Inventory-routing problem with pickups and deliveries of RTI in closed-loop supply chain

Galina IASSINOVSKAIA, UCL-Mons, Belgium
Sabine LIMBOURG, QuantOM, University of Liege, Belgium
Fouad RIANE, CIGM, Faculté des Sciences et Technique de Settat, Settat, Morocco
Returnable transport items

RTI: any means of gathering together goods, transporting them, storing them, to protect them within chains, which is reusable, such as pallets, reusable packaging such as crates, containers, roll containers.

- Roll container
- Crate
- Dolly
- Pallets
- Barrels
- Bottles

International Council for Reusable Transport Items
Why RTI?

• The increased concerns about the environmental impacts of industrial activities and the search for economic advantages have given birth to the concept of a closed-loop supply chain.

• Areas of research and opportunities in the CLSC field are various:
  – remanufacturing
  – component cannibalization
  – recycling
  – sustainable packaging
  – ...

Packaging and packaging waste directives


• The Directive‘ 94 was amended in 2004 (European Parliament and Council, 2004/12/EC)

• Main points:
  – Prevention: if packaging waste can be prevented, then there is no need to recycle or recover packaging and packaging waste.
  – Producer responsibility: there is a responsibility on the producers of packaging to consider what is going to happen to the packaging at the end of its life cycle. The packaging can then be designed for easy disposal or recovery with a minimal environmental impact.
Problem faced by Renault

- Producer distributes its products using RTIs to a set of clients.
- Producer collects of the empty RTI for reuse.
- Each partner has a storage area composed of:
  - the empty RTI stock
  - the loaded RTI stock
- Characterized by:
  - the maximum storage capacity
  - out-of-stocks are not allowed
- Homogeneous fleet of vehicles that can carry simultaneously empty and loaded RTI

Inventory-routing problem with pickups and deliveries (IRPPD)
State of art

• Coelho, Cordeau, and Laporte, 2013, Thirty Years of Inventory Routing Transportation Science, Articles in Advance, pp. 1–19, INFORMS


No LP for the vehicle routing and inventory management in a closed-loop supply chain
Variables

- \( y_{ijvt} = 1 \) iff \((i,j)\) is used on the route of \( v \) in period \( t \)
- \( I_{it}^L \) (\( I_{it}^E \)): inventory level of loaded (empty) RTI at \( i \) at the end of \( t \)
- \( q_{it} \) (\( r_{it} \)): loaded (empty) RTI quantity delivered (returned) to (from) client \( i \) in period \( t \)
- \( x_{ijt} \) (\( z_{ijt} \)): loaded (empty) RTI quantity transported from \( i \) to \( j \) in period \( t \)
- \( f_t \): RTI quantity filled from the producer in period \( t \)
- \( n_t \): new RTI quantity bought and filled from the producer in period \( t \)
- \( T_{vt} \): represents the overtime worked by the vehicle \( v \)
Objective

Inventory routing problem with Simultaneous Delivery and Pickup in a closed-loop (IRPSDPCL)

\[
\text{minimize } \sum_{i \in N} \sum_{j \in N} \sum_{t \in T} \left( \alpha \sum_{v \in V} y_{ijvt} + \beta (w_L x_{ijt} + w_E z_{ijt}) \right) d_{ij} \\
+ \sum_{i \in N} \sum_{t \in T} (h_i^L l_i^L + h_i^E l_i^E) + \sum_{t \in T} c (f_t + n_t) + \sum_{t \in T} b n_t + \sum_{v \in V} \sum_{t \in T} \delta T_{tv}
\]
Constraints

Vehicle capacity

\[ x_{ijt} + z_{ijt} \leq Q \times \sum_{v \in V} y_{ijvt} \]
\[ \forall (i,j) \in A, \forall v \in V, \forall t \in T \]

Inventory balance for the loading and empty RTI at each client and at the depot (closed-loop chain)

\[ I_{it}^L = I_{it-1}^L + q_{it} - u_{it} \]
\[ I_{it}^E = I_{it-1}^E - r_{it} + u_{it} \]
\[ I_{0t}^L = I_{0t-1}^L + F_t - \sum_{i \in N} q_{it} \]
\[ I_{0t}^E = I_{0t-1}^E - F_t + n_t + \sum_{i \in N} r_{it} \]
\[ \forall i > 0 \in N, \forall t > 0 \in T \]
Constraints

Inventory capacity of loaded and empty RTI held by each client throughout all periods (no out-of-stocks)

- \( 0 \leq I^L_{it} \leq C^L_i \)
- \( 0 \leq I^E_{it} \leq C^E_i \)

\[ \forall i > 0 \in N, \forall t > 0 \in T \]

Producer capacity (number of empty plus the number of bought RTI)

- \( f_t \leq I^E_{0t-1} + n_t \)

\[ \forall t > 0 \in T \]

