A semi-analytical sensitivity analysis for multibody systems described using Level Sets

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Introduction – Optimization of a connecting rod

- A component based approach

- Multibody system based approach

  - Experience - Empirical load case - Standard
  - Dynamic factor amplification for safety ➔ Not optimal

Geometrical modeling
Multibody system dynamics
Different levels of coupling

- **Weak coupling**
  - Coupling with pre / post processing
  - Define equivalent static load cases \(\text{(Kang, Park and Arora, 2005)}\)
  - Optimization of isolated components

- **Strong coupling**
  - Deals with time response
  - Functions may depend on time
  - Engineering approach

→ A **global-local** approach:
  - The optimization problem can account for global criteria while optimizing local components.

Example: Mass minimization of a vehicle suspension arm while a criteria on the comfort of the driver has to be fulfilled.
Level Set description of the geometry

- Fixed mesh grid
- LSF Parameterization:
  - Combination of parameterized geometric shapes (Van Mieghem and Duysinx 2007)
  - A LSF for each geometric features (global basis function).
  - Signed-distance function or analytical function
- The mapping:
  - Eulerian approach (density-based approach)
  - Association of a pseudo-density to each finite element as in TO
  - The element densities are defined based on the value of LSF at nodes
- Example: Square plate with a hole

Smooth transition
Goals of the work and motivations

- Intermediate type of optimization between shape optimization and topology optimization.
  - Fixed mesh grid: No mesh distortion (No velocity field for SA)
  - The geometry is based on CAD entities: can easily be manufactured.
  - Remove, separate, merge entities: Modification of the topology
  - Design variables: parameters of the level sets (rather small number)

- Not the most accurate mapping but... The method presented aims at determining the optimal layout of components when the dynamics of the system is accounted for i.e.:
  - Inertia effects
  - Coupled vibrations
  - Interaction between components...

The MBS problem is already highly non-linear ➔ Keep the optimization problem simple but efficient as a pre-design tool.
Other methods (EQSL) can then be used for more detailed optimization.
General form of the optimization problem

- Design problem casted in a mathematical programming problem
  \[
  \begin{align*}
  \text{minimize} & \quad \varphi(\mathbf{x}) \\
  \text{subject to} & \quad \text{Equilibrium equation} \\
  & \quad c_j(\mathbf{x}) \leq \bar{c}_j, \quad j = 1, \ldots, n_c, \\
  & \quad \underline{x}_v \leq x_v \leq \bar{x}_v, \quad v = 1, \ldots, n_v,
  \end{align*}
  \]

- Provides a general and robust framework to the solution procedure
- Various efficient solvers can be used (ConLin, MMA, IpOpt,...)

- Formulation using the strong coupling:
  \[
  \begin{align*}
  \text{minimize} & \quad \varphi(\mathbf{x}, \mathbf{s}) \\
  \text{subject to} & \quad M(\mathbf{q})\ddot{\mathbf{q}} + \Phi_q^T(\mathbf{q}, t)\lambda = g(\dot{\mathbf{q}}, \mathbf{q}, t), \\
  & \quad \Phi(\mathbf{q}, t) = 0, \\
  & \quad c_j(\mathbf{x}, \mathbf{s}, t) \leq \bar{c}_j, \quad j = 1, \ldots, n_c, \\
  & \quad \underline{x}_v \leq x_v \leq \bar{x}_v, \quad v = 1, \ldots, n_v.
  \end{align*}
  \]

\[
\mathbf{s} = [\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \lambda]
\]
The optimization problem formulation

- The formulation is a key point for this type of problems:
  - Highly non-linear behavior
- Impact on the design space: Extremely important for gradient-based algo.

\[ \text{Local formulation} \]
\[
\begin{align*}
\text{minimize} & \quad m(x) \\
\text{subject to} & \quad \Delta l(x, t_n) \leq \Delta l_{max}
\end{align*}
\]

\[ \text{Global formulation} \]
\[
\begin{align*}
\text{minimize} & \quad m(x) \\
\text{subject to} & \quad \frac{1}{t_{end}} \sum_{n=1}^{t_{end}} \Delta l(x, t_n) \leq \Delta l_{max}
\end{align*}
\]

- Tight control vs number of constraints
- Genetic algorithms
  - Do not necessarily give better results
  - Computation time much more important
**Equation of FEM-MBS dynamics**

- Approach based on the non-linear finite element method (Flexibility is naturally taken into account)

- Motion of the flexible bodies is represented by **absolute nodal coordinates** \( \mathbf{q} \) (Geradin & Cardona, 2001)

- Dynamic equations of multibody system

  \[
  \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) = \mathbf{g}^{\text{ext}} - \mathbf{g}^{\text{int}} - \mathbf{g}^{\text{gyr}}
  \]

- Subject to kinematic constraints of the motion

  \[
  \Phi(\mathbf{q}, t) = 0
  \]

