



# Inventory-routing problem with pickups and deliveries of RTI in closed-loop supply chain

Galina IASSINOVSKAIA, UCL-Mons, Belgium

Sabine LIMBOURG, QuantOM, University of Liege, Belgium

Fouad RIANE, CIGM, Faculté des Sciences et Technique de Settat, Settat, Morocco

**QuantOM**

Centre for Quantitative Methods and Operations Management



**comex**

combinatorial optimization:  
metaheuristics & exact methods



**LOUVAIN**  
School of Management<sup>1</sup>

# Returnable transport items

*RTI: any means of gathering together goods, transporting them, storing them, to protect them within chains, which is reusable, such as pallets, reusable packaging such as crates, containers, roll containers.*

International Council for Reusable Transport Items

**Roll container**



**Crate**



**Dolly**



**Pallets**



**Barrels**



**Bottles**



# Why RTI?



- The increased concerns about the environmental impacts of industrial activities and the search for economic advantages have given birth to the concept of a closed-loop supply chain.
- Areas of research and opportunities in the CLSC field are various:
  - remanufacturing
  - component cannibalization
  - recycling
  - sustainable packaging
  - ...

# Packaging and packaging waste directives

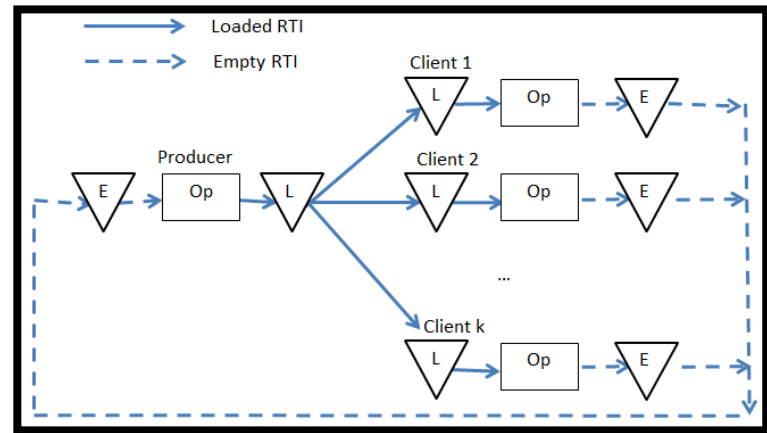


- Packaging waste is legislated by the European Packaging and Packaging Waste Directive' 94 (European Parliament and Council, 1994/62/EC).
- The Directive' 94 was amended in 2004 (European Parliament and Council, 2004/12/EC)
- Main points:
  - Prevention: if packaging waste can be prevented, then there is no need to recycle or recover packaging and packaging waste.
  - Producer responsibility : there is a responsibility on the producers of packaging to consider what is going to happen to the packaging at the end of its life cycle. The packaging can then be designed for easy disposal or recovery with a minimal environmental impact.

# Problem

## Problem faced by Renault

- Producer distributes its products using RTIs to a set of clients.
- Producer collects of the empty RTI for reuse.
- Each partner has a storage area composed of
  - the empty RTI stock
  - the loaded RTI stock
- Characterized by
  - the maximum storage capacity
  - out-of-stocks are not allowed
- Homogeneous fleet of vehicles that can carry simultaneously empty and loaded RTI



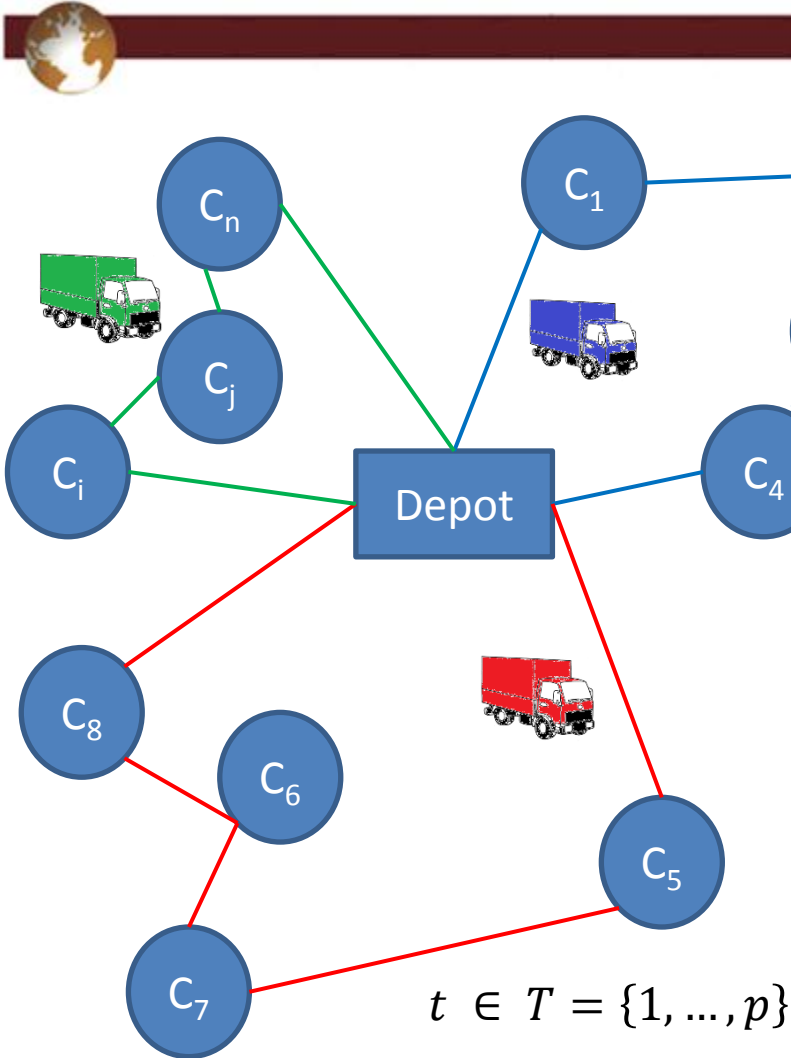
➡ Inventory-routing problem with pickups and deliveries (IRPPD)

