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Mechanical behaviour of chalk reservoir: Numerical modelling of water sensitivity and time dependence effects

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The paper proposes two numerical models ot the mechanical behaviour of chalk. First, an elasto-plastic constitutive law reproduces the different plastic mechanisms of the chalk and the influence of pore fluids by introducing the concept of suction. Secondly, a model able to reproduce time dependence behaviour. Eventually, the paper gives a simulation of an academic reservoir during depletion and waterflooding phase.
Der Artikel stellt zwei numerische Modelle des mechanischen Verhaltens der Kreide vor. Das erste ist ein elasto-plastisches Modell, das verschiedene plastische Mechanismen der Kreide und den Einfluss von Porenflüssigkeit, unter dem Begriff der Suktion, reproduziert. Das zweite Modell berücksichtigt darüberhinaus das zeitabhängige Verhalten. Schliesslich zeigt die Veröffentlichung eine Simulation einer theoretischen Lagerstätte während der Entleerung und der Wasserinjektion.
L'article propose deux modèles numériques du comportement mécanique de la craie. Le premier est une loi élasto-plastique qui reproduit les différents mécanismes plastiques de la craie et l'influence du fluide interstitiel par l'introduction de la notion de succion. Le second est un modèle prenant en compte les effets différés. Enfin, l'article présente une simulation d'un réservoir académique durant les phases de déplétion et d'injection d'eau.

Introduction

Compaction of chalky reservoirs during oil extraction, implying seabed subsidence, and other important problems are related to the mechanical properties of chalk.
The most remarcable behaviour of chalk is its water-sensitivity: water injection in the reservoir induce additional subsidence. Though many studies have been already performed on chalks, the basis mechanism of the water sensitivity was not defined. The ongoing EC Research Program Pasachalk aims to give a better understanding of the chalk behaviour. The origin of the research comes from the comparison between experimental results obtained on Lixhe chalk and on Jossigny silt which showed that the influence of water on pure high porosity chalk is similar to that on partially saturated soils (Delage *et al.* 1996). Hence the idea appeared to apply the knowledge, the approach, and the tools of the partially saturated soil mechanics to the understanding, description, and modelling of chalk behaviour during changes in saturation fluids, such as when waterflooding.
This paper presents the developed constitutive model, which is a cap type plasticity model coupled with the Barcelona one (Alonso *et al.* 1990) for taking the suction effect into account.
The model parameters are calibrated based on the experimental results. The validation of the model is performed on a waterflooding experiment.
A second model on viscous is also presented ; this is an elasto-visco-plastic model following Perzyna's approach. First numerical results are shown and some comparisons with experiment are given.
Unsaturated flow formulation is necessary here as far as suction has to be known. The flow model used is based on works in relation with the problem of nuclear waste disposal (Collin *et al.* 2002a). For each fluid (Water and oil), balance equations and state equations are written. In partial saturation conditions, the permeability and the storage law have to be modified: a generalised Darcy's law defines the fluid motion (Bear 1972). Numerous couplings existing between mechanics and flows are considered.

Elasto-plastic model

Experiments performed on chalk samples have shown two plastic mechanisms: the pore collapse for high mean stresses (contractant behaviour) and the frictional failure for low mean stresses. The pore collapse could be caused by the breakdown of physico-chemical bonds between the grains inducing some grain-to-grain slip (Monjoie *et al.* 1990). The frictional failure corresponds to a plastic distortion inducing an increase of porosity.
The two-evidenced plastic mechanisms are modeled by two yield surfaces combined within a cap model: the modified Cam-Clay model is used for pore collapse whereas an internal friction model for friction failure (Collin *et al.* 2002b). Experimental results show that the chalk strength under extension can be overestimated using an internal friction model, a third yield surface is then adopted to limit traction stresses.
Obviously, the so-defined yield curve is not continuously derivable at the intersections, leading to numerical difficulties. However, recent publications provide an elegant way to solve this problem (Simo & Hugues 1998).

