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A semi-analytical sensitivity analysis for multibody systems described using Level Sets

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Introduction – Optimization of a connecting rod

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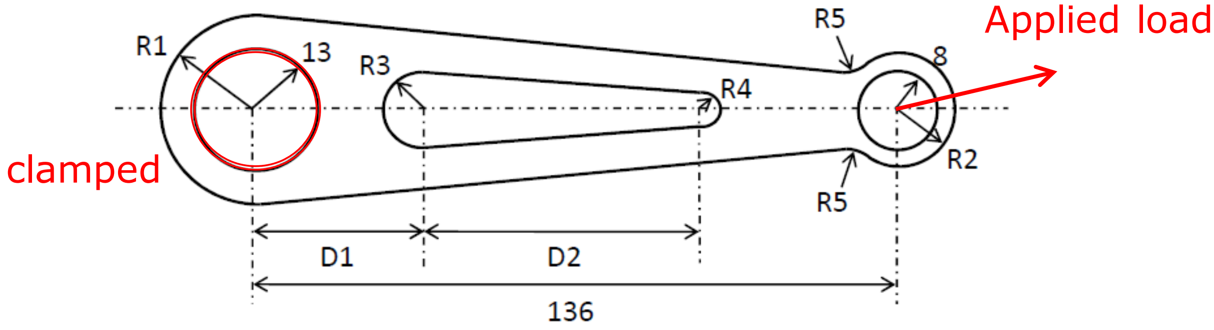
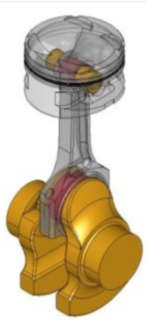
- A component based approach



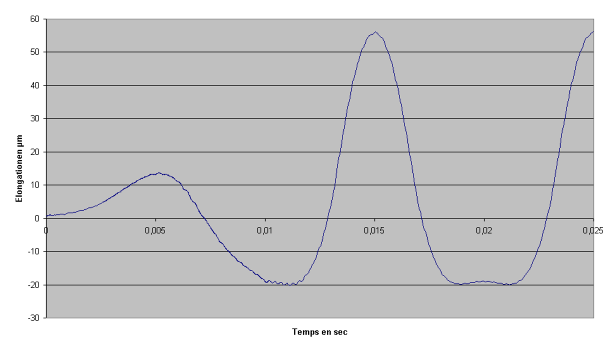
- Experience - Empirical load case - Standard
- Dynamic factor amplification for safety

➔ Not optimal

- Multibody system based approach



Geometrical modeling



Multibody system dynamics

Loop

Optimization process

Different levels of coupling

- Weak coupling
  - Coupling with pre / post processing
  - Define equivalent static load cases (Kang, Park and Arora, 2005)
  - Optimization of isolated components

- Strong coupling
  - Deals with time response
  - Functions may depend on time
  - Engineering approach

➔A global-local approach:
  - The optimization problem can account for global criteria while optimizing local components.

Example: Mass minimization of a vehicle suspension arm while a criteria on the comfort of the driver has to be fulfilled.

Level Set description of the geometry

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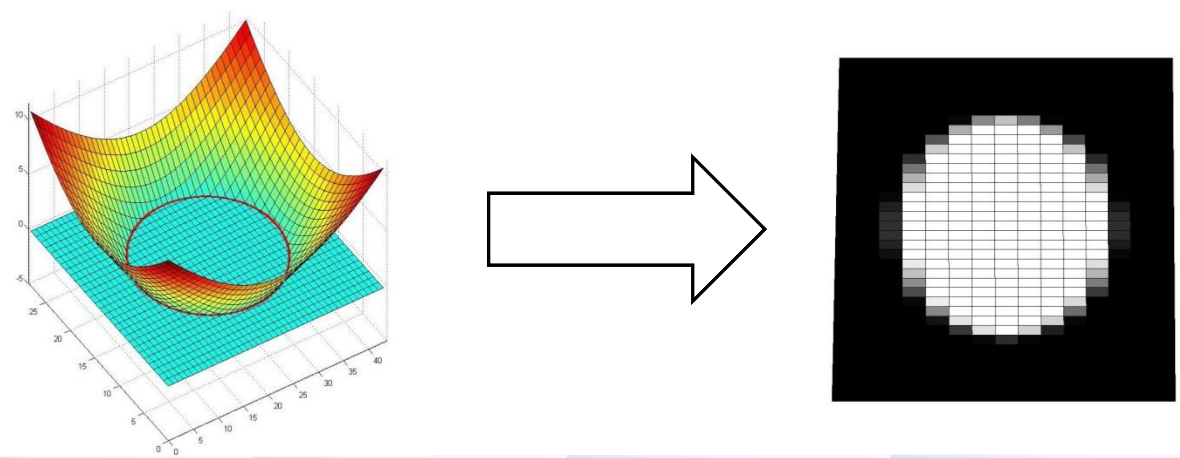
- Fixed mesh grid
- LSF Parameterization:
  - Combination of parameterized geometric shapes (Van Miegroet and Duysinx 2007)
  - A LSF for each geometric features (global basis function).
  - Signed-distance function or analytical function
- The mapping:
  - Eulerian approach (density-based approach)
  - Association of a pseudo-density to each finite element as in TO
  - The element densities are defined based on the value of LSF at nodes
- Example: Square plate with a hole



Smooth transition

Goals of the work and motivations

- Intermediate type of optimization between shape optimization and topology optimization.
  - Fixed mesh grid: No mesh distortion (No velocity field for SA)
  - The geometry is based on CAD entities: can easily be manufactured.
  - Remove, separate, merge entities: Modification of the topology
  - Design variables: parameters of the level sets (rather small number)

- Not the most accurate mapping but… The method presented aims at determining the optimal layout of components when the dynamics of the system is accounted for i.e.:
  - Inertia effects
  - Coupled vibrations
  - Interaction between components…

The MBS problem is already highly non-linear→ Keep the optimization problem simple but efficient as a pre-design tool.

Other methods (EQSL) can then be used for more detailed optimization.

