# High-order response statistics of a wind-excited oscillator with nonlinear velocity feedback

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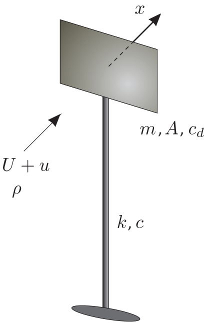
University of Liège (Belgium) - University of Genova (Italy)

XIII Conference of the Italian Association for Wind Engineering

IN VENTO 2014
22-25 JUNE 2014, GENOVA

# Context

Governing equation: $m\ddot{x} + c\dot{x} + kx = \frac{1}{2}\rho A c_d \left(U + u - \dot{x}\right)^2$



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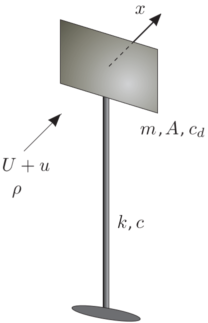

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Scaling: $x^\star = I_u \frac{\rho A c_d U^2}{k}$ ; $t^\star = \sqrt{\frac{m}{k}} = \frac{1}{\omega_0}$

$$\tilde{x}'' + 2\xi_s \tilde{x}' + \tilde{x} = \frac{1}{2I_u}\left(1 + I_u \tilde{u} - 2 I_u \xi_a \tilde{x}'\right)^2$$

$\xi_s = \frac{c}{2m\omega_0}$ ; $\xi_a = \frac{\rho A c_d U}{2m\omega_0}$ ; $I_u = \frac{\sigma_u}{U}$

Structural Damping

Aerodynamic Damping

Turbulence Intensity

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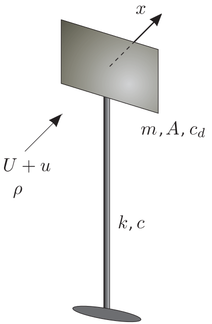
# Context



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| | | | |
|---|---|---|---|
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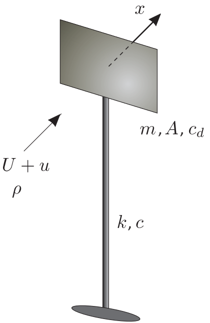
| Structural Damping | Aerodynamic Damping | Turbulence Intensity | Turbulence Frequency |

$$S_{\tilde{u}}\left(\tilde{\omega};\alpha\right) = \frac{1}{\alpha}\frac{0.546}{\left(1 + 1.64\frac{|\tilde{\omega}|}{\alpha}\right)^{5/3}}$$

# Context

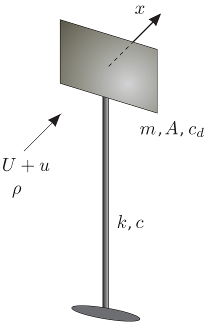


$$\tilde{x}'' + 2\left(\xi_s + \underbrace{\boxed{\xi_a}}_{\text{Aerodynamic Damping}}\right)\tilde{x}' + \tilde{x} = \underbrace{\boxed{\frac{1}{2I_u}}}_{\text{Average}} + \underbrace{\boxed{\tilde{u}}}_{\text{Turbulent}} + \underbrace{\boxed{\frac{I_u}{2}\tilde{u}^2}}_{\text{Quadratic turbulent}} - \underbrace{\boxed{2I_u\xi_a\tilde{u}\tilde{x}'}}_{\text{Parametric}} + \underbrace{\boxed{2I_u\xi_a^2\tilde{x}'^2}}_{\text{Quadratic Velocity}}$$

$$\underbrace{\xi_s = \frac{c}{2m\omega_0}}_{\text{Structural Damping}} \quad ; \quad \underbrace{\xi_a = \frac{\rho A c_d U}{2m\omega_0}}_{\text{Aerodynamic Damping}} \quad ; \quad \underbrace{I_u = \frac{\sigma_u}{U}}_{\text{Turbulence Intensity}} \quad ; \quad \underbrace{\alpha = \frac{U}{L\omega_0}}_{\text{Turbulence Frequency}}$$

# Context

$$\tilde{x}'' + 2\left(\xi_s + \xi_a\right)\tilde{x}' + \tilde{x} = \frac{1}{2I_u} + \tilde{u} + \boxed{\frac{I_u}{2}\tilde{u}^2} - \underbrace{\boxed{2I_u\xi_a\tilde{u}\tilde{x}'}}_{\text{Parametric}} + \underbrace{\boxed{2I_u\xi_a^2\tilde{x}'^2}}_{\text{Quadratic Velocity}}$$



