Calculation of third order joint acceptance function for line-like structures

N. Blaise, T. Canor & V. Denoël

University of Liège (Belgium)

XIII Conference of the Italian Association for Wind Engineering
in·vento·2014
JUNE 22-25 · Genova · ITALY
UNIVERSITÀ DEGLI STUDI DI GENOVA - DICCA
DIPARTIMENTO DI INGEGNERIA CIVILE, CHIMICA E AMBIENTALE
www.invento2014.org
ANIV
ASSOCIAZIONE NAZIONALE PER L'INGEGNERIA DEL VENTO
italian section IAWE
INTERNATIONAL ASSOCIATION FOR WIND ENGINEERING

Université de Liège

Line-like structure

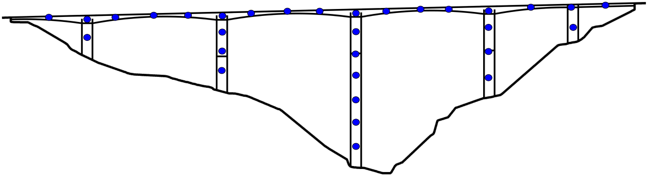
- Gorsexio bridge (16 kms west of Genoa, 144 meters span, 177 meters high)

- Wind pressure measurements to perform first a Gaussian and next a non-Gaussian static analysis

Wind pressure measurements

Refined grid

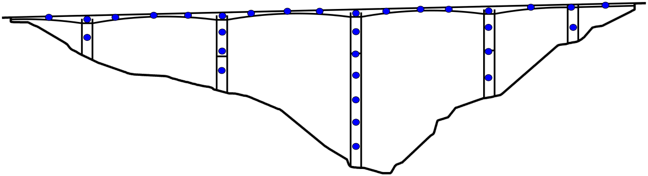
- Wind properties : turbulence intensity, correlation, etc...
- Topography, structure geometry, etc...
- Structural responses of interest : global or local



Wind pressure measurements

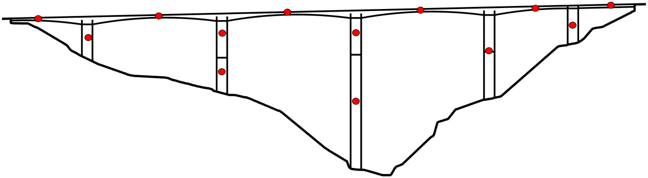
Refined grid

- Wind properties : turbulence intensity, correlation, etc...
- Topography, structure geometry, etc...
- Structural responses of interest : global or local



Coarse grid

- Inaccurate estimation of cumulants of the structural responses
- Scale effects (Central limit-theorem)



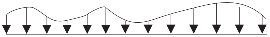
Scale effects

- Aerodynamic fluctuating resultant force

$$f(t) = \int_0^{\ell} p(x,t)\mathrm{d}x$$

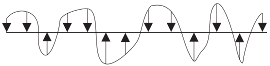
  - zero-mean fluctuating non-gaussian aerodynamic pressures $p(x,t)$ (s.d. σ_p)

- Cumulants of $f(t)$ - level of correlation in the aerodynamic field
  - High



$$\sigma_f^2 \simeq \ell^2 \sigma_p^2 \quad \gamma_{3,f} \simeq \gamma_{3,p} \quad \gamma_{e,f} \simeq \gamma_{e,p}$$

  - Low



$$\sigma_f^2,\ \gamma_{3,f},\ \gamma_{e,f} \rightarrow 0$$

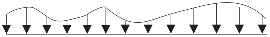
Scale effects

- Aerodynamic fluctuating resultant force

$$f(t) = \int_0^{\ell} p(x,t)\mathrm{d}x \rightarrow f(t) = \sum_i^N p_i(t)\Delta x_i$$

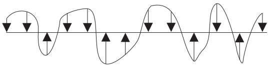
  - zero-mean fluctuating non-gaussian aerodynamic pressures $p(x,t)$ (s.d. σ_p)

- Cumulants of $f(t)$ - level of correlation in the aerodynamic field
  - High



$$\sigma_f^2 \simeq \ell^2 \sigma_p^2 \quad \gamma_{3,f} \simeq \gamma_{3,p} \quad \gamma_{e,f} \simeq \gamma_{e,p}$$

  - Low

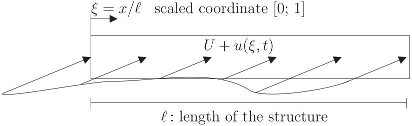


$$\sigma_f^2,\ \gamma_{3,f},\ \gamma_{e,f} \rightarrow 0$$

Evolutions of the cumulants of $f(t)$ and of structural responses ?