As far as the suction effect is concerned, the model adopts the approach developed in the Barcelona Basic Model (Alonso *et al*. 1990) where the suction is considered as an independent variable. Suction modifies yield surfaces and produces reversible and irreversible deformations.
The mechanical model is expressed in terms of the following stress invariants and suction:

$$I_\sigma = \sigma_{ii} \tag{1}$$

$$II_{\hat\sigma} = \sqrt{\frac{1}{2}\hat\sigma_{ij}\hat\sigma_{ij}}\,, \hat\sigma_{ij} = \sigma_{ij} - \frac{I_\sigma}{3}\delta_{ij} \tag{2}$$

$$III_{\hat\sigma} = \frac{1}{3}\hat\sigma_{ij}\hat\sigma_{jk}\hat\sigma_{ki} \tag{3}$$

$$\beta = -\frac{1}{3}\sin^{-1}\left(\frac{3\sqrt{3}}{2}\frac{III_{\hat\sigma}}{II_{\hat\sigma}^{\ 3}}\right) \tag{4}$$

$$s = p_o - p_w \tag{5}$$

Where β is the Lode angle, s is the suction, p_o and p_w are the oil and water pressures.

General formulation

The general elastoplastic relations are formulated in rate form. The strain rate is composed of a mechanical part (superscript m) and of suction one (superscript s). Each contribution is partitioned in an elastic (superscript e) and a plastic component (superscript p):

$$\dot\varepsilon_{ij} = \dot\varepsilon_{ij}^{m,e} + \dot\varepsilon_{ij}^{s,e} + \dot\varepsilon_{ij}^{m,p} + \dot\varepsilon_{ij}^{s,p} \tag{6}$$

The mechanical elastic part is related to the Jaumann objective stress rate through Hooke's law. For the plastic parts, a general framework of non-associated plasticity is adopted in order to limit dilatancy. In that case, the plastic flow rate is derived from a plastic potential ga:

$$\dot\varepsilon_{ij}^{m,p} = \dot\lambda^p \frac{\partial g_\alpha}{\partial \sigma_{ij}}, \tag{7}$$

where λ^p is a scalar multiplier and g_α is the plastic potential related to the plastic mechanism α.
Elastic and plastic deformations related to suction changes are defined following expressions given in Barcelona Basic Model. Irreversible deformations are induced when the suction becomes higher than a suction level s_0.

$$\dot\varepsilon_{ij}^{s,e} = \frac{\kappa_s}{(1+e)}\frac{\dot s}{(s+p_{at})}\delta_{ij} = h_{ij}^e \dot s \tag{8}$$

$$\dot\varepsilon_{ij}^{s,p} = \frac{\lambda_s - \kappa_s}{(1+e)}\frac{\dot s}{(s+p_{at})}\delta_{ij} = h_{ij}^p \dot s \tag{9}$$

Where e is the void ratio, p_{at} is the atmospheric pressure, κ_s and λ_s are elastic and plastic coefficients.
Combining equations (6) to (9), one obtains the following expression:

$$\tilde\sigma_{kl} = C_{klij}^e\left(\dot\varepsilon_{ij} - h_{ij}^e\dot s - \dot\lambda^p\frac{\partial g_\alpha}{\partial\sigma_{ij}} - h_{ij}^p\dot s\right) \tag{10}$$

Considering a general hardening/softening plastic law depending on the internal variable ζ, the consistency condition and equation (10) give the expression of multiplier λ^p and the stress rate can be computed:

$$\tilde\sigma_{kl} = (C_{klij}^e - C_{klij}^p)\dot\varepsilon_{ij} - M_{kl}\dot s \tag{11}$$

The first term of the right part is the classical expression of an elastoplastic formulation. The second term is related to the suction.