Loaded RTI are delivered and empty RTI are returned

- \( \sum_{i \in N, i \neq j} (x_{ijt} - x_{jit}) = q_{jt} \)
- \( \sum_{i \in N, i \neq j} (z_{ijt} - z_{jit}) = r_{jt} \)

\[ \forall j > 0 \in N, \forall t > 0 \in T \]
Constraints

Proper vehicle routes:

- if a vehicle $v$ visits client $j$ in period $t$, it has to leave client $j$ in period $t$ at the most a vehicle visits a client per period
  \[
  \sum_{i \in N, i \neq j} y_{ijvt} - \sum_{i \in N, i \neq j} y_{jivt} = 0
  \]
  \[\forall j \in N, \forall v \in V, \forall t \in T\]

- at the most a vehicle visits a client per period
  \[
  \sum_{i \in N} \sum_{v \in V} y_{ijvt} \leq 1
  \]
  \[\forall j > 0 \in N, \forall t \in T\]

- vehicle leaves the depot once per period or stays at the depot
  \[
  \sum_{j > 0 \in N} y_{0jvt} \leq 1
  \]
  \[\forall j > 0 \in N, \forall v \in V, \forall t \in T\]

Overtime

\[
\sum_{i \in N} \sum_{j \in N} \left(\frac{d_{ij}}{s} + g\right) y_{ijvt} \leq L + T_{vt}
\]
\[\forall v \in V, \forall t \in T\]
Constraints

• Quantity of empty RTI returned is held in the inventory
  \[ r_{it} \leq I_{it}^E \quad \forall i > 0 \in N, \forall t > 0 \in T \]

• Quantity of loaded RTI delivered was held in the inventory in the previous period
  \[ q_{it} \leq I_{it-1}^L \quad \forall i > 0 \in N, \forall t > 0 \in T \]

• Quantity of empty RTI returned at the producer is held in its inventory
  \[ \sum_{i \in N} r_{it} \leq I_{0t}^E \quad \forall t > 0 \in T \]

• Quantity of loaded RTI delivered at the producer was held in the inventory in the previous period
  \[ \sum_{i \in N} q_{it} \leq I_{0t-1}^L \quad \forall t > 0 \in T \]

• + Non-negativity and binary conditions
### Small instance

<table>
<thead>
<tr>
<th></th>
<th>Producer</th>
<th>C 1</th>
<th>C 2</th>
<th>C 3</th>
<th>C 4</th>
<th>C 5</th>
<th>C 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily demands</td>
<td></td>
<td>30</td>
<td>10</td>
<td>30</td>
<td>10</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Capacity of loaded RTIs</td>
<td>300</td>
<td>30</td>
<td>10</td>
<td>30</td>
<td>10</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Capacity of empty RTIs</td>
<td>300</td>
<td>30</td>
<td>10</td>
<td>30</td>
<td>10</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Inventory levels (t=0)</td>
<td></td>
<td>150</td>
<td>30</td>
<td>10</td>
<td>30</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>of loaded RTIs</td>
<td></td>
<td>50</td>
<td>30</td>
<td>10</td>
<td>30</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Inventory costs of</td>
<td>0.015</td>
<td>0.035</td>
<td>0.035</td>
<td>0.035</td>
<td>0.035</td>
<td>0.035</td>
<td>0.035</td>
</tr>
<tr>
<td>loaded RTIs (€/item)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inventory costs of</td>
<td>0.01</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>empty RTIs (€/item)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- 2 vehicles with capacity=120
- Fixed transportation cost: 0.8 €/km
- Variable transportation cost: 0.02€/km.item
- The total travel time is limited to 8 hours
- Overtime cost=2*regular cost
- Stop: 30 minutes
- Truck’s speed = 50 km/h
- Production cost= 0.02€/item,
- New RTI= 1€
- Number of periods=5
Results

If inventory capacity = daily demand ➔ VRP
## Variation of the inventory capacity

<table>
<thead>
<tr>
<th></th>
<th>Capacity in terms of demand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Transportation cost</td>
<td>2452</td>
</tr>
<tr>
<td>Inventory cost</td>
<td>48</td>
</tr>
<tr>
<td>Production cost</td>
<td>9</td>
</tr>
<tr>
<td>New RTI cost</td>
<td>200</td>
</tr>
<tr>
<td><strong>Total cost</strong></td>
<td><strong>2709</strong></td>
</tr>
<tr>
<td>Time (s)</td>
<td>1</td>
</tr>
</tbody>
</table>
Exact method: Results

Inventory capacity = 3 * daily demand
The total travel time of each vehicle per period is limited to 8 hours
Exact method: Results