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**All four dimensionless numbers are small**

# Monte Carlo Simulation

V.Denoël
L.Carassale

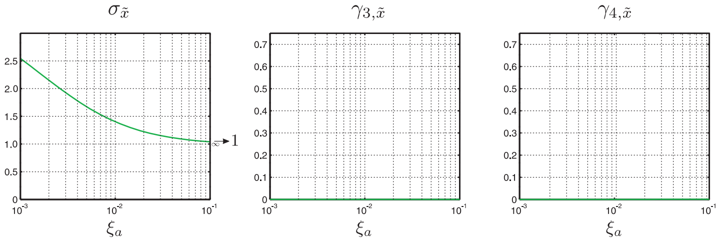
Context

Volterra Model & Validation

Spectral Analysis

Conclusions

$$\tilde{x}'' + 2\left(\xi_s + \xi_a\right)\tilde{x}' + \tilde{x} = \frac{1}{2I_u} + \tilde{u} + \boxed{\frac{I_u}{2}\tilde{u}^2} - \boxed{2I_u\xi_a\tilde{u}\tilde{x}'} + \boxed{2I_u\xi_a^2\tilde{x}'^2}$$



$$\xi_s = 0.001$$
$$I_u = 0.2$$
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$$S_{\tilde{u}} = \frac{1}{\alpha}\frac{0.546}{\left(1 + 1.64\frac{|\tilde{\omega}|}{\alpha}\right)^{5/3}}$$

**Linear Model**

# Monte Carlo Simulation

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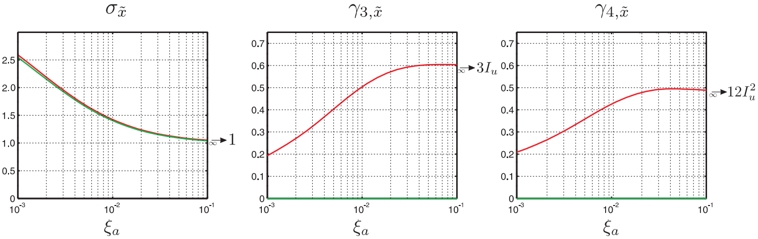

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Linear Model

Linear Model, with quadratic turbulence

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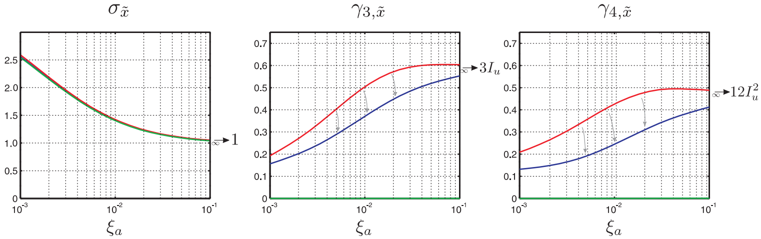

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Linear Model

Linear Model, with quadratic turbulence

Nonlinear Model

# Motivation & Objectives

**Monte Carlo simulation**:

▷ straightforward answer, but ...

**Parametric excitation and quadratic structural velocity**

▷ Other attemps with moment equations (mitigated success)

▷ What is their real influence on the response ?

▷ Any clear understanding ?

→ **Rapid & accurate estimation of 3rd and 4th order response ?**

**Our solution...**

(a) Build a Volterra model

(b) Analyse the 2-nd order Volterra model

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# What is a Volterra series model ?

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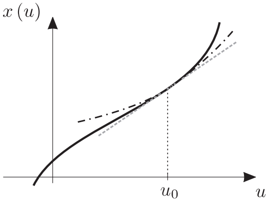
## Taylor series expansion



$$x(u) = x(u_0) + x_1(u) + x_2(u) + \cdots$$

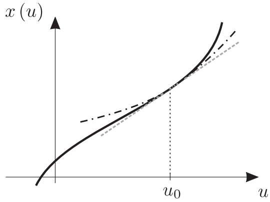
$$x_1(u) = (u - u_0)\, x'(u)$$

$$x_2(u) = \frac{1}{2}(u - u_0)^2\, x''(u)$$

## Volterra series expansion

# What is a Volterra series model ?

## Taylor series expansion



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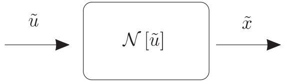

