

HEAVY-FLAVOUR PENTAQUARKS IN A CHIRAL CONSTITUENT QUARK MODEL

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Abstract

Within the chiral constituent quark model of Glozman and Riska, we discuss the stability of heavy pentaquarks, i.e. hadrons containing four light quarks and a heavy antiquark. The spin-dependent part of the Hamiltonian is dominated by the short-range part of the Goldstone-boson-exchange interaction. We find that these systems are not bound, having an energy above the lowest dissociation threshold into a baryon and a meson.

The question of the existence of hadrons other than mesons $q\bar{q}$ and baryons q^3 remains open. Multiquark states as tetraquarks $q^2\bar{q}^2$, pentaquarks $q^4\bar{q}$ or hexaquarks q^6 , could exist

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in QCD. Based on a model with a chromomagnetic interaction, Jaffe [?] predicted the existence of several multiquark states and among them the $uuddss$ hexaquark, with $J^P = 0^+$ and $I = 0$, known as the H -particle. Since then, production and decay properties of such exotic hadrons have been derived within several models and extensive efforts have been made to find them experimentally [?]. The recent high-sensitivity search at Brookhaven [?] gives no evidence for the production of deeply bound H -particles, as predicted by Jaffe.

This is the present situation in the light sector. However, from theoretical general arguments [?,?,?], one expects an increase in the stability of the multiquark system containing a heavy-flavour quark Q . Experiments are being planned at Fermilab and CERN to search for new heavy hadrons and in particular for doubly charmed tetraquarks [?,?,?]. Very recently, a search for the pentaquark $P_{cs}^0 = \bar{c}suud$ and $P_{cs}^- = \bar{c}sddu$ production, performed at Fermilab, has been reported [?]. Decay properties into the channel $\phi\pi p$ have been analysed assuming a life time ranging from 0.1 to 1 ps and a mass between 2.75 and 2.91 GeV.

This is an incentive to study the stability under strong decays of pentaquarks $q^4\bar{Q}$ where $q = u, d$ or s and $Q = c$ or b . From a theoretical point of view, the field is about ten years old. In the constituent quark model based on one-gluon exchange (OGE) interaction between quarks, the existence of a stable compact pentaquark was first predicted in Refs. [?,?] provided that at least one of the light quarks is strange and $Q = c$ or b . But with a proper treatment of the kinetic energy and a realistic breaking of the flavour $SU_F(3)$ symmetry, the pentaquark turned out to be unstable [?,?]. A systematic study of all pentaquarks has been performed in Ref. [?] where over a dozen are found to be candidates for stability. Among them those with strangeness $S = -1$ or -2 were the most favourable. Calculations using an instanton [?] or a Skyrme model [?] indicate that pentaquarks, which are not necessarily strange, can appear as bound states or as near-threshold resonances, depending on the parameters of the model. On the other hand, the possible existence of heavy-flavour pentaquarks as molecular-type resonances, where the quarks interact by the long-range one-pion exchange interaction, has also been considered [?]. In those calculations, only the $N\bar{B}^*$ system was found to be bound. Finally, a study of charmed multiquarks in a bag model,

where no bound state is found, has also recently appeared [?].

The present study can be viewed as a natural extension of our previous investigations [?] on the stability of heavy-flavour tetraquarks within the chiral constituent quark model of Glozman and Riska [?,?,?,?]. In this model, the hyperfine splitting in hadrons is due to the short-range part of the Goldstone boson exchange (GBE) interaction, instead of the one-gluon exchange (OGE) used in conventional models. In Ref. [?], we found that the GBE interaction strongly binds the heavy tetraquark system $QQ\bar{q}\bar{q}$ where $Q = c$ or b ; this is at variance with the models based on OGE interaction where $cc\bar{q}\bar{q}$ is unstable. Here we also find results somehow different from the previous literature. We show that the GBE interaction induces a short-range repulsion, of several hundreds of MeV in both P_c and P_b systems. However, the GBE interaction also generates a long-range attraction due to its Yukawa-type potential. Together with a correlated two pseudoscalar-meson exchange, this will induce a long- and a medium-range attraction which could favour a deuteron-size (molecular type) bound state. In the following, we restrict ourselves to the study of compact objects.