CamClay pore collapse model

The Modified Cam-Clay yield surface is defined by the following expression:

$$f_1 \equiv II_{\hat\sigma}^2 + m^2\left(I_\sigma + \frac{3c}{\tan\phi_C}\right)(I_\sigma - 3p_0) = 0 \tag{12}$$

Where c is the cohesion, ϕ_C is the friction angle in compression path, p_0 is the preconsolidation pressure, which defines the size of the yield surface, and m is a coefficient introduced to take into account the effect of the third stress invariant.
The plastic flow is supposed to be associated and the internal variable is the pre-consolidation pressure p_0, which is related to the volumetric plastic deformations devp following the kinematic equation:

$$dp_0 = \frac{1+e}{\lambda - \kappa}p_0 d\varepsilon_v^p \tag{13}$$

where λ is the compression coefficient and κ is the elastic coefficient.

Internal friction model

A more sophisticated model can be built from the Drucker-Prager's cone by introducing a dependence on the Lode's angle β, in order to match more closely the Mohr-Coulomb criterion. It consists of a smoothed Mohr-Coulomb plasticity surface. The formulation based on the idea of Van Eekelen (1980) is used. It can be written in a very similar way to the Drucker-Prager's criterion:

$$f_2 \equiv II_{\hat\sigma} - m\left(I_\sigma + \frac{3c}{\tan\phi_C}\right) = 0 \tag{14}$$

A non-associated plasticity is considered here using a plastic potential definition similar to Eq. 14 where the dilatancy angle y is used instead of the frictional angle.
The internal variables of the model are the frictional angles ϕ_C (for compression paths), ϕ_E (for extension paths) and the cohesion c which are related to the equivalent plastic strain.

Suction effect on yield surface

Several phenomena are usually evidenced for unsaturated soils:

1. The preconsolidation pressure p_0 and the material stiffness increase with suction. This is described by the LC concept of the Barcelona model (Alonso *et al*, 1990):

$$p_0(s) = p_c \left(\frac{p_0^*}{p_c} \right)^{\frac{\lambda(0)-\kappa}{\lambda(s)-\kappa}} \tag{15}$$

where p_0^* is the preconsolidation pressure for null suction, p_c is a reference pressure, $\lambda(0)$ is the compression coefficient at zero suction, $\lambda(s)$ is the compression coefficient at suction s.

However, chalk experiment does not show an increase of plastic stiffness with suction i.e. water saturated chalk and oil satured chalk present the same plastic compressibility (Schroeder *et al*. 2000). Another expression of LC curve has been adopted:

$$p_0(s) = p_0(0) + \Delta p_0 \frac{s}{s+s^*} \tag{16}$$

Where Δp_0 is the preconsolidation pressure difference between water and oil saturated sample, and s^* is a numerical parameter.

2. Cohesion increases with suction, this is model-led using Eq. 17. The influence of suction on friction angle depends on the material studied. Experiment on chalk shows that friction angle is independent of the saturating fluid.

$$c(\mathrm{s}) = c(0) + k\, s \tag{17}$$

where k is a material constant, $c(0)$ is the cohesion at saturated state.

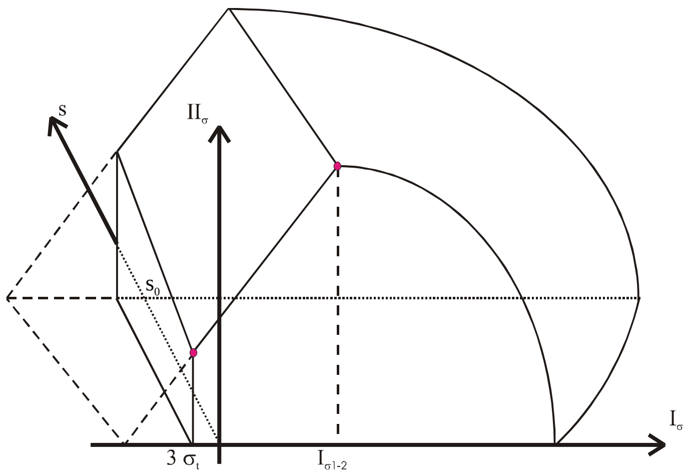

Figure 1. Cap model in the stress space

3. Suction changes may create irreversible strains. In the Barcelona model, this is modelled thanks a yield surface, the SI "Suction Increase" curve. When suction becomes higher than a suction level s_0, plastic strains are created. This yield criterion is introduced in our constitutive law:

$$f_4 \equiv s - s_0 = 0 \tag{18}$$

Figure 1 presents all the yield surfaces in the stress space.