General form of the optimization problem

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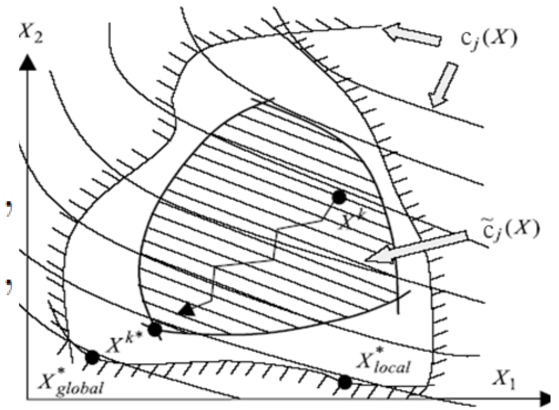
- Design problem casted in a mathematical programming problem

$$\begin{aligned} &\underset{\mathbf{x}}{\text{minimize}} && \varphi(\mathbf{x}) \\ &\text{subject to} && \text{Equilibrium equation} \\ &&& c_j(\mathbf{x}) \le \bar{c}_j, && j = 1, \ldots, n_c, \\ &&& \underline{x}_v \le x_v \le \overline{x}_v, && v = 1, \ldots, n_v, \end{aligned}$$



  - Provides a general and robust framework to the solution procedure
  - Various efficient solvers can be used (ConLin, MMA, IpOpt,…)

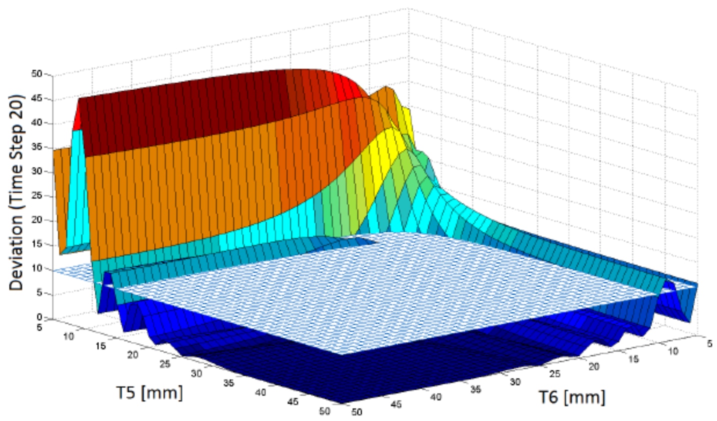
- Formulation using the strong coupling:

$$\underset{\mathbf{x},\mathbf{s}}{\text{minimize}} \quad \varphi(\mathbf{x}, \mathbf{s}) \qquad \mathbf{s} = [\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \boldsymbol{\lambda}]$$

$$\begin{aligned} &\text{subject to} && \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \boldsymbol{\Phi}_q^T(\mathbf{q}, t)\boldsymbol{\lambda} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t), \\ &&& \boldsymbol{\Phi}(\mathbf{q}, t) = \mathbf{0}, \\ &&& c_j(\mathbf{x}, \mathbf{s}, t) \le \bar{c}_j, && j = 1, \ldots, n_c, \\ &&& \underline{x}_v \le x_v \le \overline{x}_v, && v = 1, \ldots, n_v. \end{aligned}$$

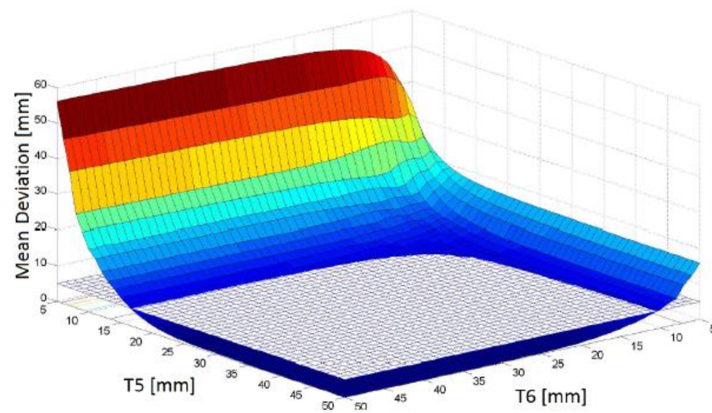
The optimization problem formulation

- The formulation is a key point for this type of problems:
  Highly non-linear behavior
- Impact on the design space: Extremely important for gradient-based algo.



Local formulation

$$\underset{\mathbf{x}}{\text{minimize}} \quad m(\mathbf{x})$$
$$\text{subject to} \quad \Delta l(\mathbf{x}, t_n) \leq \Delta l_{max}$$



Global formulation

$$\underset{\mathbf{x}}{\text{minimize}} \quad m(\mathbf{x})$$
$$\text{subject to} \quad \frac{1}{t_{end}} \sum_{n=1}^{t_{end}} \Delta l(\mathbf{x}, t_n) \leq \Delta l_{max}$$

- Tight control vs number of constraints
- Genetic algorithms
  - Do not necessary give better results
  - Computation time much more important

Equation of FEM-MBS dynamics

- Approach based on the non-linear finite element method
  (Flexibility is naturally taken into account)

- Motion of the flexible bodies is represented by absolute nodal coordinates **q** (Geradin & Cardona, 2001)

- Dynamic equations of multibody system

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) = \mathbf{g}^{ext} - \mathbf{g}^{int} - \mathbf{g}^{gyr}$$

- Subject to kinematic constraints of the motion

$$\mathbf{\Phi}(\mathbf{q}, t) = \mathbf{0}$$

- The solution is based on a Lagrange multiplier method

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{\Phi}_q^T(\mathbf{q}, t)\boldsymbol{\lambda} &= \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) \\ \mathbf{\Phi}(\mathbf{q}, t) &= \mathbf{0}, \end{aligned}$$

with the initial conditions

$$\mathbf{q}(0) = \mathbf{q}_0 \text{ and } \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}_0.$$

Time integration solver

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- Generalized-α
  - Introduction of a vector **a** of acceleration-like variables

$$(1-\alpha_m)\,\mathbf{a}_{n+1}+\alpha_m\mathbf{a}_n=(1-\alpha_f)\,\ddot{\mathbf{q}}_{n+1}+\alpha_f\ddot{\mathbf{q}}_n$$

  - Why?
    - Accurate and reliable results with a small amount of numerical damping (second-order accuracy and linear unconditional stability)
    - Larger range of numerical damping than HHT.