In the spirit of Jaffe's pioneering work [?], here we raise the question whether or not the model of Glozman and Riska can accomodate a strongly bound, compact pentaquark. We shall compare our results with those based on OGE, as mentioned above.

The model of Glozman and Riska reproduces well the light-quark baryon spectrum [?,?,?] and some extension to heavy baryons has also been proposed [?]; in particular, it gives the correct ordering of positive and negative parity states in all parts of the considered spectrum. The detailed form of the Hamiltonian used here will be given below. First it is useful to consider a simplified GBE interaction of the form (where radial dependence is neglected):

$$V_\chi = -C_\chi \sum_{i<j} \lambda_i^F \cdot \lambda_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j, \quad (1)$$

where λ_i^F ($F = 1, 2, \dots, 8$) are the quark-flavour Gell-Mann matrices (with an implicit summation over F) and $\vec{\sigma}$ are the spin matrices. The minus sign of the interaction (??) is related to the sign of the short-range part of the GBE interaction, crucial for the hyperfine splitting in baryon spectroscopy. This feature of the short-range part of the GBE interaction is dis-

cussed at length by Glozman and Riska [?]. A typical order of magnitude for the constant C_χ is about 30 MeV.

Here, we study pentaquarks with strangeness $S = 0, -1, -2$ and -3 . As in Ref. [?], we neglect the interaction due to meson exchange between a light quark and a heavy antiquark. Then the GBE interaction in a pentaquark $q^4\bar{Q}$ reduces to the interaction between the light quarks $q = u, d$ or s .

The subsystem of four light quarks must be in a colour **3** state, or alternatively $[211]_C$, in order to give rise to a colourless pentaquark. To calculate the GBE matrix elements, we assume that all light quarks are identical and that the ground-state orbital wave function is symmetric under permutation of light quarks. The corresponding Young diagram notation is $[4]_O$. Then all combined spin $[f]_S$ and flavour $[f]_F$ symmetries, allowed by Pauli principle, form the states indicated in the first column of Table ???. They have been obtained from inner product rules [?] applied to the permutation group S_4 . In column 2, the corresponding values of the spin S are indicated for each $[f]_S$ representation of $SU_S(2)$. The total angular momentum of the pentaquark is $\vec{J} = \vec{S} + \vec{s}_Q$ where $s_Q = 1/2$. The isospin I associated to a given $SU_F(3)$ representation is given in columns 3–6 for four distinct contents of the four-quark subsystem with strangeness $S = 0, -1, -2$ and -3 . The last column indicates the expectation value of the operator (??) which can be conveniently calculated from the formula:

$$\langle V_\chi \rangle = N(N - 10) + \frac{4}{3}S(S + 1) + 2C_F + 4C_C \quad (2)$$

where $N = 4$, $C_C = 4/3$ and C_F is the $SU_F(3)$ Casimir operator eigenvalue. One can see that the states in column 1 are given in an increasing order of $\langle V_\chi \rangle$. Thus $[[22]_S[211]_F]$ is most favourable (negative) for the structure $uuds\bar{Q}$ ($I = 1/2$, $J = 0$ or 1) or $udss\bar{Q}$ ($I = 0$, $J = 0$ or 1), $[[31]_S[22]_F]$ for $uudd\bar{Q}$ ($I = 0$, $J = 1/2$ or $3/2$) and $[[22]_S[31]_F]$ for $usss\bar{Q}$ ($I = 1/2$, $J = 1/2$). The column $\langle V_\chi \rangle$ also suggests that other $q^4\bar{Q}$ systems, with IJ different from the above values, would be heavier, thus there is no reason to present results for such obviously unstable states.