Université de Liège

Model of the wind velocity field



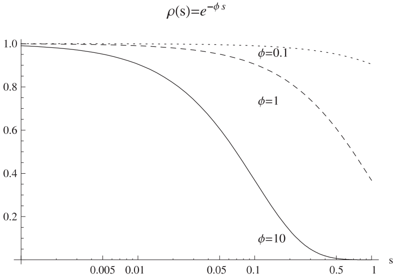
- Random stationary 1-direction wind flow
- Fluctuating gaussian turbulence component $u(\xi, t)$ with standard deviation σ_u
- Wind velocities perfectly correlated in the small direction (line-like structure)
- Turbulence intensity [0.1 ; 0.5]

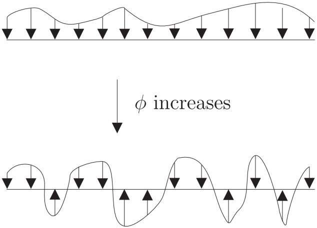
$$I_u = \frac{\sigma_u}{U}$$

Model of correlation : decaying exponential function [1]

$$\rho_{ij} = e^{-\frac{\ell}{L_u}|\xi_i - \xi_j|}$$

- Integral length scale (mean gust dimension) : $L_u \searrow$ correlation $\searrow$
- EC 1 : Reference integral length scale L_u of 300m at the reference height of 200m
- Parameter $\phi = \dfrac{\ell}{L_u}$ and change of variable $s_{ij} = |\xi_i - \xi_j|$

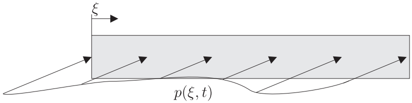




[1] Dyrbye, C. and Hansen, S.O. (1997). Wind Loads on Structures. *Wiley & Sons, Ltd* .

Université de Liège

- Bernoulli's equation $p(\xi,t) = \frac{1}{2}\rho d C \left(U + u(\xi,t)\right)^2$



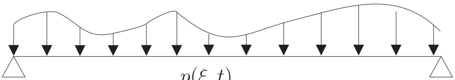
- Gaussian aerodynamic pressures (quadratic term neglected)

$$\begin{aligned} p(\xi,t) &\simeq \frac{1}{2}\rho d C U^2 + \rho d C U\, u(\xi,t) \\ &= a + b\, u(\xi,t) \end{aligned}$$

- Cross-cumulants of order 2 (covariances) between aerodynamic pressures

$$\kappa_{2p_{ij}} = b^2 \sigma^2 \rho_{ij}$$

Simply supported line-like structure



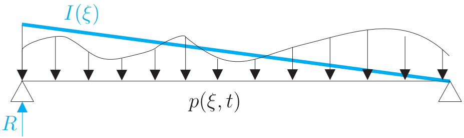
Structural response

$$R(t) = \int_0^1 I(\xi) p(\xi, t) \mathrm{d}\xi$$

- $I(\xi)$: Response influence function
- Second cumulant (variance)

$$\kappa_{2R} = \iint_0^1 I(\xi_i) I(\xi_j) \kappa_{2p}(|\xi_i - \xi_j|) \mathrm{d}\xi_i \mathrm{d}\xi_j$$

Simply supported line-like structure



Structural response

$$R(t) = \int_0^1 I(\xi) p(\xi, t) \mathrm{d}\xi$$

- Second cumulant (variance)

$$\kappa_{2R} = \iint_0^1 I(\xi_i) I(\xi_j) \kappa_{2p}(|\xi_i - \xi_j|) \mathrm{d}\xi_i \mathrm{d}\xi_j$$