- Newmark integration formulae with **a**

$$\begin{aligned}\mathbf{q}_{n+1} &= \mathbf{q}_n+h\dot{\mathbf{q}}_n+h^2\left(\frac{1}{2}-\beta\right)\mathbf{a}_n+h^2\beta\mathbf{a}_{n+1}\\ \dot{\mathbf{q}}_{n+1} &= \dot{\mathbf{q}}_n+h\,(1-\gamma)\,\mathbf{a}_n+h\gamma\mathbf{a}_{n+1},\end{aligned}$$

- Solve iteratively the linearized dynamic equation system (Newton-Raphson scheme)

$$\begin{aligned}\mathbf{M}\Delta\ddot{\mathbf{q}}+\mathbf{C}_t\Delta\dot{\mathbf{q}}+\mathbf{K}_t\Delta\mathbf{q}+\mathbf{\Phi}_\mathbf{q}^T\Delta\boldsymbol{\lambda} &= \Delta\mathbf{r}\\ \mathbf{\Phi}_\mathbf{q}\Delta\mathbf{q} &= \Delta\mathbf{\Phi}\end{aligned}$$

$$\text{where } \mathbf{r}=\mathbf{M}\ddot{\mathbf{q}}+\mathbf{\Phi}_\mathbf{q}^T\boldsymbol{\lambda}-\mathbf{g}$$

Sensitivity analysis

- General function

$$\Phi(\mathbf{q}(\mathbf{p}), \dot{\mathbf{q}}(\mathbf{p}), \ddot{\mathbf{q}}(\mathbf{p}), \mathbf{p}) \quad \longrightarrow \quad \frac{\partial \mathbf{q}}{\partial \mathbf{p}}, \frac{\partial \dot{\mathbf{q}}}{\partial \mathbf{p}}, \frac{\partial \ddot{\mathbf{q}}}{\partial \mathbf{p}} \ ?$$

  - Implicitely defined through the analysis

- Finite difference? ➔ MBS cpu-time consuming

- Direct or Adjoint Method? Here direct method (fct>dv)

- At a converged time step *t*, the residual is equal to 0:

$$R(\mathbf{q}(\mathbf{p}), \dot{\mathbf{q}}(\mathbf{p}), \ddot{\mathbf{q}}(\mathbf{p}), \mathbf{p}, t) = 0$$

- The total derivative of the residual is

$$\frac{dR}{d\mathbf{p}} = \frac{\partial R}{\partial \mathbf{q}}\frac{d\mathbf{q}}{d\mathbf{p}} + \frac{\partial R}{\partial \dot{\mathbf{q}}}\frac{d\dot{\mathbf{q}}}{d\mathbf{p}} + \frac{\partial R}{\partial \ddot{\mathbf{q}}}\frac{d\ddot{\mathbf{q}}}{d\mathbf{p}} + \frac{\partial R}{\partial \mathbf{p}}$$

Sensitivity analysis

- A semi-analytical method has been developed by O. Brüls and P. Eberhard (2008) which can be integrated in the generalized-α scheme.

$$\mathbf{M}\frac{d\ddot{\mathbf{q}}}{dp_u} + \mathbf{C}_t\frac{d\dot{\mathbf{q}}}{dp_u} + \mathbf{K}_t\frac{d\mathbf{q}}{dp_u} + \mathbf{\Phi}_{\mathbf{q}}^T\frac{d\boldsymbol{\lambda}}{dp_u} = -\frac{\partial \mathbf{r}}{\partial p_u}$$

$$\mathbf{\Phi}_{\mathbf{q}}\frac{d\mathbf{q}}{dp_u} = -\frac{\partial}{\partial p_u}\mathbf{\Phi}$$

  - Sensitivity equations are linear with respect to $\frac{d\mathbf{q}}{dp_u}$ and $\frac{d\boldsymbol{\lambda}}{dp_u}$.

- The computation of the pseudo loads is quite an issue.
  - It requires in general a lot of effort because the matrices of the mechanical system must be computed for many different values.

- In the simulation code, M, C_t and K_t are not computed independently but they are aggregated in the tangent iteration matrix (S_t).

Improving the residual derivative computation

- Rewriting the residual as follows:

$$\frac{dR}{d\mathbf{p}} = \frac{\partial R}{\partial \mathbf{q}}\frac{d\mathbf{q}}{d\mathbf{p}} + \frac{\partial R}{\partial \dot{\mathbf{q}}}\frac{d\dot{\mathbf{q}}}{d\mathbf{p}} + \frac{\partial R}{\partial \ddot{\mathbf{q}}}\frac{d\ddot{\mathbf{q}}}{d\mathbf{p}} + \frac{\partial R}{\partial \mathbf{p}} = 0$$

$$-\frac{\partial R}{\partial \mathbf{q}}\frac{d\mathbf{q}}{d\mathbf{p}} = \frac{\partial R}{\partial \dot{\mathbf{q}}}\frac{d\dot{\mathbf{q}}}{d\mathbf{p}} + \frac{\partial R}{\partial \ddot{\mathbf{q}}}\frac{d\ddot{\mathbf{q}}}{d\mathbf{p}} + \frac{\partial R}{\partial \mathbf{p}}$$

*Derivative of the residual wrt **p** holding **q** fixed.*

- Using the definition of the derivative

$$\lim_{\Delta\mathbf{p}\to\mathbf{0}} = \frac{R(\mathbf{q}(\mathbf{p}), \dot{\mathbf{q}}(\mathbf{p}+\Delta\mathbf{p}), \ddot{\mathbf{q}}(\mathbf{p}+\Delta\mathbf{p}), \mathbf{p}+\Delta\mathbf{p}, t) - R(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \mathbf{p}, t)}{\Delta\mathbf{p}}$$

=0

- Furthermore, we have

$$\dot{\mathbf{q}}(\mathbf{p}+\Delta\mathbf{p}) \approx \dot{\mathbf{q}}(\mathbf{p}) + \frac{d\dot{\mathbf{q}}}{d\mathbf{p}}\Delta\mathbf{p},$$

$$\ddot{\mathbf{q}}(\mathbf{p}+\Delta\mathbf{p}) \approx \ddot{\mathbf{q}}(\mathbf{p}) + \frac{d\ddot{\mathbf{q}}}{d\mathbf{p}}\Delta\mathbf{p}.$$

- The terms $\frac{d\dot{\mathbf{q}}}{d\mathbf{p}}$ are $\frac{d\ddot{\mathbf{q}}}{d\mathbf{p}}$ obtained from the Newmark integration formulae.

More efficient sensitivity analysis

- Gathering the previous developments, one get

$$-\frac{\partial R}{\partial \mathbf{q}}\frac{d\mathbf{q}}{d\mathbf{p}} \approx \frac{1}{\Delta \mathbf{p}} R(\mathbf{q}(\mathbf{p}), \dot{\mathbf{q}}(\mathbf{p}) + \frac{d\dot{\mathbf{q}}}{d\mathbf{p}}\Delta \mathbf{p}, \ddot{\mathbf{q}}(\mathbf{p}) + \frac{d\ddot{\mathbf{q}}}{d\mathbf{p}}\Delta \mathbf{p}, t)$$

- And after development, we end up with

$$-\mathbf{S}_t \frac{d\mathbf{q}}{d\mathbf{p}} = \frac{1}{\Delta \mathbf{p}} R(\mathbf{q}(\mathbf{p}), \dot{\mathbf{q}}(\mathbf{p}) + \frac{d\dot{\mathbf{q}}}{d\mathbf{p}}_{\text{pred}} \Delta \mathbf{p}, \ddot{\mathbf{q}}(\mathbf{p}) + \frac{d\ddot{\mathbf{q}}}{d\mathbf{p}}_{\text{pred}} \Delta \mathbf{p}, t)$$

- Only the tangent iteration matrix is needed
- The computation of the perturbed residual is suitable as the level set description of the geometry is not treated at the element level in the solver.

