IMPLICIT STRESS INTEGRATION OF THE BARCELONA BASIC MODEL IN LAGAMINE

A.C. Dieudonne[*,†], F. Collin[*], S. Levasseur[*], R. Charlier[*]

[*] ArGEnCo Department
University of Liege (ULg)
Chemin des Chevreuils 1, 4000 Liege, Belgium
e-mail: ac.dieudonne@ulg.ac.be

[†] F.R.I.A., Fonds de la Recherche Scientifique – FNRS
Rue d'Egmont 5, 1000 Brussels, Belgium

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Abstract. *The paper presents the stress integration scheme used for the implementation of the Barcelona Basic Model in the finite element code Lagamine. The proposed scheme is based on the return mapping algorithm with a backward Euler (fully implicit) time discretization. The accuracy of the stress-strain integration is improved by using a sub-stepping procedure. Verification of the model implementation within the elastic domain is provided by comparing the numerical responses of the model along various stress paths with the corresponding analytical solutions. Additionally, the numerical responses over the whole stress range are compared to the ones provided by Alonso et al. (1990)[i].*

1 INTRODUCTION

In the last decades, particular attention has been paid to the behaviour of compacted clays, especially in relation to their use as engineered barriers in deep geological repositories for nuclear waste. The Barcelona Basic Model[i] (Figure 1) is one of the first elastoplastic models developed to describe the behaviour of unsaturated clays. Although other constitutive models were later developed, they generally keep the core of the Barcelona Basic Model (BBM) which has become a reference model in the field of unsaturated soil mechanics.

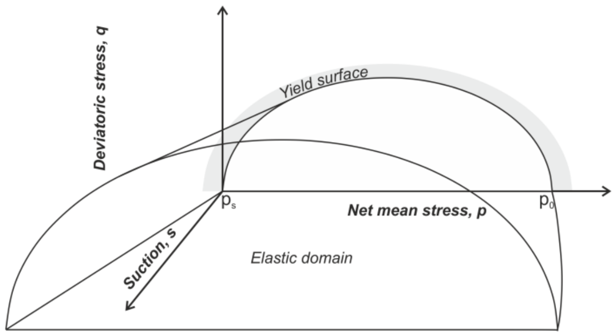


Figure 1: Three-dimensional representation of the yield surfaces in the BBM[i]

The Barcelona Basic Model was implemented in the finite element code Lagamine[ii] which has been developed at the University of Liege. Lagamine is a general-purpose code able to deal with complex non-linear constitutive models, multiphysical coupling, multiscale approaches and strain localization in the fields of metal forming, reservoir engineering and environmental geotechnics.

2 NUMERICAL INTEGRATION IN LAGAMINE

The Barcelona Basic Model is formulated in the form of incremental relationships between stress and strains. Given known incremental displacements or strains (which are given as input of the routine), the integration routine aims at integrating numerically the constitutive relationships in order to update the stresses at integration points.

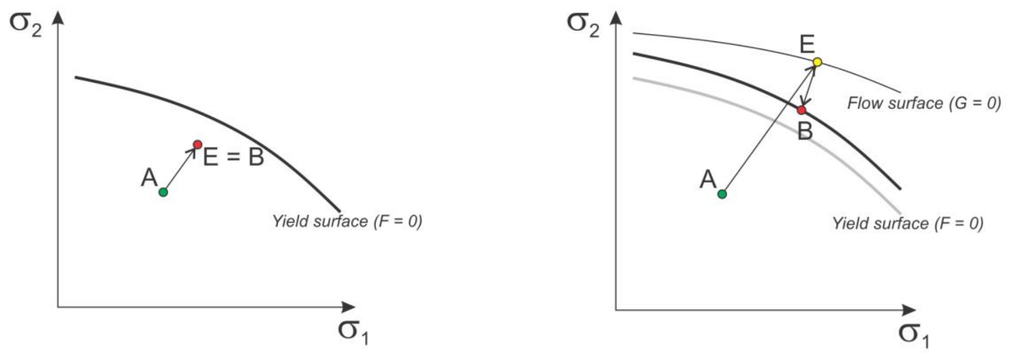
Integration schemes are generally divided into implicit and explicit algorithms, depending if the stress increment is computed using the stiffness at the start or at the end of each step. An implicit scheme, based on the work of Collin[iii] et al., was used for the implementation of the Barcelona Basic Model in Lagamine. Its main features are presented in this section.