Considered response : reaction on the left

- Response influence function

$$I(\xi) = 1 - \xi$$

Dyrbye and Hansen[1] used (i) change of variable $s_{ij} = |\xi_i - \xi_j|$ and (ii) interchange of the order of integration to replace

$$\kappa_{2R} = b^2 \sigma_u^2 \iint_0^1 I(\xi_i) I(\xi_j) \rho(s) \mathrm{d}\xi_i \mathrm{d}\xi_j$$

by two single integrals

- 2^{nd} order influence function

$$k(s) = 2 \int_0^{1-s} I(\xi) I(\xi + s) \mathrm{d}\xi$$

- 2^{nd} order joint acceptance function

$$\kappa_{2R} = b^2 \sigma_u^2 \int_0^1 k(s) \rho(s) \, \mathrm{d}s$$

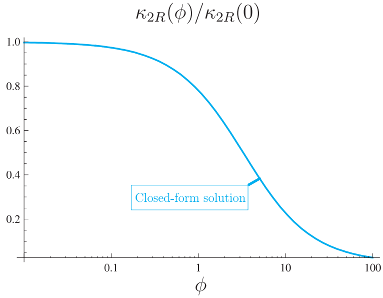
[1] Dyrbye, C. and Hansen, S. O. (1988). Calculation of joint acceptance function for line-like structures. *Journal of Wind Engineering and Industrial Aerodynamics* vol. 31-2, 351-353.

Closed-form solutions

$$k(s) = \frac{1}{3}\left(s^3 - 3s + 2\right)$$

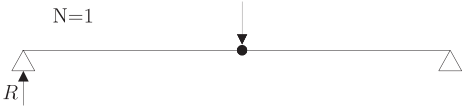
$$\frac{\kappa_{2R}(\phi)}{b^2\sigma_u^2} = \frac{2}{3\phi} - \frac{1}{\phi^2} + \frac{2}{\phi^4} - e^{-\phi}\left(\frac{2}{\phi^3} + \frac{2}{\phi^4}\right)$$

- Full correlation : $\frac{\kappa_{2R}(0)}{b^2\sigma_u^2} = \int_0^1 k(s)\mathrm{d}s = 1/4$

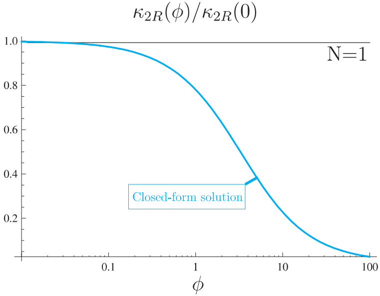


Université de Liège

Discrete aerodynamic pressure field

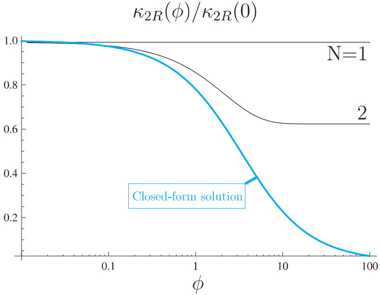


$$\kappa_{2R} \simeq \sum_{i=1}^{N} \sum_{j=1}^{N} I(\xi_i) I(\xi_j) \kappa_{2p}(|\xi_i - \xi_j|) \Delta\xi_i \Delta\xi_j$$

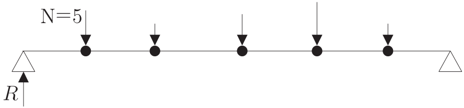


Discrete aerodynamic pressure field

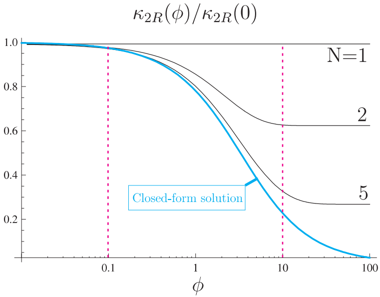
$$\kappa_{2R} \simeq \sum_{i=1}^{N} \sum_{j=1}^{N} I(\xi_i) I(\xi_j) \kappa_{2p}(|\xi_i - \xi_j|) \Delta\xi_i \Delta\xi_j$$



Discrete aerodynamic pressure field

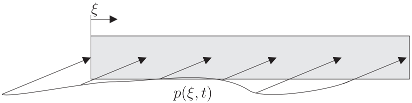


$$\kappa_{2R} \simeq \sum_{i=1}^{N} \sum_{j=1}^{N} I(\xi_i) I(\xi_j) \kappa_{2p}(|\xi_i - \xi_j|) \Delta\xi_i \Delta\xi_j$$