# State of art



- Coelho, Cordeau, and Laporte, 2013, Thirty Years of Inventory Routing Transportation Science, Articles in Advance, pp. 1–19, INFORMS
  - Andersson H, Hoff A, Christiansen M, Hasle G, Løkketangen A (2010) Industrial aspects and literature survey: Combined inventory management and routing. Comput. Oper. Res. 37(9):1515–1536.
  - Kim T. , Glock C. H. and Kwon Y. (2014). A closed-loop supply chain for deteriorating products under stochastic container return times. Omega 43, pp 30–40
- ➔ No LP for the vehicle routing and inventory management in a closed-loop supply chain

# Variables



- $y_{ijvt} = 1$  iff  $(i,j)$  is used on the route of  $v$  in period  $t$
- $I_{it}^L$  ( $I_{it}^E$ ): inventory level of loaded (empty) RTI at  $i$  at the end of  $t$
- $q_{it}$  ( $r_{it}$ ): loaded (empty) RTI quantity delivered (returned) to (from) client  $i$  in period  $t$
- $x_{ijt}$  ( $z_{ijt}$ ): loaded (empty) RTI quantity transported from  $i$  to  $j$  in period  $t$
- $f_t$ : RTI quantity filled from the producer in period  $t$
- $n_t$ : new RTI quantity bought and filled from the producer in period  $t$
- $T_{vt}$ : represents the overtime worked by the vehicle  $v$

# Objective

Inventory routing problem with Simultaneous Delivery and Pickup in a closed-loop (IRPSDPCL)

$$\begin{aligned} & \text{minimize } \sum_{i \in N} \sum_{j \in N} \sum_{t \in T} \left( \alpha \sum_{v \in V} y_{ijvt} + \beta (w_L x_{ijt} + w_E z_{ijt}) \right) d_{ij} \\ & + \sum_{i \in N} \sum_{t \in T} (h_i^L I_{it}^L + h_i^E I_{it}^E) + \sum_{t \in T} c (f_t + n_t) + \sum_{t \in T} b n_t + \sum_{v \in V} \sum_{t \in T} \delta T_{tv} \end{aligned}$$



# Constraints



## Vehicle capacity

- $x_{ijt} + z_{ijt} \leq Q * \sum_{v \in V} y_{ijvt}$   
 $\forall (i, j) \in A, \forall v \in V, \forall t \in T$

Inventory balance for the loading and empty RTI at each client and at the depot (closed-loop chain)

- $I_{it}^L = I_{it-1}^L + q_{it} - u_{it}$
  - $I_{it}^E = I_{it-1}^E - r_{it} + u_{it}$
  - $I_{0t}^L = I_{0t-1}^L + F_t - \sum_{i \in N} q_{it}$
  - $I_{0t}^E = I_{0t-1}^E - F_t + n_t + \sum_{i \in N} r_{it}$
- $\forall i > 0 \in N, \forall t > 0 \in T$

# Constraints

Inventory capacity of loaded and empty RTI held by each client throughout all periods (no out-of-stocks)

- $0 \leq I_{it}^L \leq C_i^L$
- $0 \leq I_{it}^E \leq C_i^E$

$$\forall i > 0 \in N, \forall t > 0 \in T$$

Producer capacity (number of empty plus the number of bought RTI)