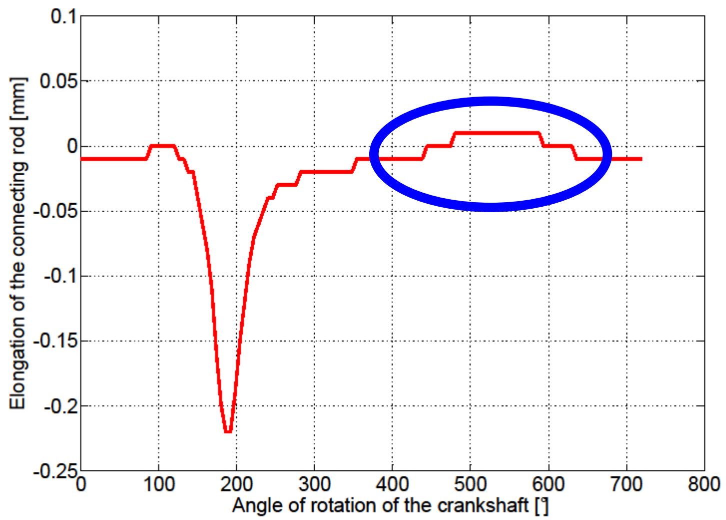
  « Perturb the design variable + Call to the residual function »

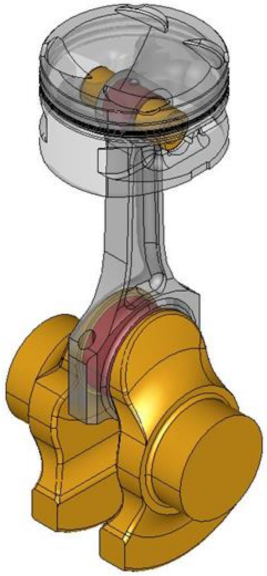
- Very fast evaluation

Numerical Applications

Connecting rod optimization

- Minimization of the connecting rod mass in a real combustion engine (Diesel).
- Elongation of the connecting rod during the exhaust phase
  ➔Collision between the piston and the valves.
- Consideration of one single complete cycle as the behavior is cyclic (720°) for the optimization
- Constraints imposed on the elongation





Local formulation

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$$\min_{\mathbf{x}} m(\mathbf{x})$$

$$s.t. \qquad \Delta l(\mathbf{x}, t_i) \leq \Delta l_{max}$$

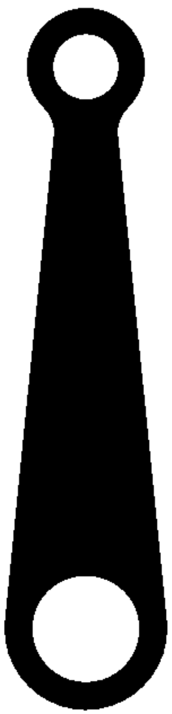
$$\text{with } i = 1, \ldots, \text{nbr time step}$$

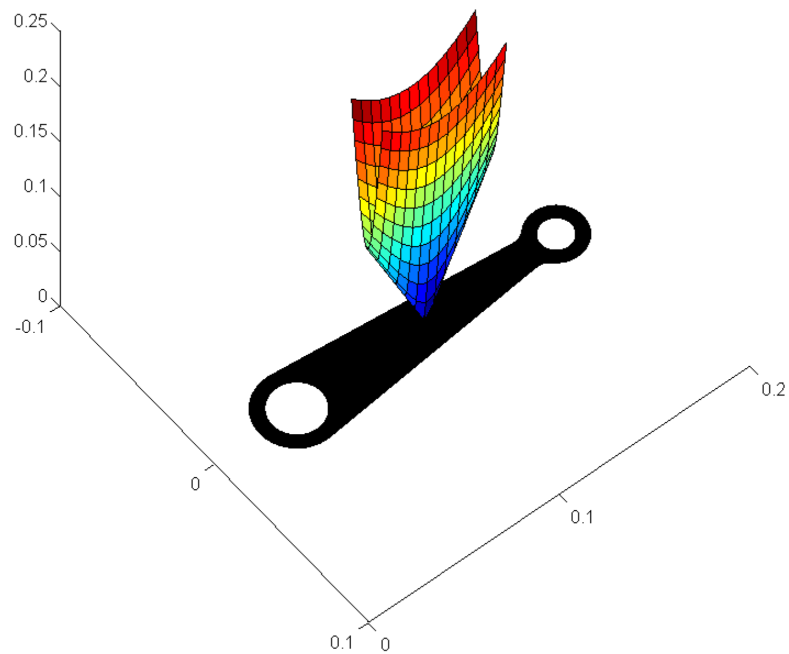
- The elongation constraints $\Delta l(\mathbf{x}, t_i)$ are considered at each time step.
  - ➔ As many constraints as the number of time steps (134)

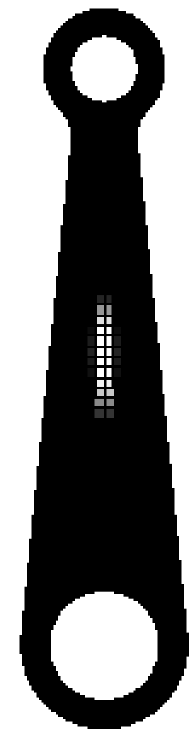
First application – 1 level set

- The level set is defined in order to have an ellipse as interface.
- 5 candidate design variables: a, b, c_x, c_y and d. Here only d is chosen.