Université de Liège

- Bernoulli's equation : $p(\xi,t) = \frac{1}{2}\rho d C \left(U + u(\xi,t)\right)^2$



- Non-Gaussian aerodynamic pressures

$$p(\xi,t) = a + b\,u(\xi,t) + c\,u^2(\xi,t)$$

  - $a = \frac{1}{2}\rho d C U^2$, $b = \rho d C U$ and $c = \frac{1}{2}\rho d C$
  - quadratic term taken into account
  - $\gamma_{3,p} \simeq 3 I_u$ and $\gamma_{e,p} \simeq 12 I_u^2$

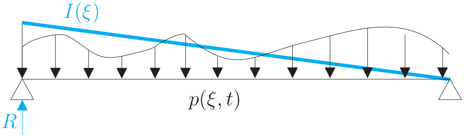
$$\gamma_{e,p} = \frac{4}{3}\gamma_{3,p}^2$$

- Cross-cumulants of order 3 between aerodynamic pressures

$$\kappa_{3p_{ijk}} \simeq 2 b^2 c \sigma_u^4 \left(\rho_{ij}\rho_{ik} + \rho_{ij}\rho_{jk} + \rho_{ik}\rho_{jk}\right)$$

Analytical approach

- Third cumulant



$$\kappa_{3R} = \iiint_0^1 I(\xi_i) I(\xi_j) I(\xi_k) \kappa_{3p}(s_{ij}, s_{ik}) \, \mathrm{d}\xi_i \mathrm{d}\xi_j \mathrm{d}\xi_k$$

$$\simeq 6 b^2 c \sigma_u^3 \iiint_0^1 I(\xi_i) I(\xi_j) I(\xi_k) \rho(s_{ij}) \, \rho(s_{ik}) \, \mathrm{d}\xi_i \mathrm{d}\xi_j \mathrm{d}\xi_k$$

Analytical approach

- Replace triple integral

$$\kappa_{3R} = 6b^2 c\sigma_u^4 \iiint_0^1 I(\xi_i)I(\xi_j)I(\xi_k)\rho\left(s_{ij}\right)\rho\left(s_{ik}\right)\mathrm{d}\xi_i\mathrm{d}\xi_j\mathrm{d}\xi_k$$

  - by three 3^{rd} order influence functions (single integrals)

$$k_1(s_1, s_2) = ...$$

$$k_2(s_1, s_2) = ...$$

$$k_3(s_1, s_2) = ...$$

  - and two 3^{rd} order joint acceptance functions (double integrals)

$$\frac{\kappa_{3R}}{6b^2 c\sigma_u^4} = 2\int_0^1\int_0^{s_1}\left[(k_1(s_1,s_2)+k_2(s_1,s_2))\rho(s_1)\rho(s_2)\right]\mathrm{d}s_2\mathrm{d}s_1 + 2\int_0^1\int_0^{1-s_1} k_3(s_1,s_2)\rho(s_1)\rho(s_2)\mathrm{d}s_2\mathrm{d}s_1$$

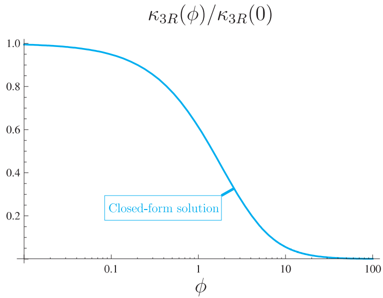
Université de Liège

Analytical approach

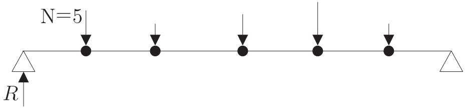
Closed-form solutions

$$\frac{\kappa_{3R}(\phi)}{6b^2c\sigma_u^4} = \frac{e^{-2\phi}\left(-4e^{\phi}(\phi+5)+e^{2\phi}(2\phi(2\phi-7)+19)+1\right)}{4\phi^4}$$