- $f_t \leq I_{0t-1}^E + n_t$   $\forall t > 0 \in T$

Loaded RTI are delivered and empty RTI are returned

- $\sum_{i \in N, i \neq j} (x_{ijt} - x_{jit}) = q_{jt}$
- $\sum_{i \in N, i \neq j} (z_{ijt} - z_{jit}) = r_{jt}$

$$\forall j > 0 \in N, \forall t > 0 \in T$$

# Constraints

Proper vehicle routes:

- if a vehicle  $v$  visits client  $j$  in period  $t$ , it has to leave client  $j$  in period  $t$  at the most a vehicle visits a client per period

$$\sum_{i \in N, i \neq j} y_{ijvt} - \sum_{i \in N, i \neq j} y_{jivt} = 0$$

$$\forall j \in N, \forall v \in V, \forall t \in T$$

- at the most a vehicle visits a client per period

$$\sum_{i \in N} \sum_{v \in V} y_{ijvt} \leq 1$$

$$\forall j > 0 \in N, \forall t \in T$$

- vehicle leaves the depot once per period or stays at the depot

$$\sum_{j > 0 \in N} y_{0jvt} \leq 1$$

$$\forall j > 0 \in N, \forall v \in V, \forall t \in T$$

Overtime

$$\sum_{i \in N} \sum_{j \in N} \left( \frac{d_{ij}}{s} + g \right) y_{ijvt} \leq L + T_{vt}$$

$$\forall v \in V, \forall t \in T$$

# Constraints

- Quantity of empty RTI returned is held in the inventory

$$r_{it} \leq I_{it}^E \quad \forall i > 0 \in N, \forall t > 0 \in T$$

- Quantity of loaded RTI delivered was held in the inventory in the previous period

$$q_{it} \leq I_{it-1}^L \quad \forall i > 0 \in N, \forall t > 0 \in T$$

- Quantity of empty RTI returned at the producer is held in its inventory

$$\sum_{i \in N} r_{it} \leq I_{0t}^E \quad \forall t > 0 \in T$$

- Quantity of loaded RTI delivered at the producer was held in the inventory in the previous period

$$\sum_{i \in N} q_{it} \leq I_{0t-1}^L \quad \forall t > 0 \in T$$

- + Non-negativity and binary conditions

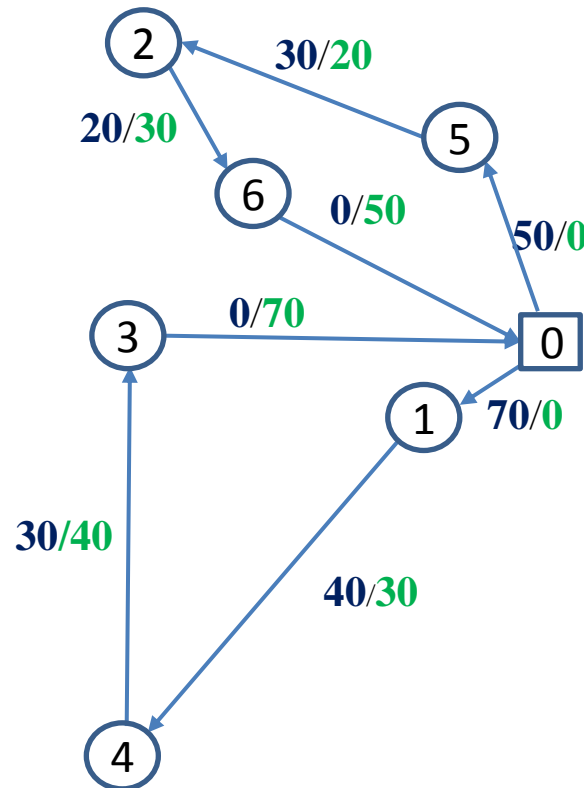
# Small instance

	Producer	C 1	C 2	C 3	C 4	C 5	C 6
Daily demands		30	10	30	10	20	20
Capacity of loaded RTIs	300	30	10	30	10	20	20
Capacity of empty RTIs	300	30	10	30	10	20	20
Inventory levels (t=0) of loaded RTIs	150	30	10	30	10	20	20
Inventory levels (t=0) of empty RTIs	50	30	10	30	10	20	20
Inventory costs of loaded RTIs (€/item)	0.015	0.035	0.035	0.035	0.035	0.035	0.035
Inventory costs of empty RTIs (€/item)	0.01	0.03	0.03	0.03	0.03	0.03	0.03

- 2 vehicles with capacity=120
- Fixed transportation cost: 0.8 €/km
- Variable transportation cost: 0.02€/km.item
- The total travel time is limited to 8 hours
- Overtime cost=2\*regular cost
- Stop: 30 minutes
- Truck's speed = 50 km/h
- Production cost= 0.02€/item,
- New RTI= 1€
- Number of periods=5