<table>
<thead>
<tr>
<th>Max travel time</th>
<th>Transportation cost</th>
<th>Inventory cost</th>
<th>Production cost</th>
<th>New RTI cost</th>
<th>Total cost</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1473</td>
<td>62.9</td>
<td>9</td>
<td>450</td>
<td>1994.9</td>
<td>726</td>
</tr>
<tr>
<td>7</td>
<td>1478.6</td>
<td>62.3</td>
<td>9</td>
<td>450</td>
<td>1999.9</td>
<td>732</td>
</tr>
</tbody>
</table>
Clustering

minimize \( \sum_{i \in N} \sum_{j \in N} \sum_{v \in V} d_{ij} \varphi_{ijv} \)

Subject to:
\( \sum_{v \in V} \chi_{iv} = 1 \quad \forall \ i \in N \)
\( \sum_{i \in N} \chi_{iv} u_i \leq pQ \quad \forall \ v \in V \)
\( \varphi_{ijv} \leq \chi_{iv} \)
\( \varphi_{ijv} \leq \chi_{jv} \)
\( \varphi_{ijv} \geq \chi_{iv} + \chi_{jv} - 1 \)
\( \chi_{iv}, \varphi_{ijv} \in \{0, 1\} \quad \forall \ i, j \in N, v \in V \)
Clustering

minimize \( \sum_{i \in N} \sum_{j \in N} \sum_{v \in V} d_{ij} \varphi_{ijv} \)

Subject to:
\( \sum_{v \in V} \chi_{iv} = 1 \quad \forall i \in N \)
\( \sum_{i \in N} \chi_{iv} u_i \leq pQ \quad \forall v \in V \)
\( \varphi_{ijv} \leq \chi_{iv} \)
\( \varphi_{ijv} \leq \chi_{jv} \)
\( \varphi_{ijv} \geq \chi_{iv} + \chi_{jv} - 1 \)

\( \chi_{iv}, \varphi_{ijv} \in \{0,1\} \quad \forall i, j \in N, v \in V \)
## Heuristic: Results

### Method Comparison Table

<table>
<thead>
<tr>
<th>Method</th>
<th>Transportation cost</th>
<th>Inventory cost</th>
<th>Production cost</th>
<th>New RTI cost</th>
<th>Total cost</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>1478.6</td>
<td>62.3</td>
<td>9</td>
<td>450</td>
<td>1999.9</td>
<td>732</td>
</tr>
<tr>
<td>Heuristic</td>
<td>1523.48 (+3%)</td>
<td>63.48 (+2%)</td>
<td>9.2 (+2%)</td>
<td>510 (+13%)</td>
<td>2106.16 (+5%)</td>
<td>2</td>
</tr>
</tbody>
</table>

### Diagrams

- **t = 1**
- **t = 2**
- **t = 3**
- **t = 4**
- **t = 5**
Heuristic: Results

\[ t = 1 \]

\[ t = 2 \]

\[ t = 3 \]

\[ t = 4 \]

\[ t = 5 \]

<table>
<thead>
<tr>
<th>Clients</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demands</td>
<td>30</td>
<td>10</td>
<td>30</td>
<td>10</td>
<td>20</td>
<td>20</td>
<td>25</td>
<td>10</td>
<td>15</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>15</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Time: 928s
Simulation: Resource–Action–Operation

• permanent resources describe the layout of the network
  – Supply chain (global data)
  – Actors: Depot or Clients (demand, inventory capacity, inventory level)
  – Vehicles (status: free-stop-active, quantity of empty and loaded RTIs, speed)

• temporary resources represent the exchanges of information and material flows
  – Transport orders

– Production orders
Simulation: Resource–Action–Operation

- The solution provided by the MIP
- Impact of demand variability (5% - 10% - 15%) on clients’ satisfaction
- 100 experiments for each case

<table>
<thead>
<tr>
<th></th>
<th>0%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>RTIs demand</td>
<td>600</td>
<td>600.49</td>
<td>599.87</td>
<td>600.07</td>
</tr>
<tr>
<td>RTIs consumed</td>
<td>600</td>
<td>595.44</td>
<td>585.08</td>
<td>581.18</td>
</tr>
<tr>
<td>Clients’ satisfaction</td>
<td>100%</td>
<td>99.19%</td>
<td>97.53%</td>
<td>96.85%</td>
</tr>
</tbody>
</table>
Conclusions & Perspectives

• A mixed integer linear program for a specific case of IRPPD
• The validity of this model is tested on small instances
• To solve realistic sized problems:
  – a clustering approach is used to reduce the original problem size;
  – the number of clusters corresponds to the vehicle fleet size;
  – Afterward, the IRPDP is solved for each vehicle.
• Simulation model to take stochasticity into account

• Improve the heuristic
• More experiments
• RTI : lost, damaged
• Combine the simulation and the optimisation model
Thank you for your attention!
Any questions?

Acknowledgements:
This work was initiated as part of a research project “FEDER : MEDIATIC – TRACEMEDIA” funded by the Walloon Region and the European Union