$$\tilde{x}(\tilde{t}) = \tilde{x}_0 + \tilde{x}_1(\tilde{t}) + \tilde{x}_2(\tilde{t}) + \cdots$$

$$\tilde{x}_1(\tilde{t}) = \int_{-\infty}^{+\infty} h_1(\tau)\, \tilde{u}(\tilde{t} - \tau)\, d\tau \qquad \rightarrow H_1(\omega)$$

$$\tilde{x}_2(\tilde{t}) = \iint_{-\infty}^{+\infty} h_2(\tau_1, \tau_2)\, \tilde{u}(\tilde{t} - \tau_1)\, \tilde{u}(\tilde{t} - \tau_2)\, d\tau_1 d\tau_2$$
$$\rightarrow H_2(\omega_1, \omega_2)$$

# The Associated Linear Equations

$$\tilde{x}\left(\tilde{t}\right) = \tilde{x}_0 + \tilde{x}_1\left(\tilde{t}\right) + \tilde{x}_2\left(\tilde{t}\right) \cancel{+\cdots}$$

$$\left\{\begin{array}{l} \tilde{x}_0 = \dfrac{1}{2I_u} \\[2ex] \tilde{x}_1''+2\left(\xi_s+\xi_a\right)\tilde{x}_1'+\tilde{x}_1 = \tilde{u} \\[2ex] \tilde{x}_2''+2\left(\xi_s+\xi_a\right)\tilde{x}_2'+\tilde{x}_2 = I_u\left(\dfrac{\tilde{u}^2}{2} - 2\xi_a\tilde{u}\tilde{x}_1' + 2\xi_a^2\tilde{x}_1'^2\right) \end{array}\right.$$

# The Associated Linear Equations

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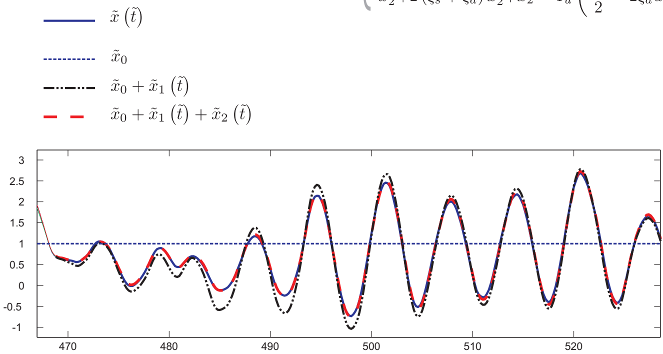

$\xi_s = 0.01$

$\xi_a = 0.05$

$\alpha = 0.005$

$I_u = 0.5$

# The Associated Linear Equations

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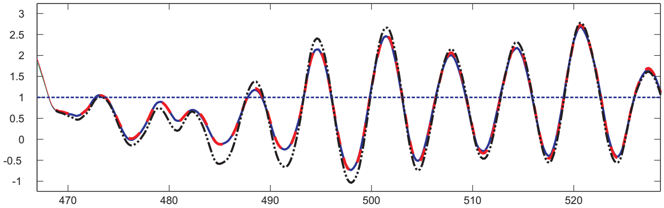
── $\tilde{x}\left(\tilde{t}\right)$

┄┄ $\tilde{x}_0$

─·─·· $\tilde{x}_0 + \tilde{x}_1\left(\tilde{t}\right)$

– – $\tilde{x}_0 + \tilde{x}_1\left(\tilde{t}\right) + \tilde{x}_2\left(\tilde{t}\right)$

- ▷ Convergence of the Volterra Series
- ▷ $\tilde{x}_2 \ll \tilde{x}_1$



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# The Associated Linear Equations

$$\tilde{x}\left(\tilde{t}\right)=\tilde{x}_0+\tilde{x}_1\left(\tilde{t}\right)+\tilde{x}_2\left(\tilde{t}\right)\cancel{+\cdots}$$

$$\left\{\begin{aligned}\tilde{x}_0&=\frac{1}{2I_u}\\ \tilde{x}_1''+2\left(\xi_s+\xi_a\right)\tilde{x}_1'+\tilde{x}_1&=\tilde{u}\\ \tilde{x}_2''+2\left(\xi_s+\xi_a\right)\tilde{x}_2'+\tilde{x}_2&=I_u\left(\frac{\tilde{u}^2}{2}-2\xi_a\tilde{u}\tilde{x}_1'+2\xi_a^2\tilde{x}_1'^2\right)\end{aligned}\right.$$