# Results

If inventory capacity = daily demand  $\Rightarrow$  VRP



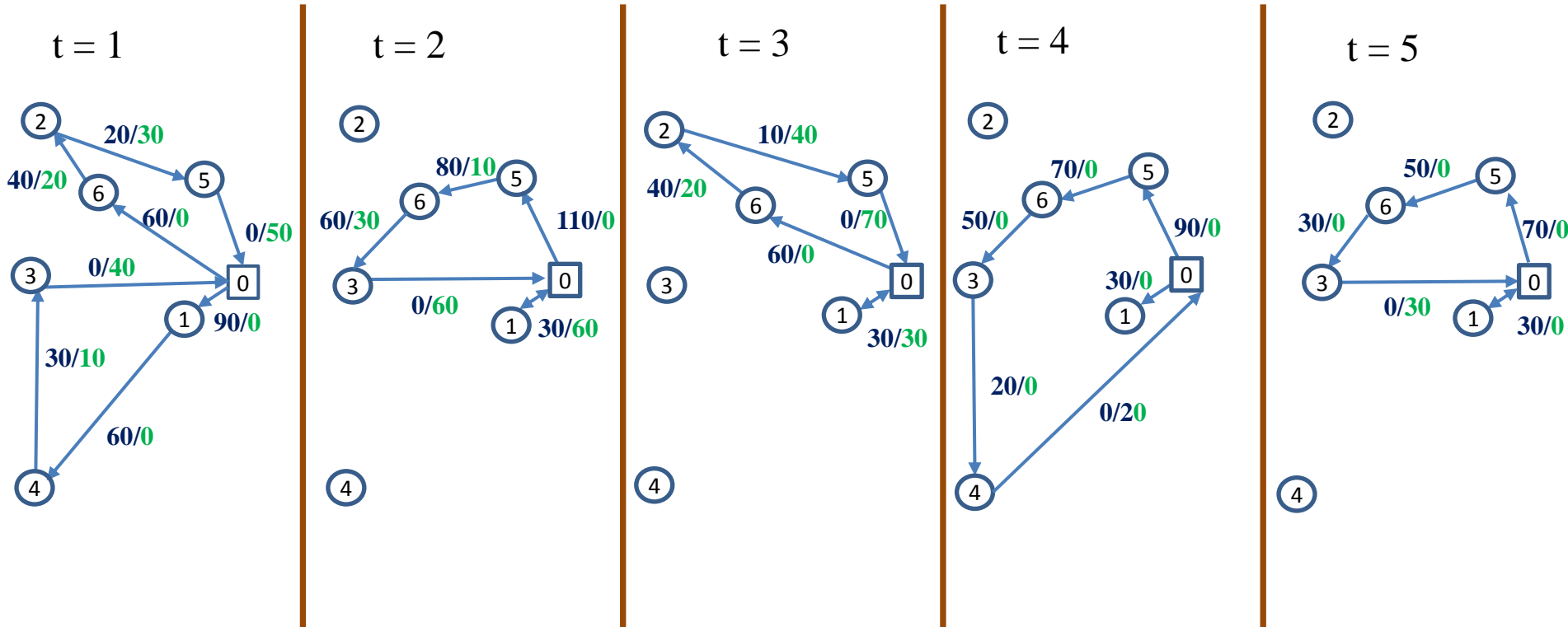
Loaded RTIs  
Empty RTIs

# Variation of the inventory capacity



	Capacity in terms of demand			
	1	2	3	4
Transportation cost	2452	1778	1473	1437.6
Inventory cost	48	55.2	62.9	63.9
Production cost	9	9	9	9
New RTI cost	200	300	450	450
<b>Total cost</b>	<b>2709</b>	<b>2142.2</b>	<b>1994.9</b>	<b>1960.5</b>
Time (s)	1	8558	726	965