Waterflooding experiment

The water-flooding test performed on Lixhe chalk initially saturated by SoltrolTM (Schroeder & al. 1998) allows validation of the developed model.

The sample with an initial porosity n = 40.55 % and permeability k_{int} = 1mDarcy has dimensions of 25 mm of diameter and 50 mm of height. Four strain gauges are glued on the sample (located respectively from the injection side at a distance of 4, 12, 22 and 30 mm), which aims to monitore the evolution of the axial deformation with the waterfront. In addition, an axial LVDT records the global axial deformation.

The initial stress state is isotropic at a level of 18 MPa, just below the expected pore collapse for 'oil-like' plug. The injection water pressure is equal to 0.9 MPa. Just before the injection front, a small swelling is measured by the strain gauges but a brutal and quasi-instantaneous compaction appears at the waterfront. The final amplitude of the compaction is around 2%-3%.

Experiment results of triaxial tests on saturated samples allow us to define the yield surface of oil-like and water-like plugs. The transition between these two cases is characterized by results of suction controlled oedometer tests.

The parameters of the mechanical model are presented in Table 1.

Table 1. Parameters of the Cap model

	Parameter	Value	Unit
Elasticity			
	K_{water}	612	MPa
	G_{water}	500	MPa
	K_{oil}	726	MPa
	G_{oil}	700	MPa
Frictional mechanism			
	ϕ_{water}	25	°
	ϕ_{oil}	25	°
	c_{water}	1.5	MPa
	c_{oil}	2.0	MPa
	k	0.167	-
CamClay + suction LC			
	$p_{0,water}$	10	MPa
	$p_{0,oil}$	22	MPa
Hardening rule			
	λ	0.195	-
	s*	0.2	MPa

The comparison of the numerical results with the experimental data is shown in Figure 2. The injected water volume evolution is similar. After 3500 sec, the waterfront reaches the top; no more oil is driven out of the sample and water is produced at the top. The computed axial strains at the four gauges present a small swelling followed by a brutal collapse of around 2.5% (Figure 3).

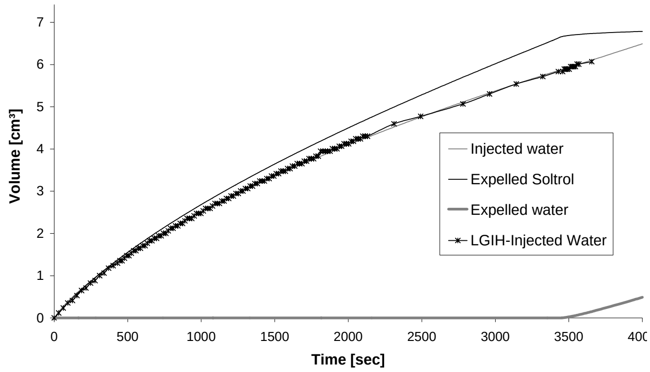

Figure 2. Fluid exchange during water flooding

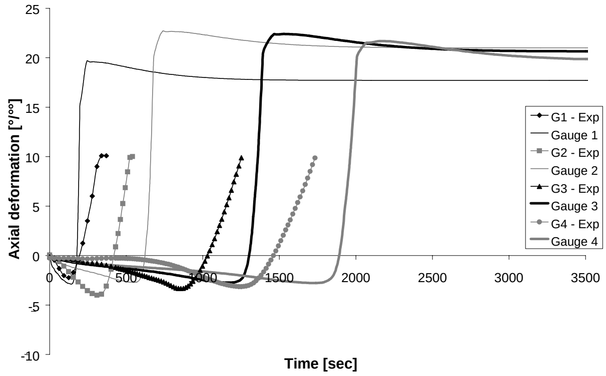

Figure 3. Axial strains at the four gauges

The experiment value of strain stops at 1%, which corresponds to the gauge failure.
The good consistence obtained between the numerical simulation and the experimental responses shows the validity of the developed model for describing the chalk behaviour during water injection.