- ▷ Convergence of the Volterra Series
- ▷ $\tilde{x}_2 \ll \tilde{x}_1$
- ▷ Cumulant = sum of a large term & a small term

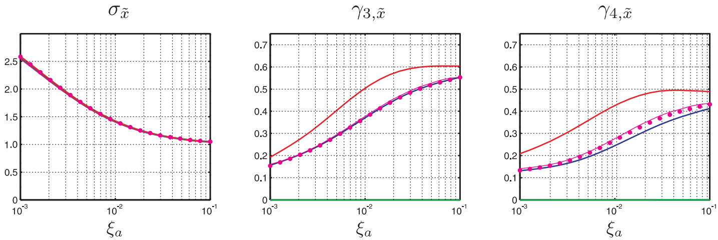
Cumulants of the response:

$$\begin{aligned}\kappa_2\left[\tilde{x}\right]&=\kappa_2\left[\tilde{x}_1\right]+\kappa_2\left[\tilde{x}_2\right]\\ \kappa_3\left[\tilde{x}\right]&=3\kappa_3\left[\tilde{x}_1,\tilde{x}_1,\tilde{x}_2\right]+\kappa_3\left[\tilde{x}_2\right]\\ \kappa_4\left[\tilde{x}\right]&=6\kappa_4\left[\tilde{x}_1,\tilde{x}_1,\tilde{x}_2,\tilde{x}_2\right]+\kappa_4\left[\tilde{x}_2\right]\end{aligned}$$

[NB: valid because $\tilde{x}_1$ and $\tilde{x}_2$ are the terms of the Volterra series ]

# Validation of the Volterra Model

$$\tilde{x}'' + 2\left(\xi_s + \xi_a\right)\tilde{x}' + \tilde{x} = \frac{1}{2I_u} + \tilde{u} + \boxed{\frac{I_u}{2}\tilde{u}^2} - \boxed{2I_u\xi_a\tilde{u}\tilde{x}'} + \boxed{2I_u\xi_a^2\tilde{x}'^2}$$



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$$S_{\tilde{u}} = \frac{1}{\alpha}\frac{0.546}{\left(1 + 1.64\frac{|\tilde{\omega}|}{\alpha}\right)^{5/3}}$$

Linear Model

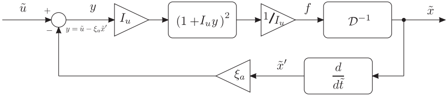
Linear Model, with quadratic turbulence

**2nd Order Volterra Model**

Nonlinear Model

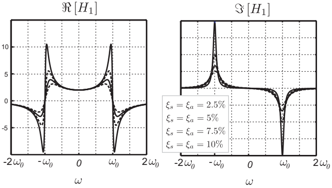
# Volterra Frequency Response Functions $H_0$, $H_1$, $H_2$

$$\tilde{x}'' + 2\left(\xi_s + \xi_a\right)\tilde{x}' + \tilde{x} = \frac{1}{2I_u} + \tilde{u} + \frac{I_u}{2}\tilde{u}^2 - \boxed{2I_u\xi_a\tilde{u}\tilde{x}'} + \boxed{2I_u\xi_a^2\tilde{x}'^2}$$



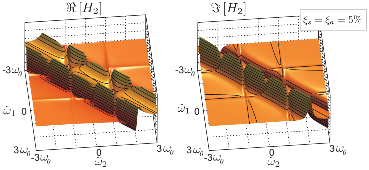
$$H_0 = \tilde{x}_0 = \frac{1}{2I_u}$$

## 1st Order



$$H_1(\tilde{\omega}) = \frac{2}{1 - \tilde{\omega}^2 + 2j\left(\xi_s + \xi_a\right)\tilde{\omega}}$$

## 2nd Order



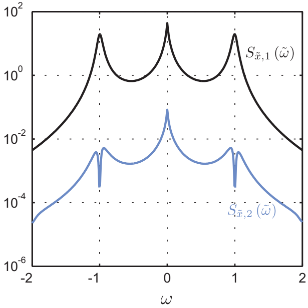
$$H_2\left(\tilde{\omega}_1, \tilde{\omega}_2\right) = \frac{I_u}{2}\frac{\left(1 - 2\xi_a j\tilde{\omega}_1 H_1\left(\tilde{\omega}_1\right)\right)\left(1 - 2\xi_a j\tilde{\omega}_2 H_1\left(\tilde{\omega}_2\right)\right)}{1 - \left(\tilde{\omega}_1 + \tilde{\omega}_2\right)^2 + 2j\left(\xi_s + \xi_a\right)\left(\tilde{\omega}_1 + \tilde{\omega}_2\right)}$$

