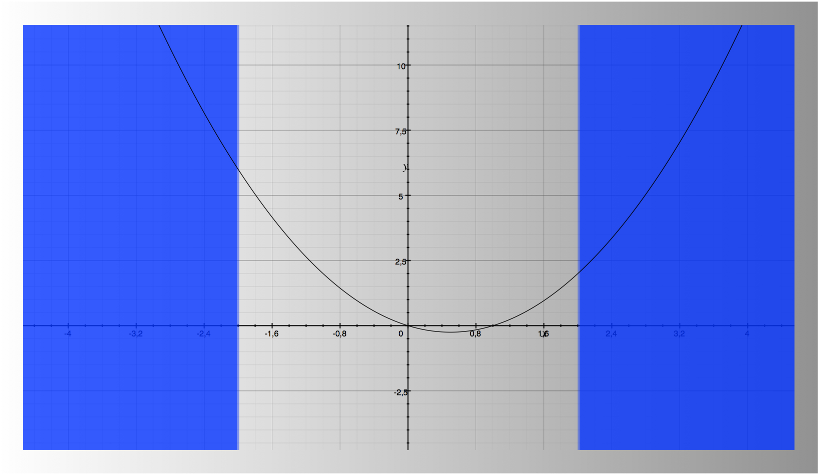
- and then **relaxed** by replacing in $\mathbb{R}^2$ the set defined by the constraint $y_2 = (y_1)^2$ by its convex hull, defined by the constraint $y_2 \geq (y_1)^2$:

$$\min_{(y_1,y_2)\in\mathbb{R}^2} \{y_2 - y_1\} \text{ s.t. } [y_2 - 4 \geq 0 \text{ and } y_2 \geq (y_1)^2]$$

which hence yields a convex program, in the form of a relaxation of the original optimization problem.

A tiny illustrative example: graphically

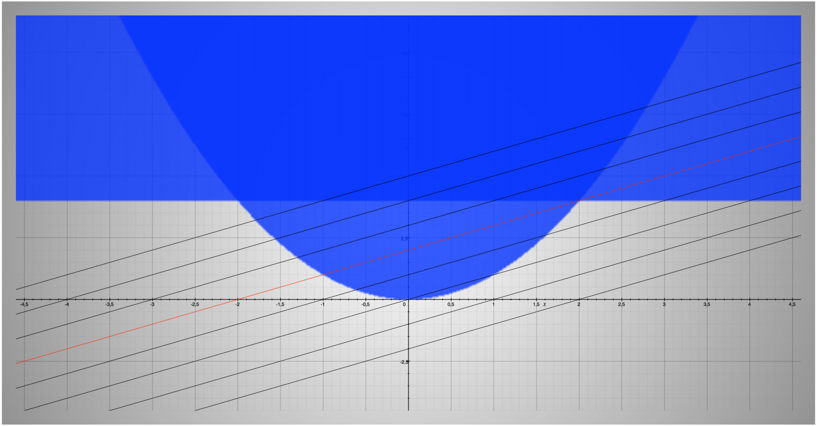
$$\min_{x \in \mathbb{R}} \{x^2 - x \equiv x(x-1)\} \text{ s.t. } [x^2 - 4 \geq 0 \equiv (x+2)(x-2) \geq 0]$$



Solution: $x^* = 2$. Optimal value f^* is $(2)^2 - 2 = 2$.

A tiny illustrative example: graphically lifted and relaxed

$$\min_{(y_1,y_2)\in\mathbb{R}^2} \{y_2 - y_1\} \text{ s.t. } [y_2 - 4 \geq 0 \;\text{ and }\; y_2 \geq (y_1)^2]$$



Solution: $(y_1 = 2, y_2 = 4)$. It satisfies the non convex constraint $y_2 = (y_1)^2$.
!!! BINGO !!!

How to apply 'lifting + relaxation' to ACOPF ?