# Exact method: Results

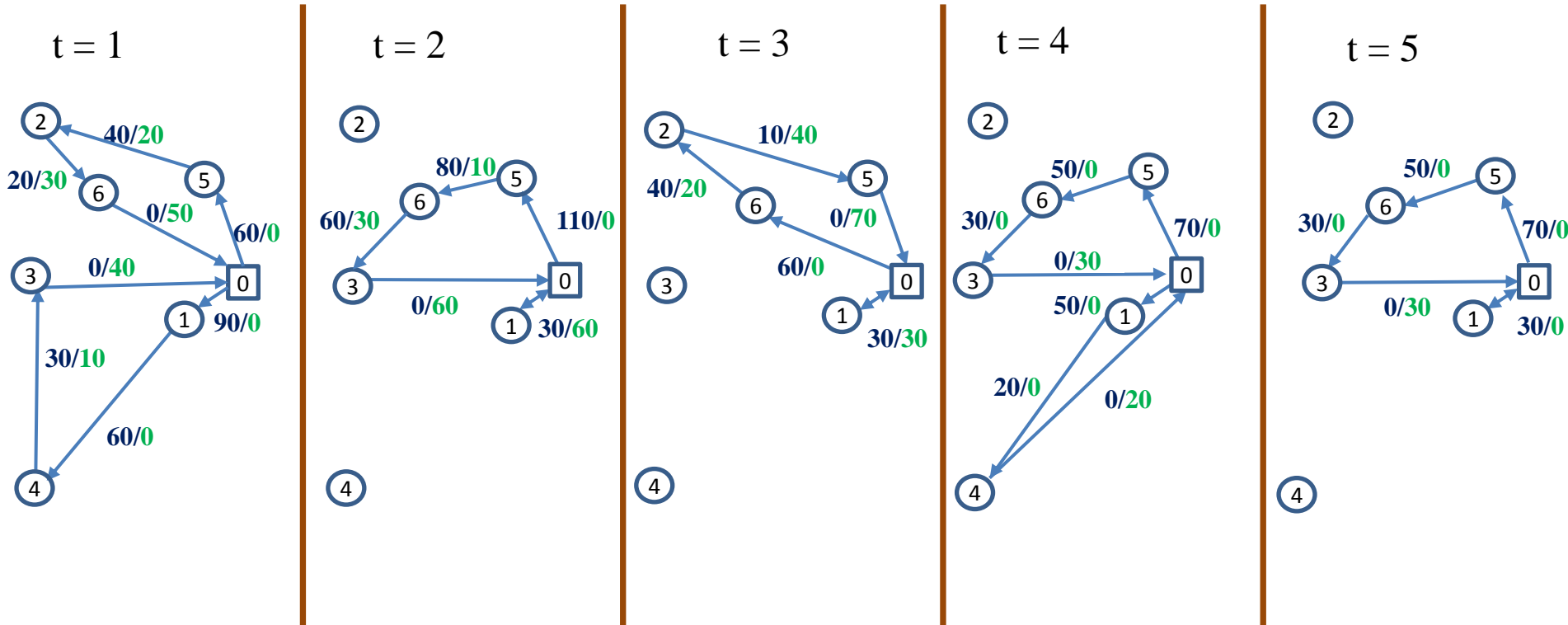


Inventory capacity =  $3 \times$  daily demand

The total travel time of each vehicle per period is limited to 8 hours

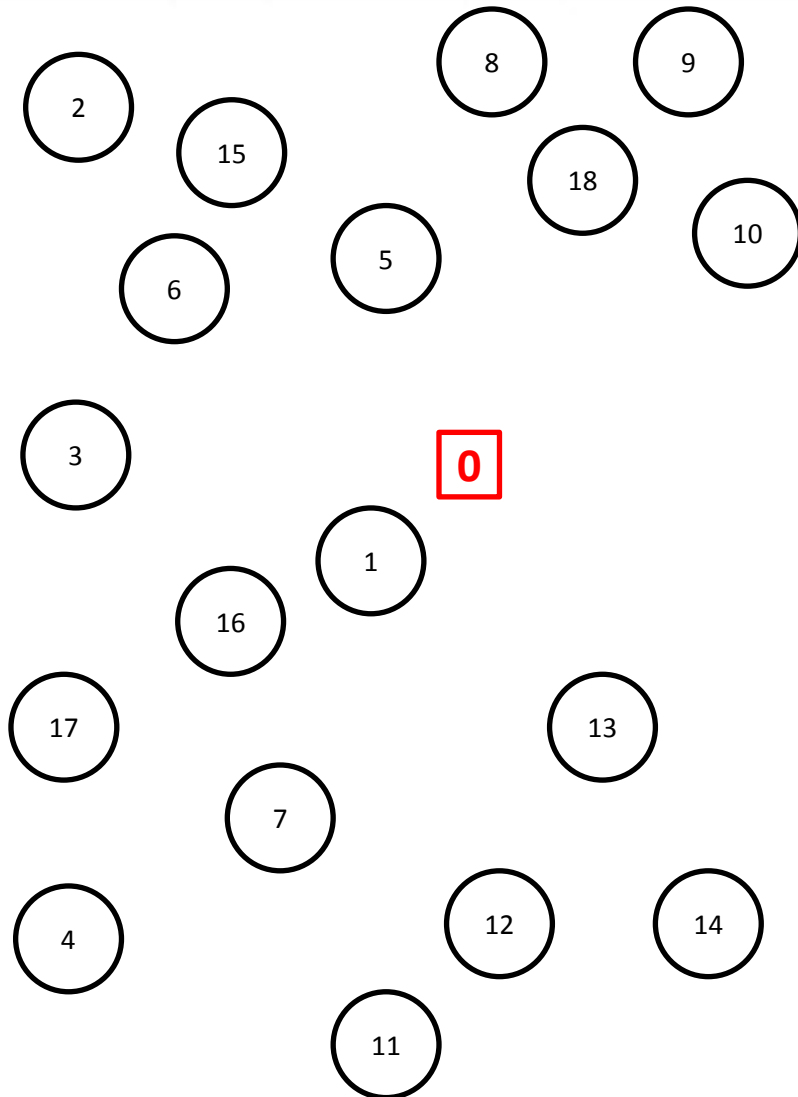


# Exact method: Results



Max travel time	Transportation cost	Inventory cost	Production cost	New RTI cost	Total cost	Time (s)
8	1473	62.9	9	450	1994.9	726
7	1478.6	62.3	9	450	1999.9	732

# Clustering



$$\text{minimize } \sum_{i \in N} \sum_{j \in N} \sum_{v \in V} d_{ij} \varphi_{ijv}$$