# Second Order Response

$$S_{\tilde{x}}(\tilde{\omega}) = \underbrace{\left|H_1(\tilde{\omega})\right|^2 S_{\tilde{u}}(\tilde{\omega})}_{S_{\tilde{x},1}(\tilde{\omega})} + \underbrace{2\int_{\mathbb{R}} \left|H_2(\omega_1, \tilde{\omega}-\omega_1)\right|^2 S_{\tilde{u}}(\omega_1)\, S_{\tilde{u}}(\tilde{\omega}-\omega_1)\, d\omega_1}_{S_{\tilde{x},2}(\tilde{\omega})}$$

$$\xrightarrow{\int_{\mathbb{R}} \cdots d\tilde{\omega}}$$

$$\kappa_2[\tilde{x}] = \kappa_2[\tilde{x}_1] + \kappa_2[\tilde{x}_2]$$



# Second Order Response

$$S_{\tilde{x}}\left(\tilde{\omega}\right) = \underbrace{\left|H_1\left(\tilde{\omega}\right)\right|^2 S_{\tilde{u}}\left(\tilde{\omega}\right)}_{S_{\tilde{x},1}\left(\tilde{\omega}\right)} + \underbrace{2\int_{\mathbb{R}} \left|H_2\left(\omega_1, \tilde{\omega}-\omega_1\right)\right|^2 S_{\tilde{u}}\left(\omega_1\right) S_{\tilde{u}}\left(\tilde{\omega}-\omega_1\right) d\omega_1}_{S_{\tilde{x},2}\left(\tilde{\omega}\right)}$$

$$\Big\downarrow \int_{\mathbb{R}} \cdots d\tilde{\omega}$$

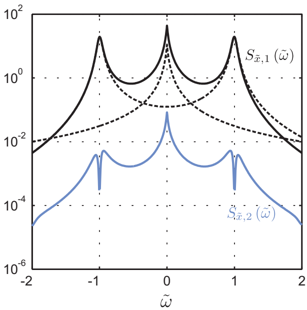
$$\kappa_2\left[\tilde{x}\right] = \underbrace{\kappa_2\left[\tilde{x}_1\right]} + \kappa_2\left[\tilde{x}_2\right]$$

$$1 \qquad + \qquad \frac{\pi S_{\tilde{u}}\left(1\right)}{2\xi}$$

BACKGROUND + RESONANT

$$\kappa_2\left[x_1\right] = \left(\frac{\rho A c_d U}{k}\right)^2 \left[\sigma_u^2 + \frac{\pi\omega_0}{2\xi} S_u\left(\omega_0\right)\right]$$

[[ no more integral ]]



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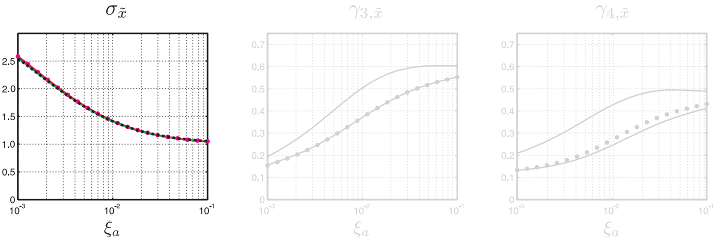
# Second Order Response

$$\tilde{x}'' + 2\left(\xi_s + \xi_a\right)\tilde{x}' + \tilde{x} = \frac{1}{2I_u} + \tilde{u} + \boxed{\frac{I_u}{2}\tilde{u}^2} - \boxed{2I_u\xi_a\tilde{u}\tilde{x}'} + \boxed{2I_u\xi_a^2\tilde{x}'^2}$$