$$\Phi(x,y) = \frac{(x-c_x)^2}{a^2} + \frac{(y-c_y)^2}{b^2} - d = 0$$

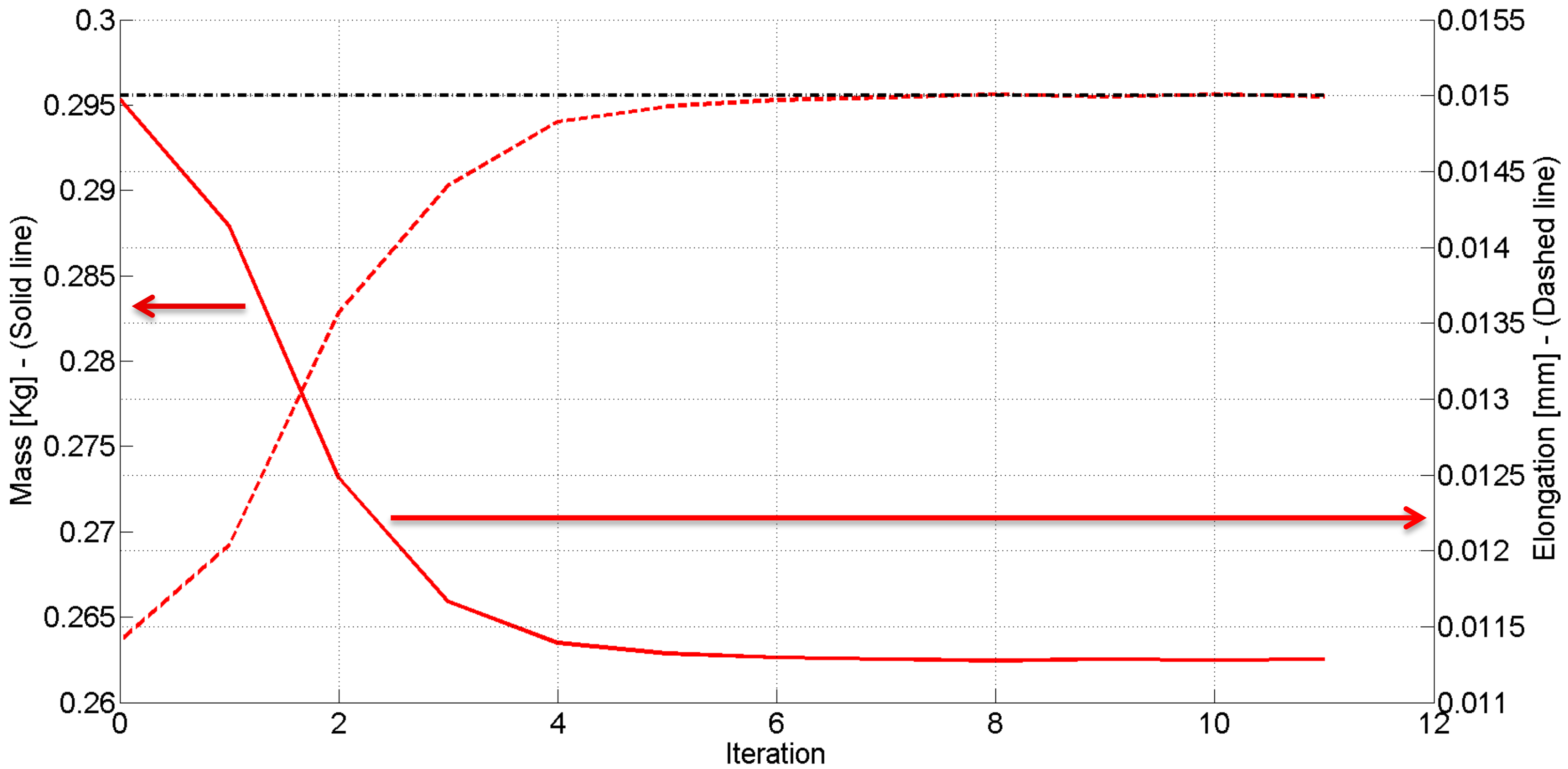






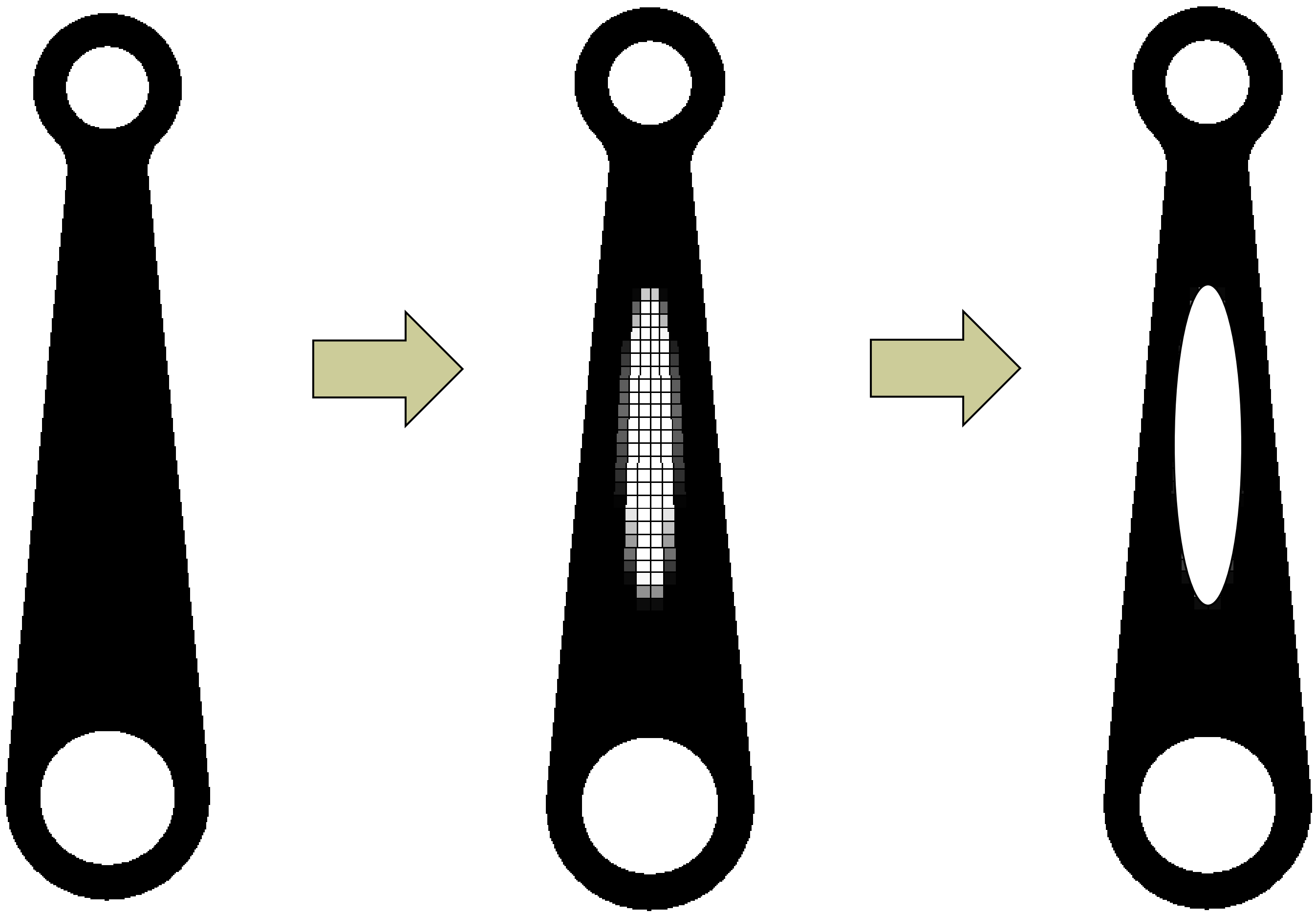
Results

- Convergence obtained after 12 iterations
- Monotonous behavior of the optimization process