- Lifting + relaxation (≡ YOGA) is a powerful general approach to construct algorithms for solving difficult optimization problems.
- For ACOPF, the questions are
  - How to lift and relax in a computationally efficient way ?
  - Under which conditions does a certain 'lifting + relaxation' scheme provide the global optimum ?
- We will introduce two recently proposed 'lifting & relaxing' approaches to ACOPF:
  - View ACOPF as a Quadratically Constrained Quadratic Programming problem (QCQP)
    - The SDP relaxation (see citations in the paper...)
  - View ACOPF as a Polynomially Constrained Polynomial Programming problem (PCPP)
    - The MOMENT relaxation (see citations in the paper...)

ACOPF viewed as a QCQP

- The ACOPF can often be rewritten as a Quadratically Constrained Quadratic Program (QCQP) in the form:

$$\min_{x \in \mathbb{C}^n} \quad x^H C_0 x \tag{1a}$$

$$\text{subject to} \quad x^H C_l x \leq b_l, \quad l = 1, \ldots, L, \tag{1b}$$

where $x \in \mathbb{C}^n$, and for $l = 0, \ldots, L$, $b_l \in \mathbb{R}$ and $C_l \in \mathbb{S}^n$.

⚠ If C_l, $l = 0, \ldots, L$, are all positive semidefinite then (1) is convex. But, ACOPF is non-convex, because of (e.g.):

1. Power flow equations. Quadratic equality constraints rewrite as:

$$\left[x^H C_{pf} x = b_{pf}\right] \equiv \left[x^H C_{pf} x \leq b_{pf} \quad \textbf{and} \quad x^H (-C_{pf}) x \leq -b_{pf}\right]$$

2. Integratility constraints for binary variables. They rewrite as:

$$[x_i \in \{-1, 1\}] \equiv \left[(x_i)^2 = 1\right]$$

SDP relaxation of a QCQP (1)

- Defining $X =: xx^H$, and using $x^H C_l x = \text{tr } C_l xx^H =: \text{tr } C_l X$ we can lift the QCQP (1) as the following equivalent problem where the optimization is now over Hermitian matrices:

$$\min_{X \in \mathbb{S}^n} \quad \text{tr } C_0 X \tag{2a}$$

$$\text{subject to} \quad \text{tr } C_l X \leq b_l, \quad l = 1, \ldots, L \tag{2b}$$

$$X \succeq 0 \tag{2c}$$

$$\text{rank } X = 1 \tag{2d}$$

- The key observation is that the objective function and the constraints are linear in X in (2a)–(2b) and that the constraint $X \succeq 0$ in (2c) is convex (since $\mathbb{S}^n_+$ is a convex cone).
- The rank constraint in (2d) ensures that $\exists x \in \mathbb{C}^n : X = xx^H$. It is the only nonconvex constraint of problem (2).

SDP relaxation of a QCQP (2)

- Relaxing the rank constraint results in a semidefinite program (SDP):

$$\min_{X \in \mathbb{S}^n} \quad \text{tr } C_0 X \tag{3a}$$

$$\text{subject to} \quad \text{tr } C_l X \le b_l, \quad l = 1, \ldots, L \tag{3b}$$

$$X \succeq 0. \tag{3c}$$

SDP is a convex relaxation of QCQP that can be efficiently computed.

- A strategy for solving QCQP (1) is therefore to solve SDP (3) for an optimal X^{opt} and check its rank.
  - If rank $X^{\text{opt}} = 1$ then X^{opt} is optimal for (2) as well and an optimal solution x^{opt} of QCQP (1) can be recovered from X^{opt} through spectral decomposition $X^{\text{opt}} = x^{\text{opt}}(x^{\text{opt}})^H$.
  - If rank $X^{\text{opt}} > 1$ then, in general, no feasible solution of QCQP can be directly obtained from X^{opt} but the optimal objective value of SDP provides a lower bound on that of QCQP. If the SDP (3) is infeasible, then it is a certificate that the original QCQP (1) is infeasible.