- The solution is based on a Lagrange multiplier method

  \[
  \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \Phi^T(\mathbf{q}, t)\lambda = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t)
  \]

  \[
  \Phi(\mathbf{q}, t) = 0,
  \]

  with the initial conditions

  \[
  \mathbf{q}(0) = \mathbf{q}_0 \text{ and } \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}_0.
  \]
Time integration solver

- **Generalized-α**
  - Introduction of a vector \( \mathbf{a} \) of acceleration-like variables
    \[
    (1 - \alpha_m) \mathbf{a}_{n+1} + \alpha_m \mathbf{a}_n = (1 - \alpha_f) \mathbf{\ddot{q}}_{n+1} + \alpha_f \mathbf{\ddot{q}}_n
    \]
  - Why?
    - Accurate and reliable results with a small amount of numerical damping (second-order accuracy and linear unconditional stability)
    - Larger range of numerical damping than HHT.

- **Newmark integration formulae with \( \mathbf{a} \)**
  \[
  \mathbf{q}_{n+1} = \mathbf{q}_n + h\mathbf{\dot{q}}_n + h^2 \left( \frac{1}{2} - \beta \right) \mathbf{a}_n + h^2 \beta \mathbf{a}_{n+1}
  
  \mathbf{\dot{q}}_{n+1} = \mathbf{\dot{q}}_n + h (1 - \gamma) \mathbf{a}_n + h \gamma \mathbf{a}_{n+1},
  \]

- **Solve iteratively the linearized dynamic equation system (Newton-Raphson scheme)**
  \[
  \mathbf{M} \Delta \mathbf{\ddot{q}} + \mathbf{C}_t \Delta \mathbf{\dot{q}} + \mathbf{K}_t \Delta \mathbf{q} + \mathbf{\Phi}_q^T \Delta \mathbf{\lambda} = \Delta \mathbf{r}
  
  \mathbf{\Phi}_q \Delta \mathbf{q} = \Delta \Phi
  \]
  where \( \mathbf{r} = \mathbf{M} \mathbf{\ddot{q}} + \mathbf{\Phi}_q^T \mathbf{\lambda} - \mathbf{g} \)
**Sensitivity analysis**

- **General function**
  \[
  \Phi(q(p), \dot{q}(p), \ddot{q}(p), p) \rightarrow \frac{\partial q}{\partial p}, \frac{\partial \dot{q}}{\partial p}, \frac{\partial \ddot{q}}{\partial p} ?
  \]

- Implicitely defined through the analysis

- **Finite difference? → MBS cpu-time consuming**

- **Direct or Adjoint Method?** Here [direct method](#) (fct>dv)

- **At a converged time step \( t \), the residual is equal to 0:**
  \[
  R(q(p), \dot{q}(p), \ddot{q}(p), p, t) = 0
  \]

- **The total derivative of the residual is**
  \[
  \frac{dR}{dp} = \frac{\partial R}{\partial q} \frac{dq}{dp} + \frac{\partial R}{\partial \dot{q}} \frac{d\dot{q}}{dp} + \frac{\partial R}{\partial \ddot{q}} \frac{d\ddot{q}}{dp} + \frac{\partial R}{\partial p}
  \]
Sensitivity analysis

- A semi-analytical method has been developed by O. Brüls and P. Eberhard (2008) which can be integrated in the generalized-$\alpha$ scheme.

\[
\begin{align*}
M \frac{d\ddot{q}}{dp_u} + C_t \frac{dq}{dp_u} + K_t \frac{dq}{dp_u} + \Phi_q^T \frac{d\lambda}{dp_u} &= - \frac{\partial r}{\partial p_u} \\
\Phi_q \frac{dq}{dp_u} &= - \frac{\partial}{\partial p_u} \Phi
\end{align*}
\]

- Sensitivity equations are linear with respect to $\frac{dq}{dp_u}$ and $\frac{d\lambda}{dp_u}$.

- The computation of the pseudo loads is quite an issue.
  - It requires in general a lot of effort because the matrices of the mechanical system must be computed for many different values.

- In the simulation code, $M$, $C_t$ and $K_t$ are not computed independently but they are aggregated in the tangent iteration matrix ($S_t$).
Improving the residual derivative computation

- Rewriting the residual as follows:

\[
\frac{dR}{dp} = \frac{\partial R}{\partial q} \frac{dq}{dp} + \frac{\partial R}{\partial \dot{q}} \frac{d\dot{q}}{dp} + \frac{\partial R}{\partial \ddot{q}} \frac{d\ddot{q}}{dp} + \frac{\partial R}{\partial p} = 0
\]

Derivative of the residual wrt \( p \) holding \( q \) fixed.