Subject to:

$$\sum_{v \in V} \chi_{iv} = 1 \quad \forall i \in N$$

$$\sum_{i \in N} \chi_{iv} u_i \leq pQ \quad \forall v \in V$$

$$\varphi_{ijv} \leq \chi_{iv}$$

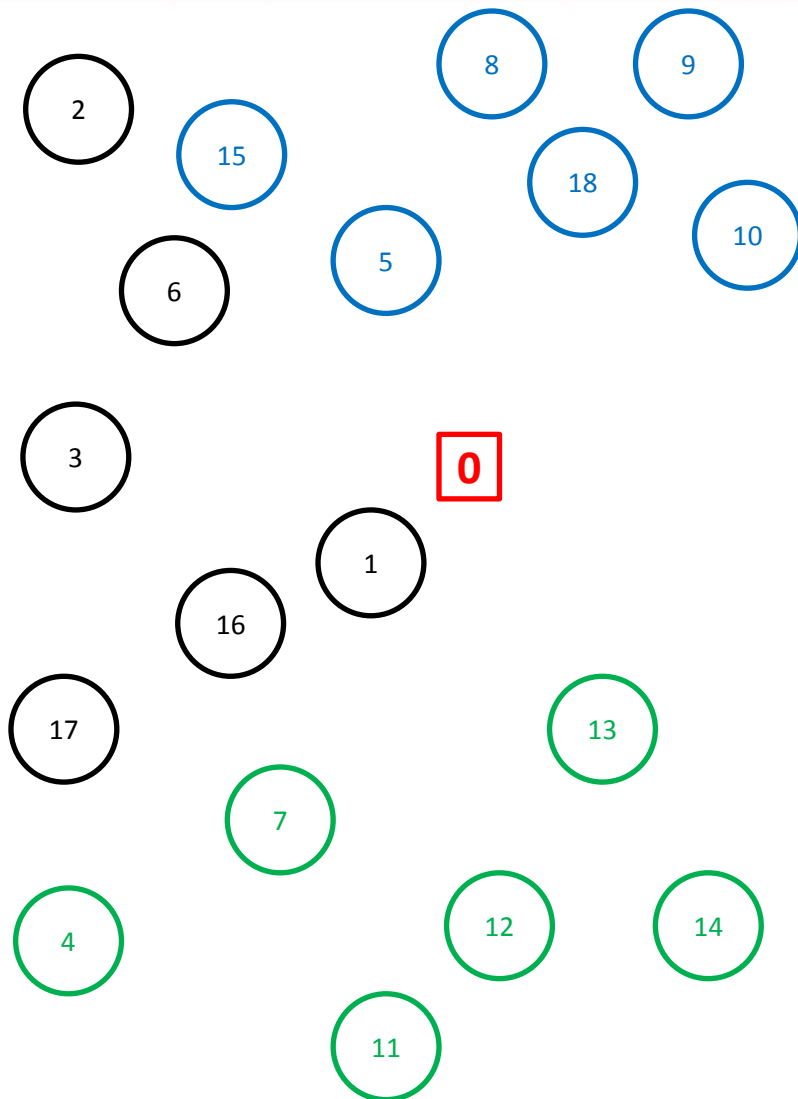
$$\varphi_{ijv} \leq \chi_{jv}$$

$$\varphi_{ijv} \geq \chi_{iv} + \chi_{jv} - 1$$

$$\chi_{iv}, \varphi_{ijv} \in \{0,1\}$$

$$\forall i, j \in N, v \in V$$

# Clustering



minimize  $\sum_{i \in N} \sum_{j \in N} \sum_{v \in V} d_{ij} \varphi_{ijv}$

Subject to:

$$\sum_{v \in V} \chi_{iv} = 1 \quad \forall i \in N$$

$$\sum_{i \in N} \chi_{iv} u_i \leq pQ \quad \forall v \in V$$

$$\varphi_{ijv} \leq \chi_{iv}$$

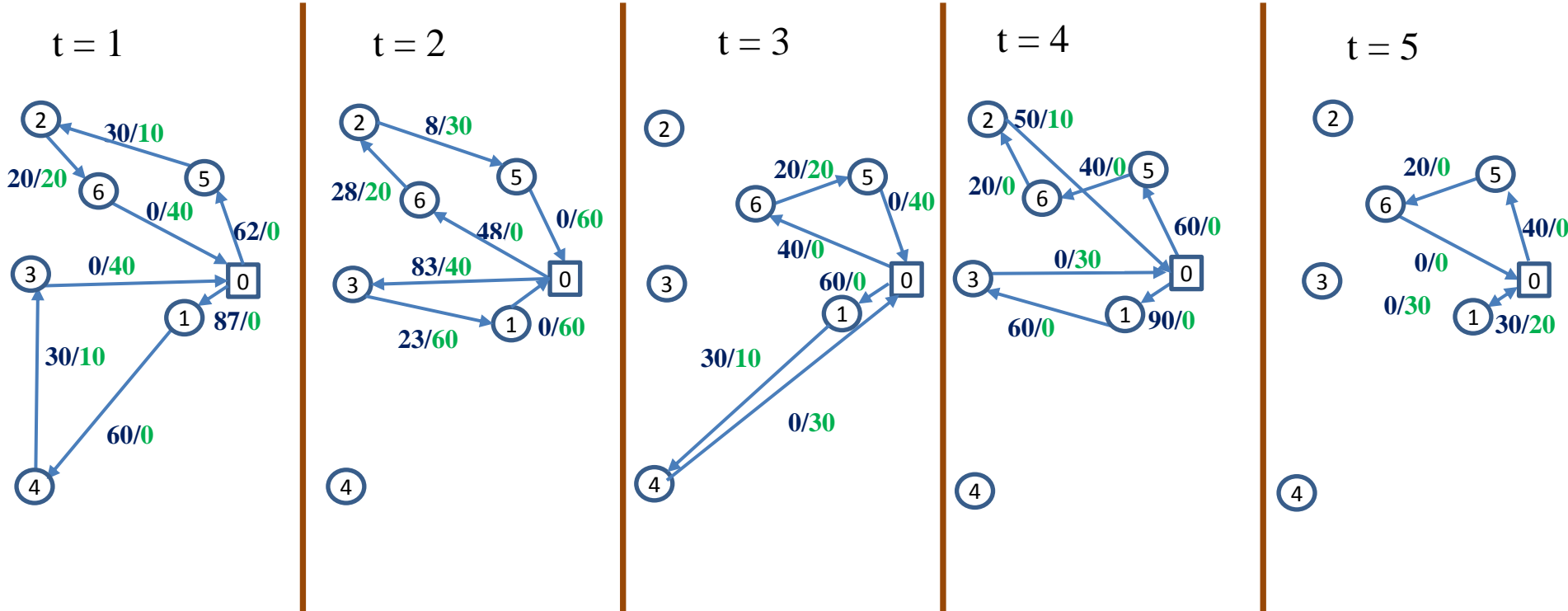
$$\varphi_{ijv} \leq \chi_{jv}$$

$$\varphi_{ijv} \geq \chi_{iv} + \chi_{jv} - 1$$

$$\chi_{iv}, \varphi_{ijv} \in \{0,1\}$$

$$\forall i, j \in N, v \in V$$

# Heuristic: Results

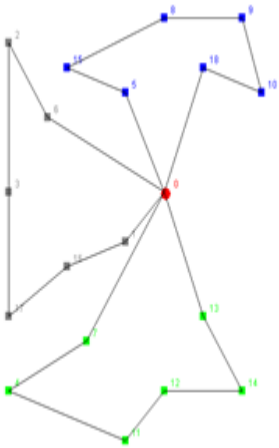


Method	Transportation cost	Inventory cost	Production cost	New RTI cost	Total cost	Time (s)
Exact	1478.6	62.3	9	450	1999.9	732
Heuristic	1523.48 (+3%)	63.48 (+2%)	9.2 (+2%)	510 (+13%)	2106.16 (+5%)	2