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Linear Model

Linear Model, with quadratic turbulence

2nd Order Volterra Model

Nonlinear Model

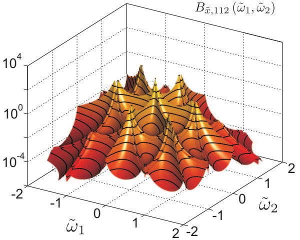
**Analytical Approximation**

# Third Order Response

$$
\begin{aligned}
B_{\tilde{x}}(\tilde{\omega}_1,\tilde{\omega}_2) = \underbrace{\begin{array}{l}
2H_1(-\tilde{\omega}_1-\tilde{\omega}_2)\,H_1(\tilde{\omega}_1)\,H_2(\tilde{\omega}_1+\tilde{\omega}_2,-\tilde{\omega}_1)\,S_{uu}(\tilde{\omega}_1+\tilde{\omega}_2)\,S_{\tilde{u}}(\tilde{\omega}_1)\\
+\;2H_1(-\tilde{\omega}_1-\tilde{\omega}_2)\,H_1(\tilde{\omega}_2)\,H_2(\tilde{\omega}_1+\tilde{\omega}_2,-\tilde{\omega}_2)\,S_{uu}(\tilde{\omega}_1+\tilde{\omega}_2)\,S_{\tilde{u}}(\tilde{\omega}_2)\\
+\;2H_1(\tilde{\omega}_1)\,H_1(\tilde{\omega}_2)\,H_2(-\tilde{\omega}_1,-\tilde{\omega}_2)\,S_{uu}(\tilde{\omega}_1)\,S_{\tilde{u}}(\tilde{\omega}_2)
\end{array}}_{B_{\tilde{x},112}(\tilde{\omega}_1,\tilde{\omega}_2)} \underbrace{+\cdots}_{B_{\tilde{x},2}(\tilde{\omega}_1,\tilde{\omega}_2)}
\end{aligned}
$$

$$\iint_{\mathbb{R}^2}\cdots d\tilde{\omega}_1 d\tilde{\omega}_2$$

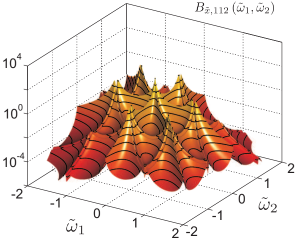
$$\kappa_3[\tilde{x}] = 3\kappa_3[\tilde{x}_1,\tilde{x}_1,\tilde{x}_2] + \kappa_3[\tilde{x}_2]$$



# Third Order Response

$$
\begin{aligned}
B_{\tilde{x}}(\tilde{\omega}_1,\tilde{\omega}_2) = & \underbrace{\begin{aligned}
& 2H_1(-\tilde{\omega}_1-\tilde{\omega}_2)\,H_1(\tilde{\omega}_1)\,H_2(\tilde{\omega}_1+\tilde{\omega}_2,-\tilde{\omega}_1)\,S_{uu}(\tilde{\omega}_1+\tilde{\omega}_2)\,S_{\tilde{u}}(\tilde{\omega}_1) \\
+ & 2H_1(-\tilde{\omega}_1-\tilde{\omega}_2)\,H_1(\tilde{\omega}_2)\,H_2(\tilde{\omega}_1+\tilde{\omega}_2,-\tilde{\omega}_2)\,S_{uu}(\tilde{\omega}_1+\tilde{\omega}_2)\,S_{\tilde{u}}(\tilde{\omega}_2) \\
+ & 2H_1(\tilde{\omega}_1)\,H_1(\tilde{\omega}_2)\,H_2(-\tilde{\omega}_1,-\tilde{\omega}_2)\,S_{uu}(\tilde{\omega}_1)\,S_{\tilde{u}}(\tilde{\omega}_2)
\end{aligned}}_{B_{\tilde{x},112}(\tilde{\omega}_1,\tilde{\omega}_2)} \underbrace{+\cdots}_{B_{\tilde{x},2}(\tilde{\omega}_1,\tilde{\omega}_2)}
\end{aligned}
$$

$$\iint_{\mathbb{R}^2} \cdots d\tilde{\omega}_1 d\tilde{\omega}_2$$



$$\kappa_3[\tilde{x}] = \underbrace{3\kappa_3[\tilde{x}_1,\tilde{x}_1,\tilde{x}_2]} + \kappa_3[\tilde{x}_2]$$

$$3I_u \quad + \quad 6\pi I_u(2\xi_s+\xi_a)\,S_{\tilde{u}}(1)\int_{\mathbb{R}} \frac{S_{\tilde{u}}(\tilde{\omega})}{\tilde{\omega}^2+4\xi^2}d\tilde{\omega}$$

