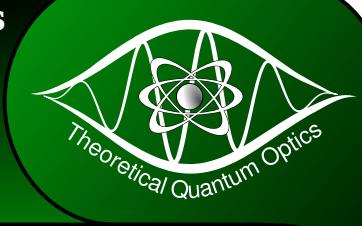


Generation of artificial magnetic fields using dipole-dipole interactions

Alexandre Cesa, John Martin

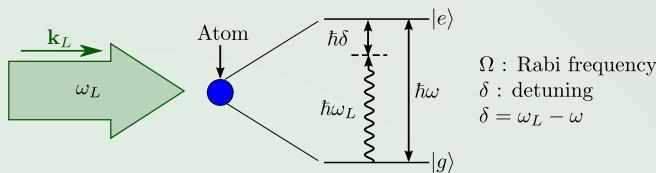
Institut de Physique Nucléaire, Atomique et de Spectroscopie,
Université de Liège, 4000 Liège, Belgium



Introduction

Many works on artificial gauge potentials induced by atom-light interaction adopt a single-particle approach. The predicted potentials are then supposed to be valid for a system of many weakly interacting atoms. So far, the consequences of atom-atom interactions on the generation of artificial gauge fields has little been studied [1]. The aim of this work is to study the artificial gauge fields arising from the interaction of two Rydberg atoms driven by a common laser field [2].

Artificial gauge potentials without atom-atom interactions [3]



Consider a single two-level atom interacting with a classical laser field.

- $E_{\pm} = \pm \hbar \sqrt{|\Omega|^2 + \delta^2}$: eigenvalues of internal Hamiltonian (\hat{H}_{2l})
 $|\chi_{\pm}(\Omega(\mathbf{r}), \delta(\mathbf{r}))\rangle$: eigenstates of \hat{H}_{2l} ; depend parametrically on the atomic position \mathbf{r}
- Total hamiltonian : $\hat{H}_{\text{lat}} = \hat{\mathbf{p}}^2/(2m) \otimes \hat{1}^{\text{int}} + \hat{1}^{\text{ext}} \otimes \hat{H}_{2l}$
- Global wave function in position representation :

$$\langle \mathbf{r} | \psi(t) \rangle = \sum_{j=\pm} \psi_j(\mathbf{r}, t) |\chi_j(\mathbf{r})\rangle$$

- Adiabatic evolution of the internal state ($j = \pm$) :

$$\begin{aligned} \langle \mathbf{r} | \psi(0) \rangle &= \psi_j(\mathbf{r}, 0) |\chi_j(\mathbf{r})\rangle \Rightarrow \langle \mathbf{r} | \psi(t) \rangle \approx \psi_j(\mathbf{r}, t) |\chi_j(\mathbf{r})\rangle \\ i\hbar \partial_t |\psi(t)\rangle &= \hat{H}_{\text{lat}} |\psi(t)\rangle \downarrow \\ i\hbar \frac{\partial}{\partial t} \psi_j(\mathbf{r}, t) &= \left[\left(\hat{\mathbf{p}} - \mathbf{A}^j(\mathbf{r}) \right)^2 / 2m + \phi^j + E_j \right] \psi_j(\mathbf{r}, t) \end{aligned} \quad (1)$$

- Eq. (1) is formally equivalent to Schrödinger's equation for a particle of unit charge immersed in EM fields described by the artificial potentials $\mathbf{A}^j(\mathbf{r})$ and $\phi^j(\mathbf{r})$ given by [3] :

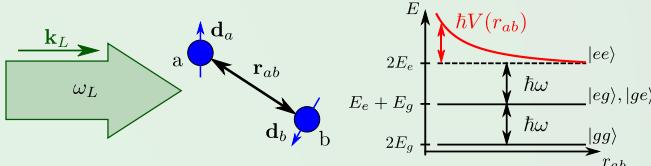
$$\begin{aligned} \mathbf{A}^{\pm}(\mathbf{r}) &= i\hbar \langle \chi_{\pm}(\mathbf{r}) | \nabla_{\mathbf{r}} \chi_{\pm}(\mathbf{r}) \rangle = -\langle \hat{\mathbf{p}} \rangle_{\chi_{\pm}} \\ \phi^{\pm}(\mathbf{r}) &= \hbar^2 |\langle \chi_{\pm}(\mathbf{r}) | \nabla_{\mathbf{r}} \chi_{\pm}(\mathbf{r}) \rangle|^2 / 2m = (\Delta \hat{\mathbf{p}})^2_{\chi_{\pm}} / 2m \end{aligned}$$

new formulation [2]

- $-(\hat{\mathbf{p}} - \mathbf{A}^{\pm})^2 / 2m \Leftrightarrow$ kinetic energy of slow C.M. motion
- ϕ^{\pm} originates from the quantum fluctuations of momentum
- Ω and δ homogeneous $\Rightarrow \mathbf{A}^{\pm}$ homogeneous $\Rightarrow \mathbf{B}^{\pm} = 0$
- Classical electromagnetic field + many noninteracting atoms \Rightarrow same artificial gauge potentials as in the single atom case

Two interacting Rydberg atoms

Consider a pair of Rydberg atoms driven by a common laser field [4] in the case of spatially uniform Ω and δ for which the artificial gauge fields vanish in the absence of atom-atom interactions



- Dipole-dipole interactions \Rightarrow energy shift $\hbar V = \hbar C_3 / r_{ab}^3$ of $|ee\rangle$
 - Two-atom internal Hamiltonian :
- $$\hat{H}_{\text{d-d}} = \hat{H}_{2l,a} + \hat{H}_{2l,b} + \hbar V(r_{ab}) |ee\rangle \langle ee|$$
- Two-atom internal eigenstates $|\chi_i(V(r_{ab}))\rangle$ ($i = 0, 1, \pm$) of energy E_i ; depend parametrically on the atomic positions
 - Definition of a crossover interatomic distance r_c :
- Dipole-dipole Interaction energy $\hbar V(r_c) = \hbar \sqrt{|\Omega|^2 + \delta^2}$ Atom-light Interaction energy