2.1 Objectivity of the stress tensor

In large deformation analysis, the incremental formulation requires to define an objective stress rate. Indeed the stress rate should be frame independent, so that any rigid-body motion does not induce stress within the material. Here, the objectivity of the stress tensor is satisfied by adopting the Jaumann objective stress rate which is introduced before integration of the stress-strain constitutive relationships.

2.2 Return mapping algorithm

The return mapping algorithm is based on an elastic predictor and an eventual plastic corrector if the yield criterion is violated (Figure 2). First, an elastic trial stress state (point E) is computed assuming a purely elastic behaviour (the plastic flow is frozen). If the elastic predictor does not violate the yield criterion, then the updated stress state is obtained (Figure 2a). On the other hand, if the yield criterion is violated, a plastic corrector should be applied to return back on the (updated) yield surface (Figure 2b). The updated stress state is on the yield surface (point B). Different techniques exist to return back on the yield surface. The closest-point projection method is used.



(a) (b)

Figure 2: Return mapping algorithm. (a) The yield criterion is not vioalted. The updated stresses are given by the elastic predictor. (b) The yield criterion is violated. A plastic corrector should be applied to return back on the yield surface and update the stresess.

2.3 Mixed control

The Barcelona Basic Model is formulated in terms two stress variables, namely net stress and suction. However, in most conventional finite element codes (such as Code_Brigh or Lagamine), the increment of suction is given as input of the integration routine, like increment of displacements are. Vaunat et al.[iv] introduced generalized stress and strain variables and presented an implicit stress integration scheme with mixed control, where the constitutive equations are driven partly by the strains (suction) and partly by the stresses (mechanical stresses). Such a formulation is used for the integration of the BBM in Lagamine.

2.4 Sub-stepping procedure

During large time steps where yielding occurs, difficulties may appear in returning back to the yield surface. In such situations, a possible strategy is to use smaller time steps in the global resolution algorithm. However, this solution is not reasonable since the most restrictive integration point will control the global problem. An alternative approach consists in subdividing locally the current time step into several substeps (which can be different for each integration point).

A sub-stepping procedure is used in the integration of the Barcelona Basic Model within Lagamine. A time step Δt is divided into $NINTV$ sub-steps, such as $\delta t = \Delta t / NINTV$. The value of $NINTV$ may either be constant, or computed according to the current normal strain rate.

3 VERIFICATION

The verification process consists in controlling that a computational model accurately represents the underlying mathematical model. In order to verify the stress integration of the Barcelona Basic Model, the response of the model along various stress paths were compared to the corresponding analytical solutions within the elastic domain. An example of such a procedure is shown in Figure 3 for a wetting path under oedometer conditions (constant vertical stress). Additionally, the numerical responses over the whole stress range were compared to the ones provided by Alonso et al. (1990)[i].

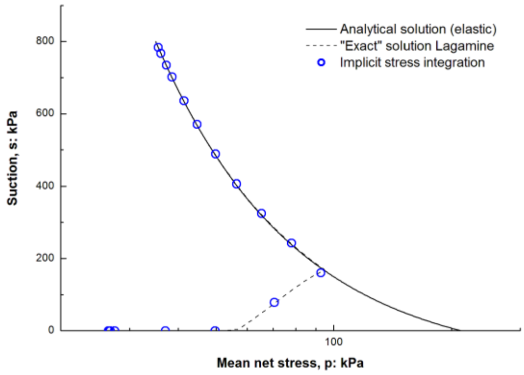


Figure 3: Verification of the model response along wetting under oedometer conditions (constant vertical stress) with the corresponding analytical solution in the elastic domain. The “exact” solution Lagamine refers to the answer of the model obtained for very small loading steps.

4 CONCLUSIONS

The Barcelona Basic Model was implemented within the finite element code Lagamine using a fully implicit stress integration scheme. The implementation was verified using analytical solutions within the elastic domain and compared to responses provided in the literature.

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