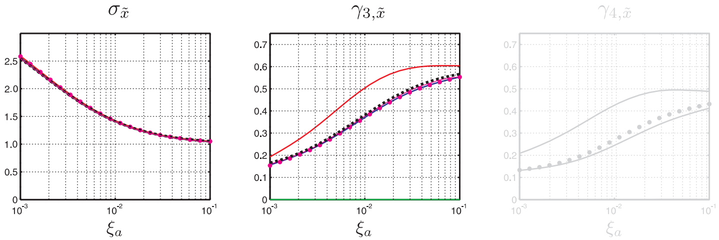
BCKGR + BI-RESONANT

[[ double integral :: single integral ]]

# Third Order Response

V.Denoël
L.Carassale

$$\tilde{x}'' + 2\left(\xi_s + \xi_a\right)\tilde{x}' + \tilde{x} = \frac{1}{2I_u} + \tilde{u} + \boxed{\frac{I_u}{2}\tilde{u}^2} - \boxed{2I_u\xi_a\tilde{u}\tilde{x}'} + \boxed{2I_u\xi_a^2\tilde{x}'^2}$$



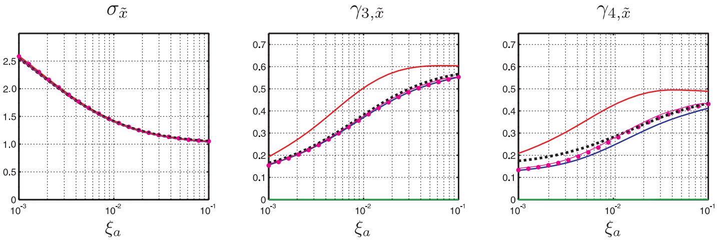
$\xi_s = 0.001$
$I_u = 0.2$
$\alpha = 0.005$

$$S_{\tilde{u}} = \frac{1}{\alpha}\frac{0.546}{\left(1 + 1.64\frac{|\tilde{\omega}|}{\alpha}\right)^{5/3}}$$

Linear Model
Linear Model, with quadratic turbulence
2nd Order Volterra Model
Nonlinear Model
**Analytical Approximation**

# Fourth Order Response

$$\tilde{x}'' + 2\left(\xi_s + \xi_a\right)\tilde{x}' + \tilde{x} = \frac{1}{2I_u} + \tilde{u} + \boxed{\frac{I_u}{2}\tilde{u}^2} - \boxed{2I_u\xi_a\tilde{u}\tilde{x}'} + \boxed{2I_u\xi_a^2\tilde{x}'^2}$$



$$\xi_s = 0.001$$
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$$S_{\tilde{u}} = \frac{1}{\alpha}\frac{0.546}{\left(1 + 1.64\frac{|\tilde{\omega}|}{\alpha}\right)^{5/3}}$$

Linear Model

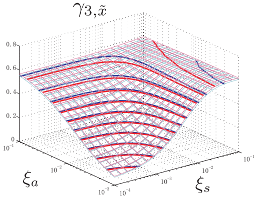
Linear Model, with quadratic turbulence

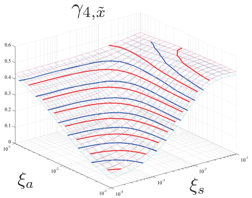
2nd Order Volterra Model

Nonlinear Model

Analytical Approximation

# Wide Scale Validation





Monte Carlo Simulation

CPU: 2500 h (100 days)

- Nonlinear equation
- Volterra series (ALEs, Ord. 1 to 5)
- Linear equation

Analytical Approach

CPU: 5 min

Nonlinear Model

Analytical Approximation

# Perspectives & Conclusions

**Parametric excitation term**

▷ decreases the third and fourth cumulants

**Quadratic structural velocity**

▷ has negligible influence on the third and fourth cumulants

**Analytical Formulation for the response of this nonlinear system**

**Fully Spectral Analysis of the (slightly) nonlinear system**

▷ avoid generation of time series

▷ proven to be $10^3 - 10^4 \times$ faster (or more)

# Thank you ...

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Read more about this topic:

- Denoël V., Carassale (to appear). *Response of an oscillator to a random quadratic velocity-feedback loading*.
- Denoël V. (2011). *On the background and biresonant components of the random response of single degree-of-freedom systems under non-Gaussian random loading*. Engineering Structures. 33(8): p. 2271-2283.

Available @ `www.orbi.ulg.ac.be`