SDP relaxation of ACOPF: discussion

- Under restrictive conditions one can show that the SDP relaxation is exact for ACOPF (e.g. for radial networks), but in general it is not exact.
- However, the relaxed solution provides a lower bound on optimality, which may be usefully exploited in practice.
- Also, given any solution provided by any alternative algorithm for ACOPF, one can use the SDP relaxation to provide sufficient conditions of global optimality (but non necessary ones).

☞ QCQP is a special case of polynomial optimization.

☞ Polynomial optimization is more general and lends itself to the application of the moment relaxation approach (see next slides).

ACOPF viewed as a PCPP

- Any thinkable ACOPF problem can be rewritten as a Polynomially Constrained Polynomial Program in the following form:

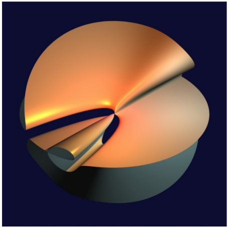
$$\min_{x \in \mathbb{R}^n} \quad f_0(x) \tag{4a}$$

$$\text{subject to} \quad f_l(x) \geq 0, \quad l = 1, \dots, L, \tag{4b}$$

where $\forall l = 0, \dots, L : f_l(x) = \sum_{\alpha \in \mathbb{N}^n} c_{l,\alpha} x^\alpha$, and $x^\alpha \equiv \prod_{i=1}^n x_i^{\alpha_i}$.

- Examples of polynomial feasible domains in 3 dimensions

$x_1^3 x_3 + x_1^2 + x_2 x_3^3 + x_3^4 = 3 x_1 x_2 x_3, \; x_1^2 + x_2^2 + x_3^2 \leq 4$



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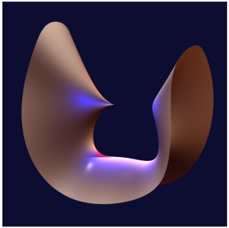
$x_1^2 - x_1^3 + x_2^2 + x_2^4 + x_3^3 - x_3^4 = 0, \; x_1^2 + x_2^2 + x_3^2 \leq 4$



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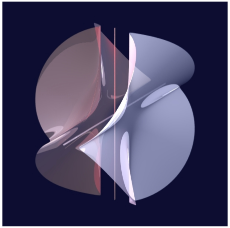
$x_1^4 - x_1^2 - x_2^2 x_3^2 = 0, \; x_1^2 + x_2^2 + x_3^2 \leq 25$



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Moment approach: general principle

- The generic optimization problem:

$$\min_{x \in \mathbb{R}^n} \quad f_0(x) \tag{5a}$$

$$\text{subject to} \quad f_l(x) \geq 0, \quad l = 1, \ldots, L, \tag{5b}$$

can be rewritten as:

$$\min_{\mu \in \mathscr{M}(\mathbb{K})_+} \quad \int_{\mathbb{K}} f_0 \, d\mu \tag{6a}$$

$$\text{subject to} \quad \int_{\mathbb{K}} d\mu = 1, \tag{6b}$$

where $\mathbb{K}$ is the feasible space defined by the constraints (5b) and $\mathscr{M}(\mathbb{K})_+$ is the space of finite (positive) Borel measures μ on $\mathbb{K}$.

- Indeed (intuitively),
  - for any positive μ on $\mathbb{K}$ such that (6b) holds, we have that $\int_{\mathbb{K}} f_0 \, d\mu \geq f^*$, where f^* is the optimal value of problem (5);
  - at the same time, if x^* is a solution of (5), then the Dirac measure $\mu^* \equiv \delta_{x^*}(x)$ is a solution of (6).