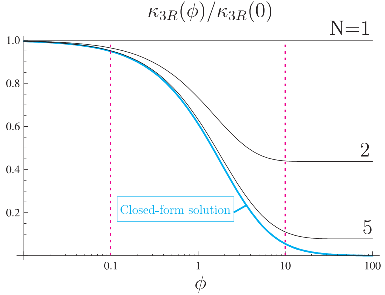
- Full correlation : $\frac{\kappa_{3R}(0)}{6b^2c\sigma_u^4} = \frac{1}{8}$



Discrete aerodynamic pressure field



$$\kappa_{3R} \simeq 6b^2 c\sigma_u^4 \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} I(\xi_i)I(\xi_j)I(\xi_k)\rho\left(s_{ij}\right)\rho\left(s_{ik}\right)\Delta\xi_i\Delta\xi_j\Delta\xi_k$$

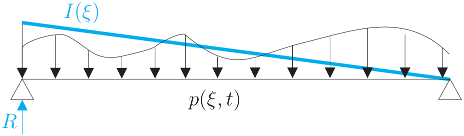


Université de Liège

- Cross-cumulants of order 4 between aerodynamic pressures

$$\kappa_{4p_{ijkl}} \simeq 4b^2c^2\sigma^6\left(12\rho_{ij}\rho_{ik}\rho_{jl}\right)$$

- Fourth cumulant of the reaction



$$\kappa_{4R} \simeq 48b^2c^2\sigma^6\iiiint_0^1 I(\xi_i)I(\xi_j)I(\xi_k)I(\xi_l)\rho\left(s_{ij}\right)\rho\left(s_{ik}\right)\rho\left(s_{jl}\right)\mathrm{d}\xi_i\mathrm{d}\xi_j\mathrm{d}\xi_k\mathrm{d}\xi_l$$

Quadruple integral

$$\frac{\kappa_{4R}}{48b^2c^2\sigma^6} \simeq \iiiint_0^1 I(\xi_i)I(\xi_j)I(\xi_k)I(\xi_l)\rho\left(|\xi_i-\xi_j|\right)\rho\left(|\xi_i-\xi_k|\right)\rho\left(|\xi_j-\xi_l|\right)\mathrm{d}\xi_i\mathrm{d}\xi_j\mathrm{d}\xi_k\mathrm{d}\xi_l$$

- Interchange of order of integration is challenging for a 4-dimensional domain
- Alternative : 8 quadruple integrals

$$\int_0^1\int_0^{\xi_i}\int_0^{\xi_i}\int_0^{\xi_j}\ldots\rho\left(\left(\xi_i-\xi_j\right)\right)\rho\left(\left(\xi_i-\xi_k\right)\right)\rho\left(\left(\xi_j-\xi_l\right)\right)\mathrm{d}\xi_i\mathrm{d}\xi_j\mathrm{d}\xi_k\mathrm{d}\xi_l$$
$$\int_0^1\int_0^{\xi_i}\int_{\xi_i}^1\int_0^{\xi_j}\ldots\rho\left(\left(\xi_i-\xi_j\right)\right)\rho\left(-\left(\xi_i-\xi_k\right)\right)\rho\left(\left(\xi_j-\xi_l\right)\right)\mathrm{d}\xi_i\mathrm{d}\xi_j\mathrm{d}\xi_k\mathrm{d}\xi_l$$
$$\int_0^1\int_0^{\xi_i}\int_0^{\xi_i}\int_{\xi_j}^1\ldots\rho\left(\left(\xi_i-\xi_j\right)\right)\rho\left(\left(\xi_i-\xi_k\right)\right)\rho\left(-\left(\xi_j-\xi_l\right)\right)\mathrm{d}\xi_i\mathrm{d}\xi_j\mathrm{d}\xi_k\mathrm{d}\xi_l$$
$$\int_0^1\int_0^{\xi_i}\int_{\xi_i}^1\int_{\xi_j}^1\ldots\rho\left(\left(\xi_i-\xi_j\right)\right)\rho\left(-\left(\xi_i-\xi_k\right)\right)\rho\left(-\left(\xi_j-\xi_l\right)\right)\mathrm{d}\xi_i\mathrm{d}\xi_j\mathrm{d}\xi_k\mathrm{d}\xi_l$$
$$\int_0^1\int_{\xi_i}^1\int_0^{\xi_i}\int_0^{\xi_j}\ldots\rho\left(-\left(\xi_i-\xi_j\right)\right)\rho\left(\left(\xi_i-\xi_k\right)\right)\rho\left(\left(\xi_j-\xi_l\right)\right)\mathrm{d}\xi_i\mathrm{d}\xi_j\mathrm{d}\xi_k\mathrm{d}\xi_l$$
$$\int_0^1\int_{\xi_i}^1\int_{\xi_i}^1\int_0^{\xi_j}\ldots\rho\left(-\left(\xi_i-\xi_j\right)\right)\rho\left(-\left(\xi_i-\xi_k\right)\right)\rho\left(\left(\xi_j-\xi_l\right)\right)\mathrm{d}\xi_i\mathrm{d}\xi_j\mathrm{d}\xi_k\mathrm{d}\xi_l$$
$$\int_0^1\int_{\xi_i}^1\int_0^{\xi_i}\int_{\xi_j}^1\ldots\rho\left(-\left(\xi_i-\xi_j\right)\right)\rho\left(\left(\xi_i-\xi_k\right)\right)\rho\left(-\left(\xi_j-\xi_l\right)\right)\mathrm{d}\xi_i\mathrm{d}\xi_j\mathrm{d}\xi_k\mathrm{d}\xi_l$$
$$\int_0^1\int_{\xi_i}^1\int_{\xi_i}^1\int_{\xi_j}^1\ldots\rho\left(-\left(\xi_i-\xi_j\right)\right)\rho\left(-\left(\xi_i-\xi_k\right)\right)\rho\left(-\left(\xi_j-\xi_l\right)\right)\mathrm{d}\xi_i\mathrm{d}\xi_j\mathrm{d}\xi_k\mathrm{d}\xi_l$$