Artificial gauge potentials and fields with dipole-dipole interactions [2]

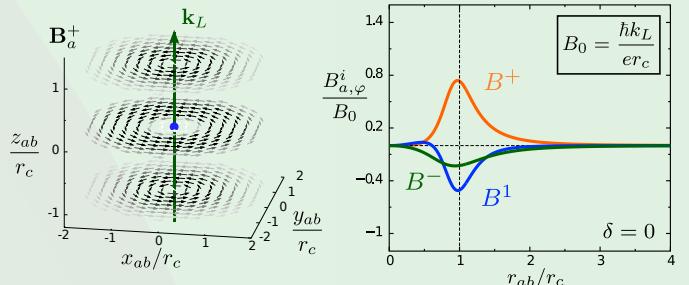
Adiabatic evolution of internal state $|\chi_i\rangle$ ($i = 1, \pm$) \Rightarrow equation for the two-atom spatial wave function $\psi_i(\mathbf{r}_a, \mathbf{r}_b, t)$ equivalent to Schrödinger's equation for two charged particles in EM fields

- Artificial gauge potentials (for atom $\alpha = a, b$, $\mathbf{e}_{\mathbf{k}_L} = \mathbf{k}_L / k_L$) :

$$\begin{aligned} \mathbf{A}_{\alpha}^i &= i\hbar \langle \chi_i | \nabla_{\mathbf{r}_{\alpha}} \chi_i \rangle = A_{\alpha}^i(r_{ab}) \mathbf{e}_{\mathbf{k}_L} \\ \phi_{\alpha}^i &= \frac{\hbar^2}{2m_{\alpha}} \sum_{j \neq i} |\langle \chi_j | \nabla_{\mathbf{r}_{\alpha}} \chi_i \rangle|^2 \end{aligned}$$

- Artificial magnetic fields experienced by atom a for $\mathbf{k}_L = k_L \mathbf{e}_z$, $\mathbf{r}_b = 0$ and spherical coordinates $\{r_{ab}, \theta, \varphi\}$:

$$\mathbf{B}_a^i(r_{ab}) = \nabla_{\mathbf{r}_a} \times \mathbf{A}_a^i = \frac{dA_a^i}{dr_{ab}} \sin \theta \mathbf{e}_{\varphi} = B_{a,\varphi}^i(r_{ab}) \mathbf{e}_{\varphi}$$

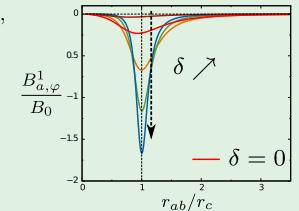


- For realistic Ω , δ , \mathbf{k}_L and $V(r_{ab})$ [4], $r_c \approx 8 \mu\text{m}$ and $B_0 \approx 2 \text{ mT}$

$$\mathbf{A}_a^i = \mathbf{A}_b^i \Rightarrow \mathbf{B}_a^i = -\mathbf{B}_b^i$$

$$\delta \nearrow (\delta \searrow) \Rightarrow |\mathbf{B}_a^i| \nearrow (\searrow)$$

- For $\delta \gg 0$, peaks of intensity of \mathbf{B}_a^i located at $r_{ab} = r_c$ or $r_{ab} = r_c / \sqrt[3]{2}$

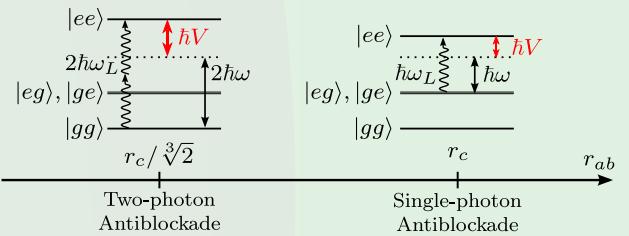


Physical interpretation

The location of the peaks of intensity of $|\mathbf{B}_a^i|$ is related to transitions between bare states. For $\delta \gg 0$, transitions between bare states are possible only when $\hbar V(r_{ab})$ compensates for $\delta \Rightarrow$ **antiblockade** [5]

- Single-photon antiblockade : $\hbar \omega + \hbar V(r_{ab}) = \hbar \omega_L$
 \Rightarrow transitions $|eg\rangle, |ge\rangle \leftrightarrow |ee\rangle$ resonant
- Two-photon antiblockade : $2\hbar \omega + \hbar V(r_{ab}) = 2\hbar \omega_L$
 \Rightarrow transition $|gg\rangle \leftrightarrow |ee\rangle$ resonant

At interatomic distances where the antiblockade is effective, the internal eigenstates $|\chi_i(r_{ab})\rangle$ present strong nonuniform variations which induce large artificial magnetic fields.



Conclusion

We have shown that the combination of atom-atom and atom-field interactions in a uniform laser field gives rise to nonuniform artificial magnetic fields. These fields are maximum where atom-atom and atom-light interactions are of the same order of magnitude. See [2] for more details.

References

- M. Kiffner *et al.*, J. Phys. B: At. Mol. Opt. Phys. **46**, 134008 (2013).
- A. Cesa and J. Martin, Phys. Rev. A **88**, 062703 (2013).
- J. Dalibard *et al.*, Rev. Mod. Phys. **83**, 1523 (2011).
- A. Gaëtan *et al.*, Nature Physics **5**, 115 (2009).
- C. Ates *et al.*, Phys. Rev. Lett. **98**, 023002 (2007).