Moment relaxation (for polynomial optimization)

- The formulation (6) generically represents the optimization problem (5) as a linear (and therefore convex), but infinite-dimensional optimization problem. We can further reformulate (6) as

$$\min_{y} \quad \sum_{\alpha\in\mathbb{N}^n} c_{0,\alpha}\, y_\alpha \tag{7a}$$
$$\text{s.t.} \quad y_0 = 1 \tag{7b}$$
$$y_\alpha = \int_{\mathbb{K}} x^\alpha d\mu,\ \ \alpha\in\mathbb{N}^n,\ \text{for some } \mu\in\mathscr{M}(\mathbb{K})_+ . \tag{7c}$$

  where now any measure $\mu\in\mathscr{M}(\mathbb{K})_+$ is represented by its (infinite-dimensional) vector of moments $y = [y_\alpha]_{\alpha\in\mathbb{N}^n}$.
- Formulation (7) is as well linear in y and infinite-dimensional. But it can be relaxed into a finite dimensional convex optimization problem, by keeping only a finite number of convex conditions over y implied by the constraints (7c), leading to a so-called moment relaxation.

Moment relaxation (Lasserre hierarchy)

- For some $\gamma \in \mathbb{N}$, define the vector of monomials up to degree γ:

$$\chi_\gamma = [1 \quad x_1 \quad \dots \quad x_n \quad x_1^2 \quad x_1 x_2 \quad \dots \quad x_n^2 \quad \dots \quad x_n^\gamma]^\top.$$

- Write the moment matrix $L_y(\chi_\gamma \chi_\gamma^\top)$, where term by term a polynomial in x is replaced by a corresponding linear combination of moments (i.e. $L_y(\sum_i \beta_i x^{\alpha_i}) \to \sum_i \beta_i y_{\alpha_i}$).
- Similarly, write the matrices $L_y\left(f_l(x)\, \chi_{\gamma-\beta_l} \chi_{\gamma-\beta_l}^\top\right), l = 1, \dots, L$, where the polynomial $f_l(x)$ has degree $2\beta_l$ or $2\beta_l - 1$.
- The order-γ moment relaxation in the Lasserre hierarchy is then

$$\min_y \quad L_y(f_0(x)) \tag{8a}$$

$$\text{s.t.} \quad y_0 = 1 \tag{8b}$$

$$L_y\left(\chi_\gamma \chi_\gamma^\top\right) \succeq 0 \tag{8c}$$

$$L_y\left(f_l(x)\, \chi_{\gamma-\beta_l} \chi_{\gamma-\beta_l}^\top\right) \succeq 0, \; l = 1, \dots, L. \tag{8d}$$

Moment relaxation (discussion)

- In the same way as in the SDP relaxation, satisfaction of a rank condition is sufficient for exactness of the moment relaxation:

$$\operatorname{rank}\left(L_y\left(\chi_1 \chi_1^\top\right)\right) = 1. \tag{9}$$

  When condition (9) is satisfied, the globally optimal decision variables are obtained from a spectral decomposition of the matrix $L_y\left(\chi_1 \chi_1^\top\right)$.

- If not, the solution provides a lower bound on the value of the original problem. Increasing the order γ tightens this relaxation, at the price of a rapidly growing computational burden.
- Assuming that the optimization problem is a QCQP, the order-1 moment relaxation in the Lasserre hierarchy is essentially equivalent to the SDP relaxation discussed previously.
- Hence, the moment relaxation approach provides a possibility to find globally optimal solutions for a broader class of ACOPF, and in particular when the SDP relaxation does not work out.

Wrap up about ACOPF convexification

✓ The two presented approaches already provide significant progress in understanding the nature and solving exactly ACOPF problems of moderate size (few hundred to few thousand nodes).

✓ They can be further developed by exploiting peculiarities of ACOPF problems (wrt generic QCQPs and PCPPs), in particular the sparsity structure induced by the network topology, both

- theoretically (e.g. by studying the space of chordal graphs, beyond the space of tree-structured topologies) and
- practically by leveraging HPC infrastructures on top of sparse formulations.

☛ Please see full paper for further explanations and references to the relevant bibliography.