Closed-form solution

$$\frac{\kappa_{4R}}{48b^2c^2\sigma^6} = \frac{3e^{-\phi}}{40\left(e^{\phi}\left((2\phi-3)\phi^2+6\right)-6(\phi+1)\right)^2}\Big[15e^{\phi}\left(2\phi^2+13\right)\left(\phi(\phi+4)+2\right)$$
$$+e^{3\phi}\left((2\phi(6\phi(16\phi-95)+1595)-4275)\phi^2+11130\right)+15(\phi+1)(2\phi+1)$$
$$-5e^{2\phi}(\phi(2\phi(\phi+3)(\phi(3\phi+22)+31)+2307)+2307)\Big]$$

- Solution may seem unhandy but approximations available (e.g., Taylor series)
- Full correlation : $\frac{\kappa_{4R}(0)}{48b^2c^2\sigma^6} = \frac{1}{16}$

Université de Liège

- Skewness coefficient

$$\begin{aligned}\gamma_{3R}(\phi) &= \kappa_{3,R}(\phi)/\kappa_{2R}^{3/2}(\phi) \\ &= \ldots\end{aligned}$$

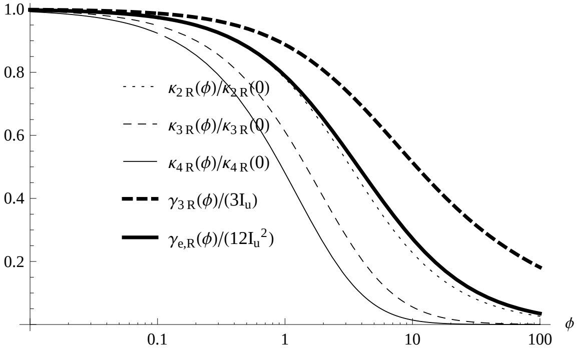
  - Full correlation : $\gamma_{3R}(0) = 3\dfrac{\sigma_u}{U} = 3I_u = \gamma_{3p}$

- Excess coefficient

$$\begin{aligned}\gamma_{eR}(\phi) &= \kappa_{4R}(\phi)/\kappa_{2R}^{2}(\phi) \\ &= \ldots\end{aligned}$$