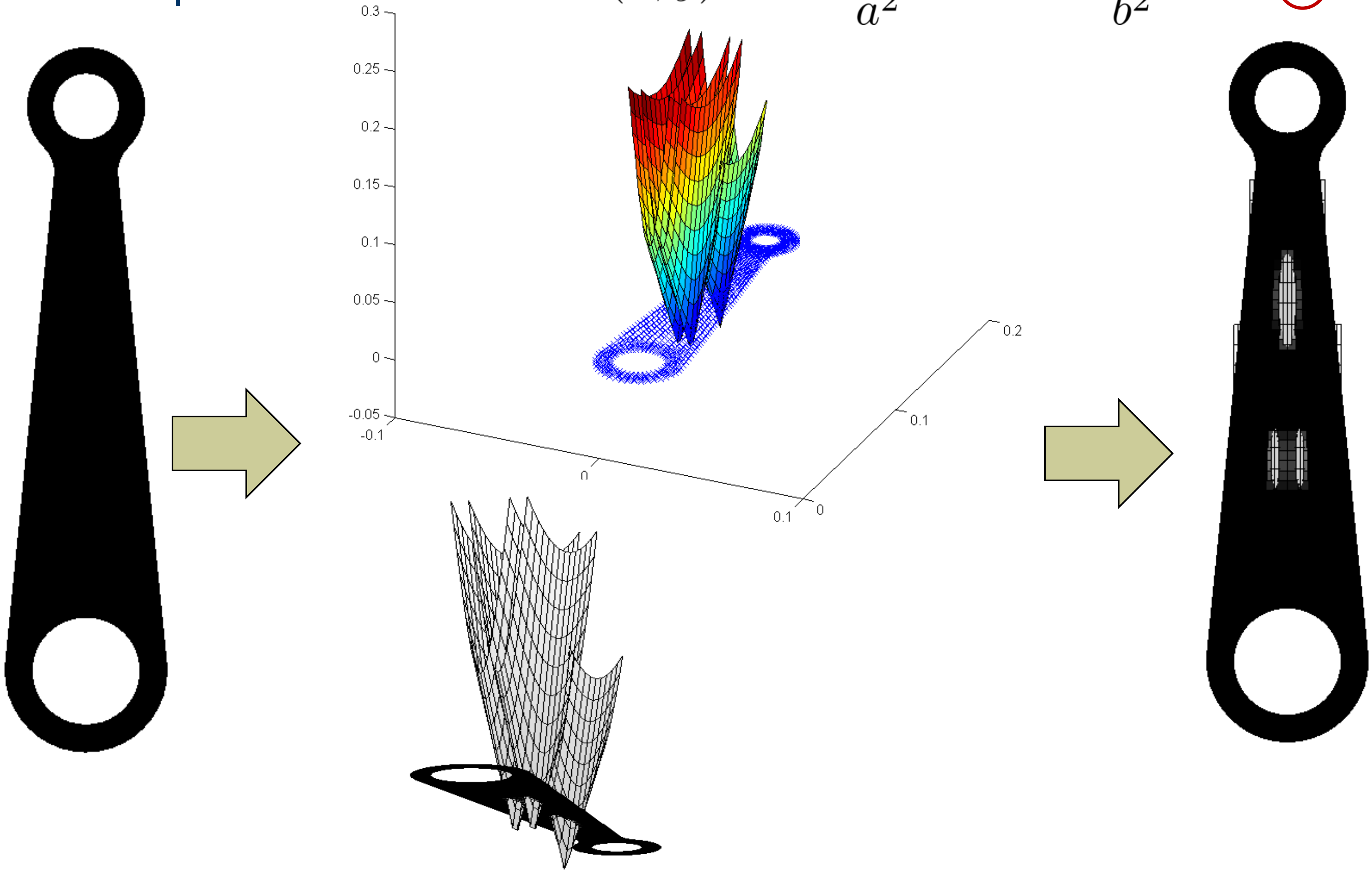
Results – Optimal design

- As the boundary is defined by a CAD entity, the connecting rod can be directly manufactured without any post processing.