☞ They can be leveraged to SCOPF and to optimization under uncertainty (see subsequent slides).

Optimization under Uncertainties: Motivation/Outline

- In many practical applications, some parameters (or even structures) of the optimization problem to be solved are not known exactly.
- In such circumstances, we would like to find a good solution that is also acceptable even in the worst case conditions, over a given uncertainty set.
- We will look at two complementary frameworks for stating and solving such problems, namely
  - Chance Constrained Optimization (CCO)
  - Robust Optimization (RO)
- ... and highlight some recent results of interest in these two fields.
- We will see that both approaches can and should benefit from the ACOPF convexification ideas presented earlier in this paper.

Chance Constrained Optimization (CCO)

- Generic CCO problem formulation:

  choose $x^* \in \mathbb{R}^d$ to minimize $f(x)$ subject to $x \in \mathscr{X}, \delta \in \Delta$ and
  $$\mathbb{P}\{\delta : x \in \mathscr{X}_\delta\} \geq 1 - \epsilon.$$

- Example application in ACOPF under uncertainty:
  - x are decisions to be taken ahead in time (say the day ahead).
  - Δ represents uncertainties still present when a decision has to be taken (say future power injections).
  - $f(x)$ is a performance measure, eg. $f(x) = \mathbb{E}_{\mathbb{P}'(\delta)} f'(x, \delta)$ (say an expected cost or the cost under nominal conditions).
  - The sets $\mathscr{X}_\delta$ represent feasibility constraints to be satisfied, as a function of the uncertainty $\delta \in \Delta$ (say security constraints over the next period of time).
  - The CCO problem then amounts to find a decision x^* such that with high probability all feasibility constraints are satisfied by x^*, and x^* is optimal in terms of the objective function.

Recent results in CCO (1): scenario sampling

- Scenario approach: (valid if $f(x)$, $\mathcal{X}$, and $\mathcal{X}_\delta, \forall \delta \in \Delta$ are convex in x):
  - Collect N samples $\delta^{(1)}, \delta^{(2)}, \ldots, \delta^{(N)}$, independent and identically distributed according to the probability measure $\mathbb{P}$.
  - Solve the (finite-dimensional) problem:

$$\begin{aligned} \mathrm{SP}_N: \quad & \min_{x \in \mathcal{X}} f(x) \\ & \text{s.t. } x \in \mathcal{X}_{\delta^{(i)}},\ i \in \{1, \ldots, N\}. \end{aligned}$$

  - A solution x_N^* of SP_N satisfies, with probability $1-\beta$ all constrains in Δ but at most an ϵ-fraction, i.e. $\mathbb{P}\{x_N^* \notin \mathcal{X}_\delta\} \le \epsilon$, provided that

$$N \ge \frac{2}{\epsilon}\left(\ln \frac{1}{\beta} + d\right)$$

that is, the obtained solution is chance constrained feasible with high confidence $1-\beta$ (d is the dimension of the space containing x).

Recent results in CCO (2): scenario sampling and discarding

- The scenario approach formulated above may yield conservative results, specially if N is very large.
- To improve the performance, one can a posteriori discard k constraints among the initially sampled set of N constraints. In this way, the solution improves, and a solid theory permits one to still guarantee chance constrained feasibility.
- More precisely, one can show that the solution obtained after discarding k constraints satisfies

$$\mathbb{P}\{x_N^* \notin \mathcal{X}_\delta\} \leq \frac{k}{N} + O\left(\frac{\sqrt{k \ln k}}{N}\right).$$

This expression shows that $\mathbb{P}\{x_N^* \notin \mathcal{X}_\delta\}$ rapidly approaches the empirical chance constraint violation $\frac{k}{N}$ as N increases, so that the approach bears very little conservatism.

Recent results in CCO: discussion

- CONVEXITY:
  - The theoretical guarantees are relying on convexity assumptions, which may not be available readily in ACOPF problems.