- Using the definition of the derivative

\[
\lim_{\Delta p \to 0} = \frac{R(q(p), \dot{q}(p + \Delta p), \ddot{q}(p + \Delta p), p + \Delta p, t) - R(q, \dot{q}, \ddot{q}, p, t)}{\Delta p} = 0
\]

- Furthermore, we have

\[
\dot{q}(p + \Delta p) \approx \dot{q}(p) + \frac{d\dot{q}}{dp} \Delta p,
\]

\[
\ddot{q}(p + \Delta p) \approx \ddot{q}(p) + \frac{d\ddot{q}}{dp} \Delta p.
\]

- The terms \( \frac{d\dot{q}}{dp} \) and \( \frac{d\ddot{q}}{dp} \) are obtained from the Newmark integration formulae.
More efficient sensitivity analysis

- Gathering the previous developments, one get

\[- \frac{\partial R}{\partial q} \frac{dq}{dp} \approx \frac{1}{\Delta p} R(q(p), \dot{q}(p) + \frac{dq}{dp} \Delta p, \ddot{q}(p) + \frac{d\ddot{q}}{dp} \Delta p, t)\]

- And after development, we end up with

\[-S_t \frac{dq}{dp} = \frac{1}{\Delta p} R(q(p), \dot{q}(p) + \frac{dq}{dp_{pred}} \Delta p, \ddot{q}(p) + \frac{d\ddot{q}}{dp_{pred}} \Delta p, t)\]

- Only the tangent iteration matrix is needed

- The computation of the perturbed residual is suitable as the level set description of the geometry is not treated at the element level in the solver.

  « Perturb the design variable + Call to the residual function »

- Very fast evaluation
Numerical Applications
Connecting rod optimization

- Minimization of the connecting rod mass in a real combustion engine (Diesel).

- Elongation of the connecting rod during the exhaust phase → Collision between the piston and the valves.

- Consideration of one single complete cycle as the behavior is cyclic (720°) for the optimization

- Constraints imposed on the elongation

![Graph showing elongation of the connecting rod with respect to angle of rotation of the crankshaft]
Local formulation

$$\min_{\mathbf{x}} m(\mathbf{x})$$

subject to

$$\Delta l(\mathbf{x}, t_i) \leq \Delta l_{max}$$

with $$i = 1, \ldots, \text{nbr time step}$$

- The elongation constraints $$\Delta l(\mathbf{x}, t_i)$$ are considered at each time step.
  - As many constraints as the number of time steps (134)
First application – 1 level set

- The level set is defined in order to have an ellipse as interface.
- 5 candidate design variables: $a$, $b$, $c_x$, $c_y$ and $d$. Here only $d$ is chosen.

$$\Phi(x, y) = \frac{(x - c_x)^2}{a^2} + \frac{(y - c_y)^2}{b^2} - d = 0$$
Results

- Convergence obtained after 12 iterations
- Monotonous behavior of the optimization process

![Graph showing mass and elongation over iterations.](image-url)
Results – Optimal design

- As the boundary is defined by a CAD entity, the connecting rod can be directly manufactured without any post processing.
Second application – 3 level sets

- 3 ellipses are defined. \[ \Phi(x, y) = \frac{(x-c_x)^2}{a^2} + \frac{(y-c_y)^2}{b^2} - d = 0 \]
Results

- Convergence obtained after 18 iterations
- The non-linearities of the design space are larger
  ➔ Oscillations
Results – Optimal design

- Modification of the topology during the evolution of the optimization process
2-dof robot: trajectory tracking constraint

- Heavy, Stiff $\Rightarrow$ No vibration
- Improve productivity $\Rightarrow$ Speed up
- Faster $\Rightarrow$ Energy consumption increase
- Reduce mass $\Rightarrow$ Vibration appear

Design variables:
5 Level Sets not independent

\[
\Phi(x, y) = \frac{(x - c_x)^2}{a^2} + \frac{(y - c_y)^2}{b^2} - d = 0
\]

\[
\begin{align*}
\text{minimize} & \quad m(x) \\
\text{subject to} & \quad \frac{1}{t_{\text{end}}} \sum_{n=1}^{t_{\text{end}}} \Delta l(x, t_n) \leq \Delta l_{\text{max}}
\end{align*}
\]
2-dof robot: trajectory tracking constraint

![Diagram of a 2-dof robot with trajectory tracking constraint]

Upper bound 1 mm
Opt. value 0.99 mm
Conclusions and perspectives

- Optimization of flexible components carried out in the framework of flexible dynamic multibody system simulation
  - Deals with the time response coming directly from the simulation
  - Enables a formulation of the optimization problem based on the task executed by the system
  - Allows a global-local approach
  - More general than the EQSL

- Determine the optimal layout of mechanical system components under dynamic loading

- The Level Set description of the geometry enables to solve optimization problems while limiting the introduction of new non-linearities.

- The simple examples show encouraging results

- The proposed sensitivity analysis enables to reduce the computation time

- Introduce other geometrical features (Nurbs with Fast Marching method)

- Extend the method to 3D
Thank You Very Much For Your Attention

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