Elasto-viscoplastic model

From triaxial tests performed at various stress rates, there is experimental evidence that viscous effect in chalk deformation may be important. An elastoviscoplastic model for fully saturated chalk has been first developped (Charlier *et al.* 2001). The Perzyna's elastic viscoplastic theory has been adopted based on the following motivations:

- the formulation is well accepted and well used
- the generality of the time-rate flow rule offers the capability of simulating time-dependent material behaviour over a wide range of loading
- it gives a possibility to take advantage of the inviscid multisurface cap-failure-tension model, developed for elastoplasticity modelling
- the formulation is readily adaptable to a numerical algorithm suitable for finite element procedure

The suction effects are now included in a new model, strains are divided in reversible and irreversible parts:

$$\underline{\dot{\varepsilon}} = \underline{\dot{\varepsilon}}^{m,e} + \underline{\dot{\varepsilon}}^{m,vp} + \underline{\dot{\varepsilon}}^{s,e} + \underline{\dot{\varepsilon}}^{s,vp} \tag{19}$$

As explained before, plastic deformation related to suction changes have not been experimentally evidenced. Total strains have thus three components and the following consitutive equation holds:

$$\underline{\dot{\sigma}} = \underline{\underline{C}}^{e}(\underline{\dot{\varepsilon}} - \underline{\dot{\varepsilon}}^{s,e} - \underline{\dot{\varepsilon}}^{m,vp}) \tag{20}$$

The irreversible strain may be described as normal to some potential *g*:

$$\underline{\dot{\varepsilon}}^{vp} = \gamma \langle \phi(f) \rangle \frac{\partial g}{\partial \underline{\sigma}} \tag{21}$$

This formulation is rather similar to the elastoplastic one, unless it is not based on the consistency condition. The amount of strain rate is described with respect to some reference surface *f*, similar to the yield surface.
The cap reference surface is based on the following equations (Shao *et al*. 1993):

$$\langle \phi_c(f_c) \rangle = \left(\frac{f_c}{3 p_0^{\;2}} \right)^{\alpha_c} \tag{22}$$

$$\gamma_c = \omega \left(\frac{I_\sigma}{p_a} \right)^{\iota} \tag{23}$$

where the reference surface f_c is similar to the yield surface of the elastoplastic model (12). The function f_c may here be analyzed as an overstress, or a measure of the amount of the stress state going outside the yield or reference surface. Moreover, the overstress is only positive when the stress state is outside the yield – reference surface (figure 4). As the Mac Aulay brackets are used in (21, 22) equations, irreversible strains exist only in this peculiar situation. The more the overstress, the more irreversible strain rate is.

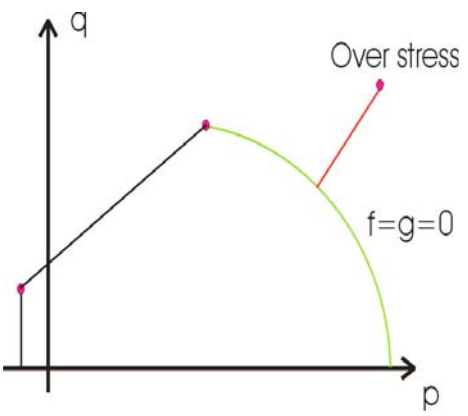

Figure 4 : overstress with respect to the reference surface.

A hardening rule has to be defined. One uses here again the plasticity hardening rule (13).
Similar development may be implemented for the other plastic mechanisms.