  $\Rightarrow$ We need to see how to combine ACOPF convexification with CCO.
- NO FREE LUNCH CAVEAT:
  - If we want small ϵ (say $\epsilon = 10^{-5}$) and work in high dimension (say $d = 10^5$) we need in the order of $N = 2 \times 10^{10}$ scenarios.

  $\Rightarrow$ We need HPC approaches to handle such problems.
- FURTHER RESEARCH is needed to leverage the CCO approach in practice!
- Please see survey paper for references on ongoing works in this direction.

Robust Optimization (RO)

- The two-stage adaptive robust decision making model can be written in the following compact form

$$\min_{\boldsymbol{x}\in X}\left\{f(x)+\max_{\boldsymbol{d}\in\mathscr{D}}\ \min_{\boldsymbol{y}\in Y(\boldsymbol{x},\boldsymbol{d})} g(x,d,y)\right\}. \tag{10}$$

- Here, the first-stage decision is $\boldsymbol{x}$ in the feasible region X, and the second-stage decision is $\boldsymbol{y}$, which adapts to the realization of uncertainty $\boldsymbol{d}$ in the uncertainty set $\mathscr{D}$ and satisfies various operational constraints in $Y(\boldsymbol{x},\boldsymbol{d})$ parametrized by the first-stage decision $\boldsymbol{x}$ and $\boldsymbol{d}$.
- Usually, the uncertainty $\boldsymbol{d}$ models load uncertainty and generation uncertainty in variable resources such as wind and solar power.
- The idea can been applied to various unit commitment and operation planning problems, coupled or not with SCOPF formulations.

Recent results in RO: discussion

- Approach can be worked out in order to reduce conservativeness, in particular by combining it with ideas from stochastic programming.
- The question is how to efficiently solve this type of problems.
- Generally, the solution to solve these problems is to write the KKT conditions of the low level problems but this is valid only for convex problems.
- In case of non convex low level problems which is the case for ACOPF or with integer variables, standard approaches could fail to find a solution; MITSOS [2] proposed an algorithm for convex non linear problems with integer variables

Wrap up about Optimization under Uncertainties

- Both CCO and RO offer complementary frameworks for dealing with uncertainties.
  - RO is closer to current practice (and risk-adverseness) of TSOs
  - CCO needs assumptions about $\mathbb{P}$, but in some circumstances it is the only sensible formulation
  - The scenario approach to CCO may actually be viewed as a randomisation approach to solve approximately an RO problem (SIC).
- Both formulations will benefit from ACOPF convexification approaches presented earlier.
- Further work is required to develop adequate models of uncertainties (in particular by taking into account soft correlations and hard physical constraints among the various dimensions of uncertainty sets).
- Further work is required to develop scalable algorithmic solutions useful in power systems practice.

Some words about possible synergies with sister fields

- IT: huge progresses in HPC and BIG DATA technologies !
  - How to leverage HPC to ACOPF applications ?
  - How to exploit BIG DATA solutions in this context ?
- CS: huge progresses in Machine Learning and Randomized Algorithmics !
  - How to use ML to build 'proxies' from measurements and simulation results ?
  - How to build on RA, to extend various Monte-Carlo types of approaches ?

Summary, and pointers to other relevant sessions at PSCC2014

- Summary
  - During the last years, HUGE progresses have been made in theory and practice in sister fields from applied mathematics and computer science
  - The opportunities for applying them to power systems are as well HUGE
  - Collaboration between power system experts and experts from these sister fields is the best way to generate significant progress both in practice and in theory.
- Relevant sessions at PSCC 2014
  - This morning, right after the coffee break:
    INVITED PAPER SESSION PS 16, Advanced Optimization Methods for Power Systems.
  - Other sessions:
    - Today after lunch: PS20 (OPF 1)
    - Tomorrow after lunch: PS28 (OPF 2)
    - Friday 12AM: Panel session on Advanced Data-Driven Modelling Techniques for Power Systems

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Thank you for your attention