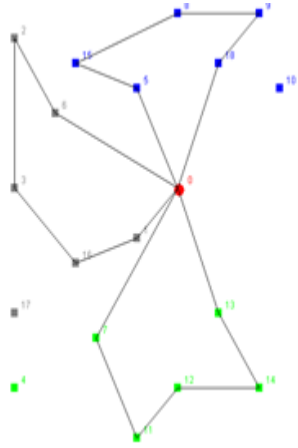
# Heuristic: Results



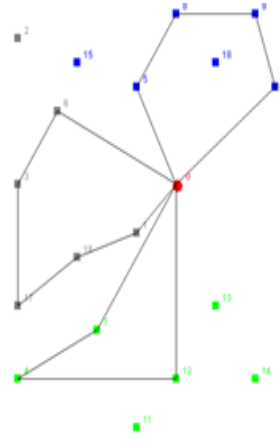
t = 1



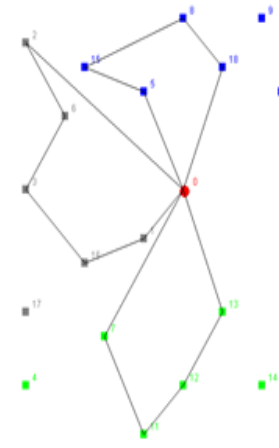
t = 2



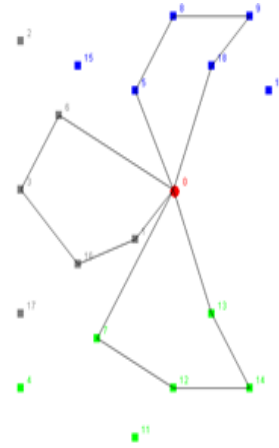
t = 3



t = 4



t = 5

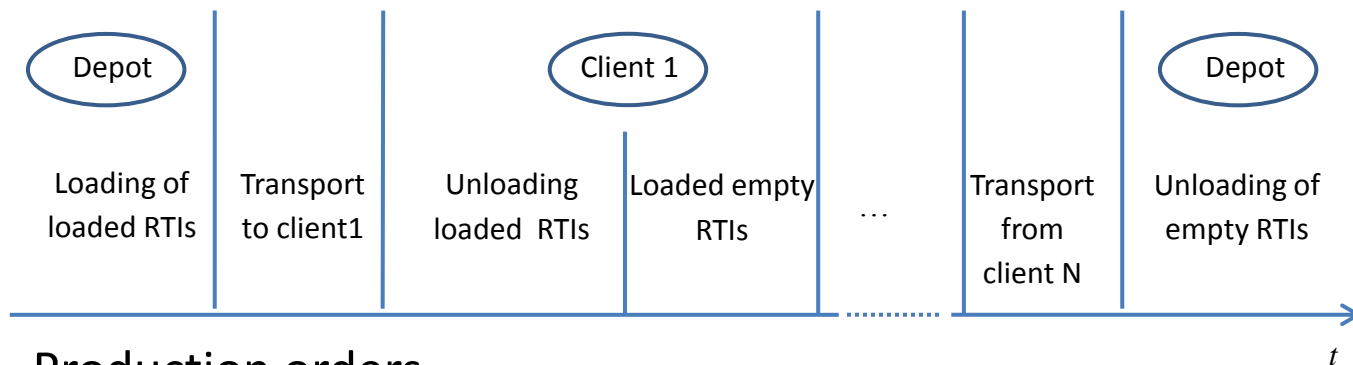


Clients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Demands	30	10	30	10	20	20	25	10	15	5	10	20	5	10	20	15	5	5

Time: 928s

# Simulation: Resource–Action–Operation

- permanent resources describe the layout of the network
  - Supply chain (global data)
  - Actors: Depot or Clients (demand, inventory capacity, inventory level)
  - Vehicles (status: free-stop-active, quantity of empty and loaded RTIs, speed)
- temporary resources represent the exchanges of information and material flows
  - Transport orders



- Production orders

# Simulation: Resource–Action–Operation

- The solution provided by the MIP
- Impact of demand variability (5% - 10% - 15%) on clients' satisfaction
- 100 experiments for each case

	0%	5%	10%	15%
RTIs demand	600	600.49	599.87	600.07
RTIs consumed	600	595.44	585.08	581.18
Clients' satisfaction	100%	99.19%	97.53%	96.85%

# Conclusions & Perspectives



- A mixed integer linear program for a specific case of IRPPD
- The validity of this model is tested on small instances
- To solve realistic sized problems:
  - a clustering approach is used to reduce the original problem size;
  - the number of clusters corresponds to the vehicle fleet size;
  - Afterward, the IRPDP is solved for each vehicle.
- Simulation model to take stochasticity into account
- *Improve the heuristic*
- *More experiments*
- *RTI : lost, damaged*
- *Combine the simulation and the optimisation model*





**Thank you for your attention!**  
**Any questions?**

**Acknowledgements:**

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