The figure 5 presents isotropic compression experimental results and the proposed constitutive model responses for a wide range of stress rate. One can observe that the model is able to reproduce quantitatively the chalk behaviour for the different rate.

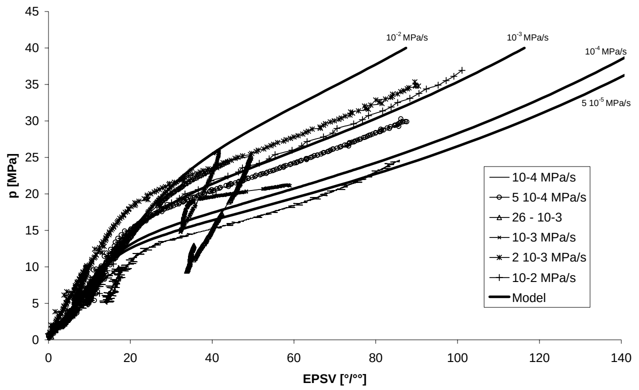

Figure 5 : Calibration curve (model and experiment – Oil saturated sample)

Coupled reservoir modelling

The reservoir model used here is similar to the academic one proposed by Homand (2000) : it is 2.5 km long and 300 m thick. The sideburden and the underburden are assumed to be rigid ; a dead load corresponding to the weight of the overburden acts on the upper edge of the reservoir. The initial total stress condition are those of Ekofisk field i.e. 62 MPa vertical stress and 55 MPa horizontal stress (Teufel *et al*. 1990).

All the reservoir edges are supposed to be impervious : the fluids inflows or outflows come only through the wells. In this computations well pressures are controlled in order to reproduce a depletion and an injection phase. The production scheme is the one proposed by Homand (2000) and presented in Figure 6.

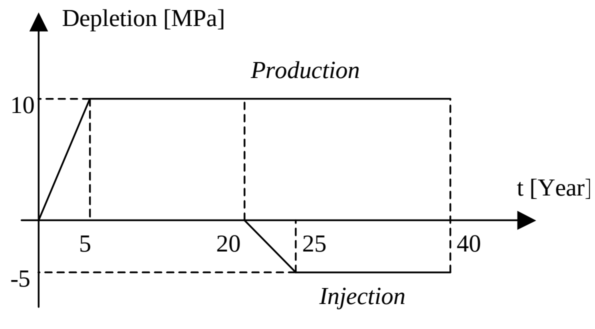

Figure 6. Production scheme

In this model, the injection well is located on the left edge and the distance between the injection and the production well is 2.25km. The initial pressure condition supposed an uniform suction value of 300 kPa (neglecting gravity effect on the reservoir thickness) corresponding to an oil pressure of 49 MPa.

The computation are performed using the elasto-plastic model and the flow model, with the same parameter used in the waterflooding simulation. Except the permeability was modified, related to the scale change between the laboratory sample and the reservoir field.

In the production phase, the oil pressure decrease at well to reach a depletion of 10 MPa after 5 years. Due to hydro-mechanical couplings, this pressure decrease leads to a compaction of the reservoir (increase of the effective stress). Figure 7 shows that compaction is low after 1 year (40 cm). However, up to 5 years, irreversible deformations appears and compaction reaches a value of 13 meters at the production well after 20 years.

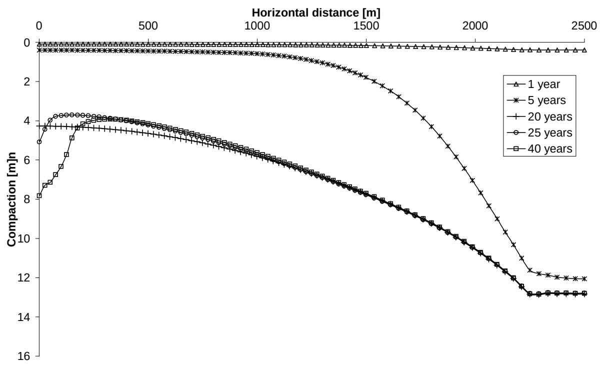

Figure 7. Compaction of the reservoir

When injection phase begins, the water-resaturation compaction is well predicted by the code. Additional deformations appear and are related to decrease of suction at the injection well.