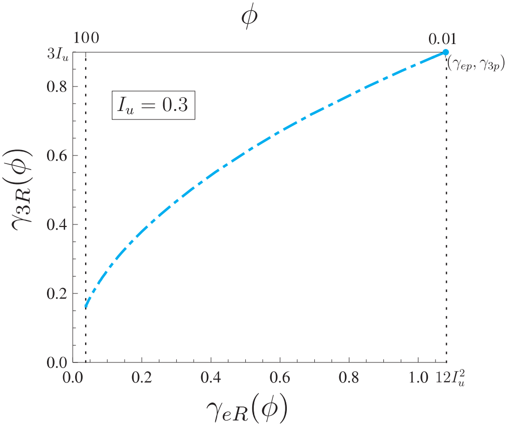
  - Full correlation : $\gamma_{eR}(0) = 12\left(\dfrac{\sigma_u}{U}\right)^2 = 12I_u^2 = \gamma_{ep}$

Evolution of the cumulants

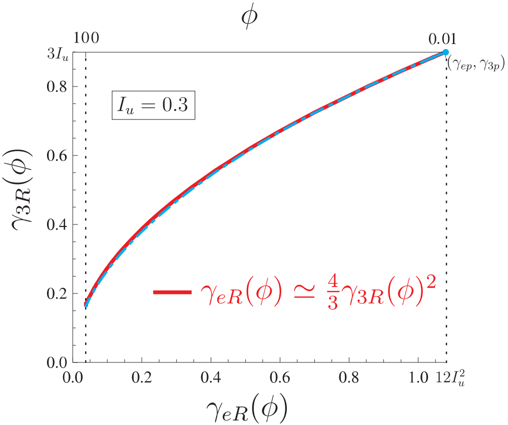


Université de Liège

$\gamma_{e,R} = f(\gamma_{3,R})$

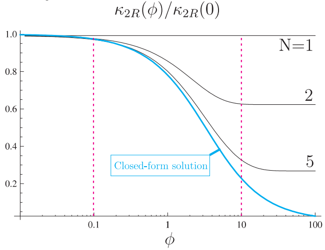


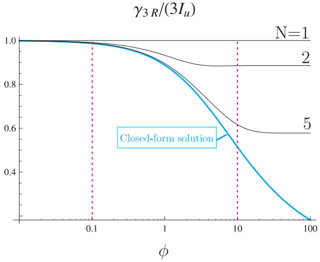
- $\gamma_{e,R} = f(\gamma_{3,R})$

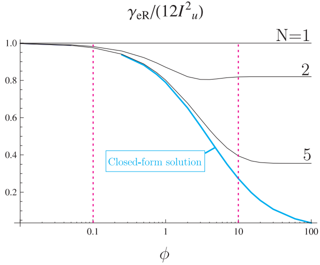


- Could use the approximation $\gamma_{eR}(\phi) \simeq \frac{4}{3}\gamma_{3R}(\phi)^2$ for this type of structural response

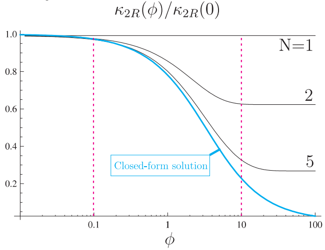
Discrete aerodynamic pressure field

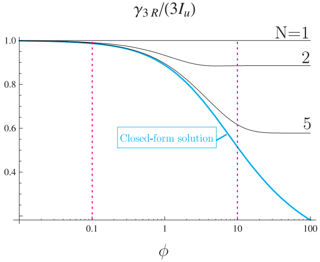


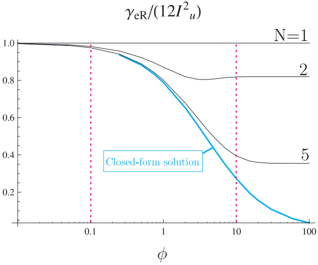




Discrete aerodynamic pressure field







Improvements of the approximations for a same number of sensors ?

Université de Liège

Perspective

- Numerical admittance[1] : correction terms on cumulants and cross-cumulants of order 2, 3 and 4 for discrete aerodynamic pressures field

[1] Denoël, V. and Maquoi, R. (2012). The concept of numerical admittance. *Archive of Applied Mechanics* vol. 82, 10-11, pages 1337-1354.

Perspective

- Numerical admittance[1] : correction terms on cumulants and cross-cumulants of order 2, 3 and 4 for discrete aerodynamic pressures field

May be used

- as is for simple structures : closed-form solutions

[1] Denoël, V. and Maquoi, R. (2012). The concept of numerical admittance. *Archive of Applied Mechanics* vol. 82, 10-11, pages 1337-1354.