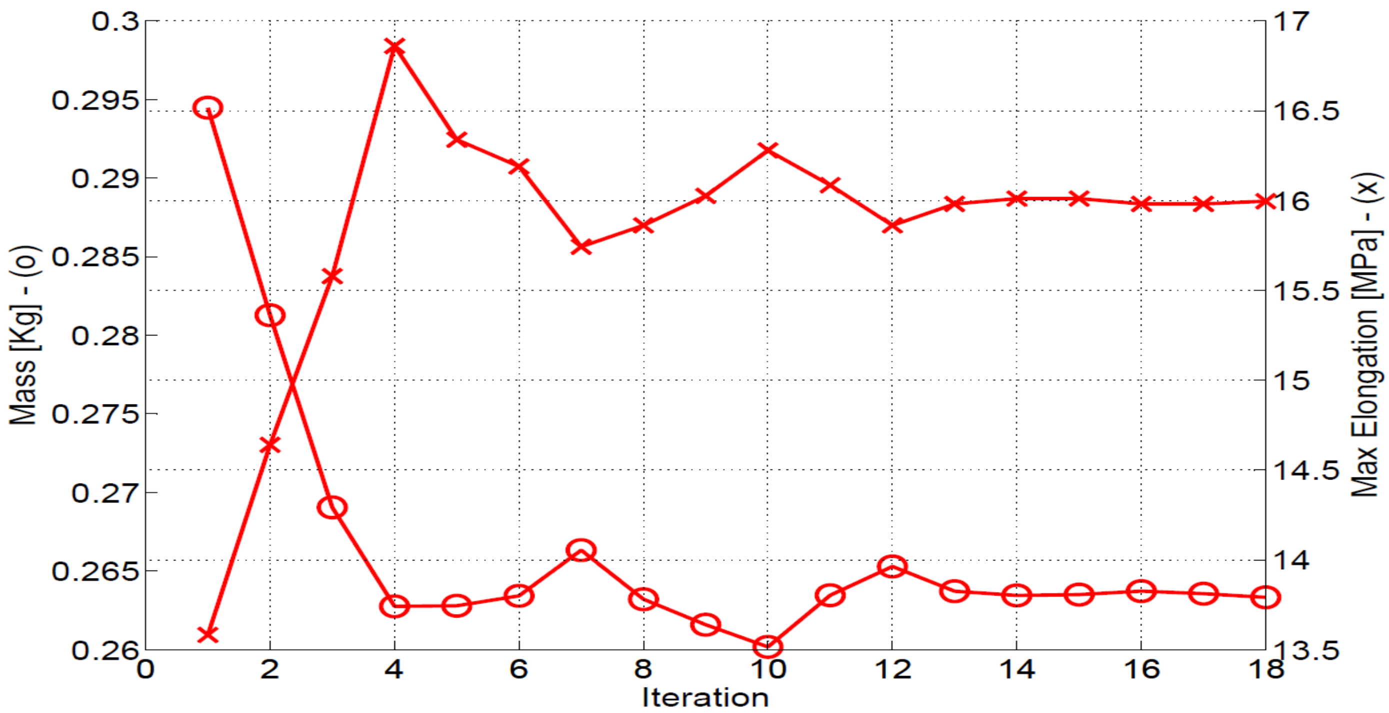
Second application – 3 level sets

- 3 ellipses are defined. $\Phi(x,y) = \frac{(x-c_x)^2}{a^2} + \frac{(y-c_y)^2}{b^2} - d = 0$



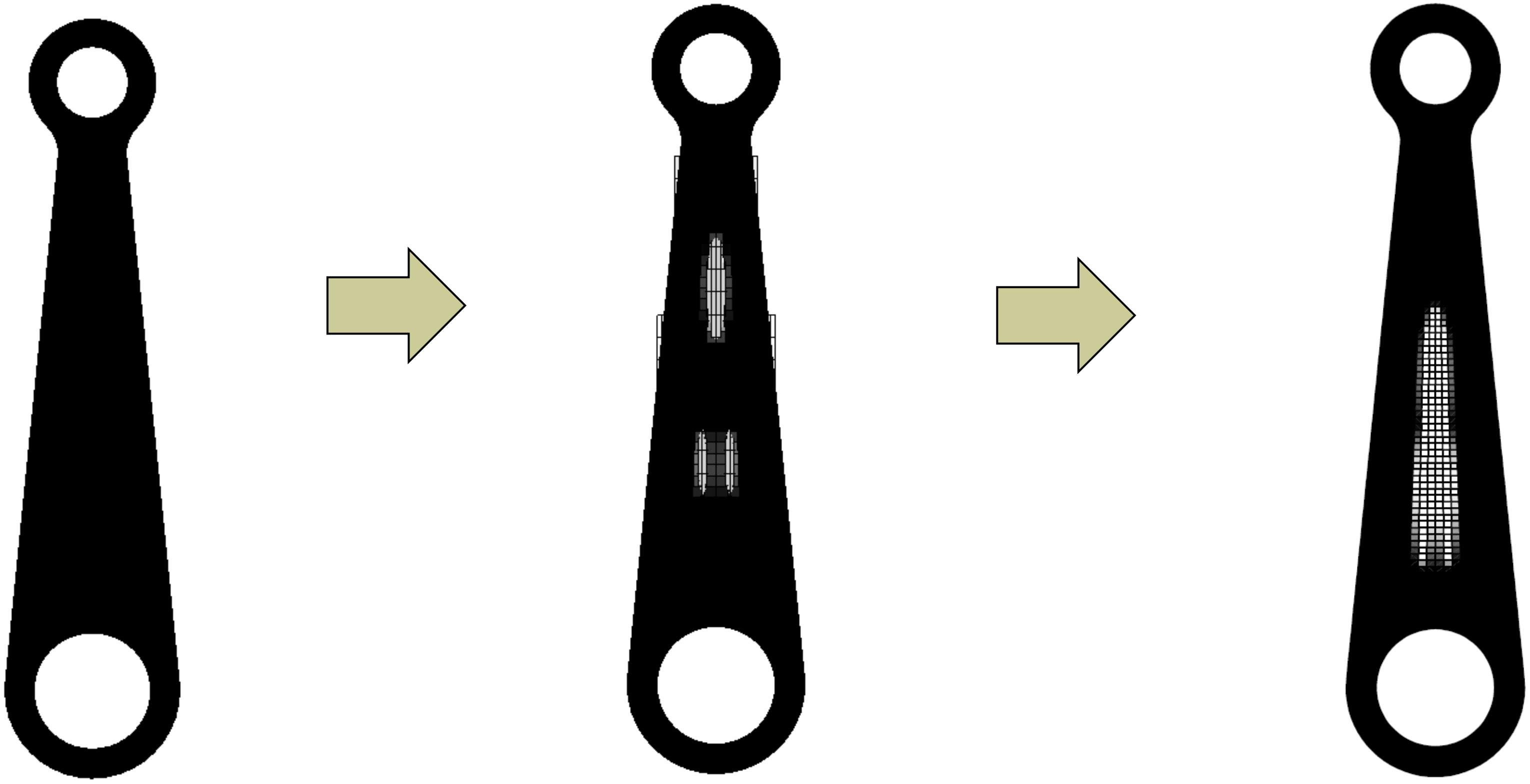
Results

- Convergence obtained after 18 iterations
- The non-linearities of the design space are larger
  - ➔ Oscillations