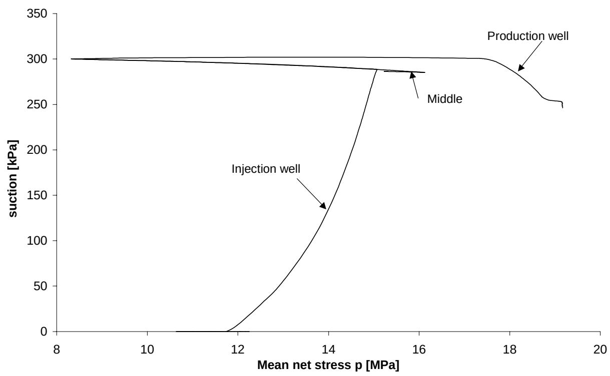

Figure 8. Stress path in the reservoir

Figure 8 presents stress paths (p-s plane) followed in finite elements located near the production well, the injection well and between them. During production phase, stress path in the three points is nearly horizontal : this means that suction does not vary a lot during production phase. When injection phase begins, one observes a brutal decrease of suction at the injection well. This leads to the additional compaction when the stress path reaches the LC curve. However, at the production well and at the midpoint, suction does not vary so much during the time scale considered in this simulation because the water front does not yet reach them.

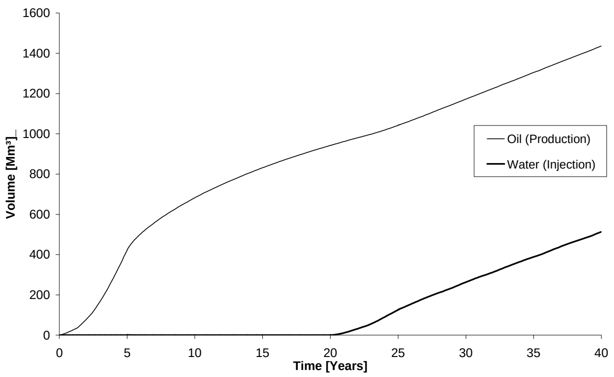

Figure 9. Oil production and water injection volumes

Oil production is clearly affected by the water injection (Figure 9). The oil production rate increases when the injection begins. Indeed the water flows out oil present in the reservoir.
The results show that the constitutive relationship developed within Pasachalk projet is able to reproduce the reservoir compaction related to water injection.

Conclusions

A multimechanism model is developed to describe the chalk behaviour during water flooding. The main assumption is that water sensitivity of chalk could be related to oil-water suction effects. The Barcelona model concepts are integrated into the PASACHALK constitutive law in order to reproduce suction dependency behaviour of chalk.
Experimental results obtained in the framework of Pasachalk project are used to define parameters. The validation of the constitutive law is achieved thanks to the modelling of waterflooding experiment. Comparisons between experimental and numerical results show that our finite element code is able to reproduce the fluid mass transfert and the mechanical water sensitive behaviour of chalk.
This numerical tool may be used for simulations of oil reservoir chalk in order to reproduce the compaction during water injection. The computations presented here correspond to an academic reservoir. The results shows that the model is able to reproduce the compaction during production phase. These deformations are related to an increase of ‘effective’ stress. During injection phase, an additionnal compaction is predicted. Even if the reservoir pressure increases, the suction decrease leads to compressive deformations. Suction is a pertinent mechanism to explain the mechanical behaviour of chalk.
Another important aspect of chalk is its time dependent behaviour.
We are currently developping an elasto-visco-plastic model including suction effects. Some preliminar results have been presented but both experimental and nurical works are still needed in order to have a constitutive law able to deal with the main issues related to chalk!

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