The team...

Thomas Canor

Vincent Denoël

...thanks you for your kind attention

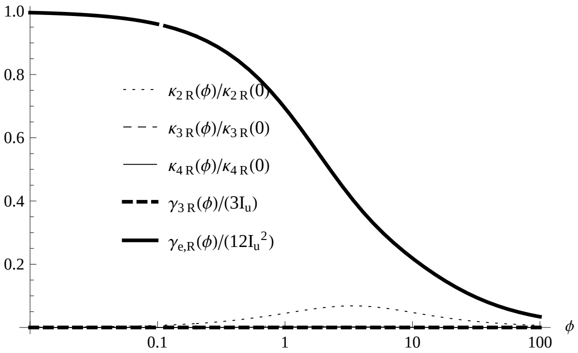
Read out more about us on : www.orbi.ulg.ac.be

Contact me at : N.Blaise@ulg.ac.be

Université de Liège

Questions ?

- Other type of structural responses : $2\xi - 1$
  - $\gamma_{3R}(\phi) = 0$ while $\gamma_{eR}(\phi) \neq 0$



Spectral analysis

$$S_{u_i u_j}(\omega, s_{ij}) = \Gamma_u(\omega, s_{ij}) S_u(\omega)$$

$$\Gamma_u(\omega, s_{ij}) = e^{-\frac{C|\omega|s_{ij}}{2U}}$$

- Structural response

$$R(t) = \int_0^1 I(\xi) p(\xi, t) \mathrm{d}\xi$$

- Power spectral density

$$S_{p_i p_j}(\omega) \simeq b_i b_j S_{u_i u_j}(\omega, s_{ij})$$

$$S_R(\omega) = \int_0^1 \int_0^1 I(\xi_i) I(\xi_j) S_p(\omega, \xi_i, \xi_j) \mathrm{d}\xi_i \mathrm{d}\xi_j$$

Spectral analysis

$$S_{u_i u_j}(\omega, s_{ij}) = \Gamma_u(\omega, s_{ij}) S_u(\omega)$$

$$\Gamma_u(\omega, s_{ij}) = e^{-\frac{C|\omega|s_{ij}}{2U}}$$

- Structural response

$$R(t) = \int_0^1 I(\xi) p(\xi, t) \mathrm{d}\xi$$

- Bi-spectrum

$$\begin{aligned} D_{p_i p_j p_k}(\omega_1, \omega_2) \simeq\ & 2 c_i b_j b_k S_{u_i u_j}(\omega_1) S_{u_i u_k}(\omega_2) + \\ & 2 b_i c_j b_k S_{u_i u_j}(\omega_1) S_{u_j u_k}(\omega_1 + \omega_2) + \\ & 2 b_i b_j c_k S_{u_i u_k}(\omega_2) S_{u_j u_k}(\omega_1 + \omega_2) \end{aligned}$$

$$D_R(\omega_1, \omega_2) = \int_0^1 \int_0^1 \int_0^1 I(\xi_i) I(\xi_j) I(\xi_k) D_p(\omega_1, \omega_2, \xi_i, \xi_j, \xi_k) \mathrm{d}\xi_i \mathrm{d}\xi_j \mathrm{d}\xi_k$$

Spectral analysis

$$S_{u_i u_j}(\omega, s_{ij}) = \Gamma_u(\omega, s_{ij}) S_u(\omega)$$

$$\Gamma_u(\omega, s_{ij}) = e^{-\frac{C|\omega|s_{ij}}{2U}}$$

- Structural response

$$R(t) = \int_0^1 I(\xi) p(\xi, t) \mathrm{d}\xi$$

- Tri-spectrum

$$T_{p_i p_j p_k p_l}(\omega_1, \omega_2, \omega_3) = \dots$$

$$T_R(\omega_1, \omega_2, \omega_3) = \int_0^1 \int_0^1 \int_0^1 \int_0^1 I(\xi_i) I(\xi_j) I(\xi_k) I(\xi_l) T_p(\omega_1, \omega_2, \omega_3, \xi_i, \xi_j, \xi_k, \xi_l) \mathrm{d}\xi_i \mathrm{d}\xi_j \mathrm{d}\xi_k \mathrm{d}\xi_l$$