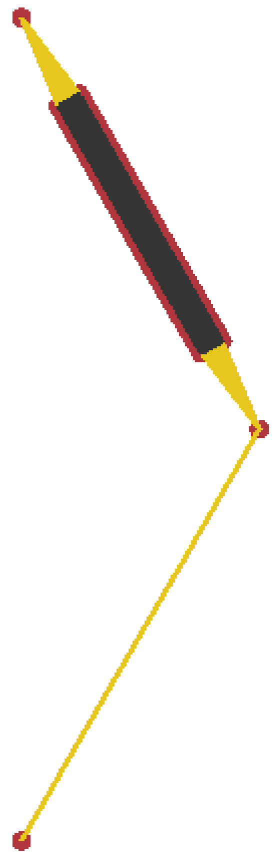


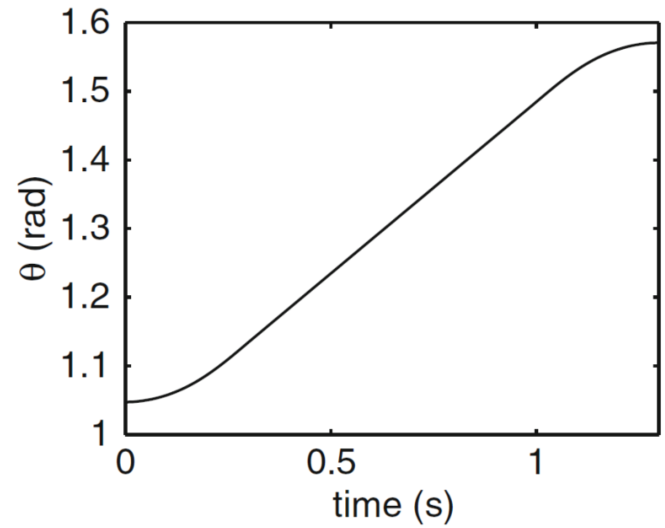
Results – Optimal design

- Modification of the topology during the evolution of the optimization process



2-dof robot: trajectory tracking constraint





$$\underset{\mathbf{x}}{\text{minimize}} \quad m(\mathbf{x})$$

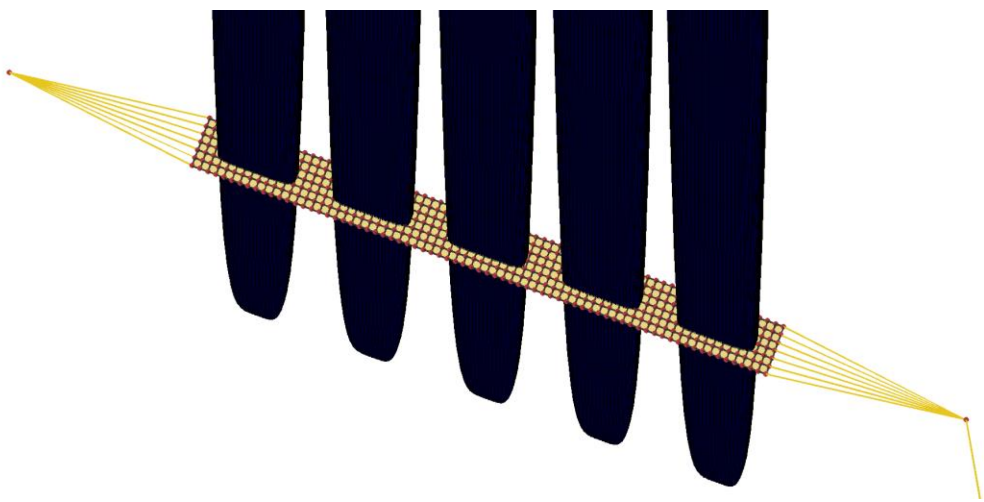
$$\text{subject to} \quad \frac{1}{t_{end}} \sum_{n=1}^{t_{end}} \Delta l(\mathbf{x}, t_n) \leq \Delta l_{max}$$

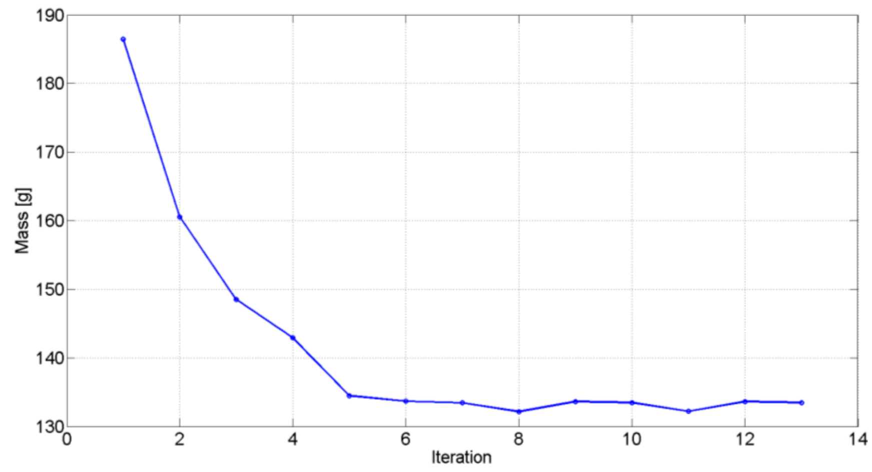
- Heavy, Stiff ➔ No vibration
- Improve productivity ➔ Speed up
- Faster ➔ Energy consumption increase
- Reduce mass ➔ Vibration appear

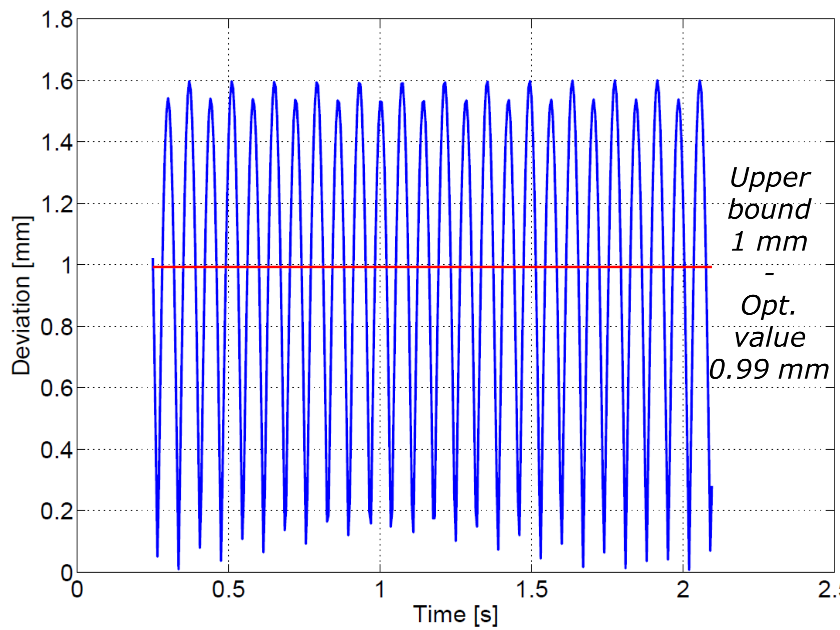
Design variables:
5 Level Sets not independent

$$\Phi(x, y) = \frac{(x - c_x)^2}{a^2} + \frac{(y - c_y)^2}{b^2} - d = 0$$

2-dof robot: trajectory tracking constraint







Conclusions and perspectives

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- Optimization of flexible components carried out in the framework of flexible dynamic multibody system simulation
  - Deals with the time response coming directly from the simulation
  - Enables a formulation of the optimization problem based on the task executed by the system
  - Allows a global-local approach
  - More general than the EQSL
- Determine the optimal layout of mechanical system components under dynamic loading
- The Level Set description of the geometry enables to solve optimization problems while limiting the introduction of new non-linearities.
- The simple examples show encouraging results
- The proposed sensitivity analysis enables to reduce the computation time
- Introduce other geometrical features (Nurbs with Fast Marching method)
- Extend the method to 3D

Thank You Very Much For Your Attention

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Contact

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