The GBE Hamiltonian has the form [?] :

$$H = \sum_i m_i + \sum_i \frac{\vec{p}_i^2}{2m_i} - \frac{(\sum_i \vec{p}_i)^2}{2\sum_i m_i} + \sum_{i<j} V_{\text{conf}}(r_{ij}) + \sum_{i<j} V_\chi(r_{ij}), \quad (3)$$

with the linear confining interaction :

$$V_{\text{conf}}(r_{ij}) = -\frac{3}{8}\lambda_i^c \cdot \lambda_j^c C r_{ij}, \quad (4)$$

and the spin-spin component of the GBE interaction in its $SU_F(3)$ form :

$$V_\chi(r_{ij}) = \left\{ \sum_{F=1}^3 V_\pi(r_{ij}) \lambda_i^F \lambda_j^F + \sum_{F=4}^7 V_K(r_{ij}) \lambda_i^F \lambda_j^F + V_\eta(r_{ij}) \lambda_i^8 \lambda_j^8 + V_{\eta'}(r_{ij}) \lambda_i^0 \lambda_j^0 \right\} \vec{\sigma}_i \cdot \vec{\sigma}_j, \quad (5)$$

with $\lambda^0 = \sqrt{2/3} \mathbf{1}$, where $\mathbf{1}$ is the 3×3 unit matrix. The interaction (??) contains $\gamma = \pi, K, \eta$ or η' meson-exchanges and the form of $V_\gamma(r_{ij})$ is given explicitly in Ref. [?] as the sum of two distinct contributions: a Yukawa-type potential containing the mass of the exchanged meson and a short-range contribution of opposite sign, the role of which is crucial in baryon spectroscopy. For a given meson γ , the meson exchange potential is :

$$V_\gamma(r) = \frac{g_\gamma^2}{4\pi} \frac{1}{12m_i m_j} \left\{ \Theta(r - r_0) \mu_\gamma^2 \frac{e^{-\mu_\gamma r}}{r} - \frac{4}{\sqrt{\pi}} \alpha^3 \exp(-\alpha^2(r - r_0)^2) \right\}, \quad \gamma = \pi, K, \eta, \eta'. \quad (6)$$

For the Hamiltonian (??)–(??), we use the parameters of Ref. [?]. These are :

$$\frac{g_{\pi q}^2}{4\pi} = \frac{g_{\eta q}^2}{4\pi} = \frac{g_{Kq}^2}{4\pi} = 0.67, \quad \frac{g_{\eta' q}^2}{4\pi} = 1.206,$$

$$r_0 = 0.43 \text{ fm}, \quad \alpha = 2.91 \text{ fm}^{-1}, \quad C = 0.474 \text{ fm}^{-2}, \quad m_{u,d} = 340 \text{ MeV},$$

$$\mu_\pi = 139 \text{ MeV}, \quad \mu_\eta = 547 \text{ MeV}, \quad \mu_{\eta'} = 958 \text{ MeV}, \quad \mu_K = 495 \text{ MeV}. \quad (7)$$

They provide a very satisfactory description of low-lying nonstrange baryons, extended to strange baryons in Ref. [?] where a dynamical three-body calculation is performed as well. The latter reference gives $m_s = 0.440 \text{ GeV}$.

First, we calculate variationally the masses of the threshold hadrons using the Hamiltonian (??)–(??). The meson mass is obtained from a trial wave function of type $\Phi \propto$

$\exp[-\alpha r^2/2]$ with $\vec{r} = \vec{r}_1 - \vec{r}_2$ by varying the parameter α . The heavy-quark mass is adjusted to reproduce the experimental average mass $\bar{M} = (M + 3M^*)/4$ of $M = D$ or B mesons. This gives $m_c = 1.35$ GeV and $m_b = 4.66$ GeV. With all quark masses now fixed, we estimate the baryon masses with a trial wave function of type $\Phi \propto \exp[-\alpha(\rho^2 + \lambda^2)/2]$ where $\vec{\rho} = \vec{r}_2 - \vec{r}_3$ and $\vec{\lambda} = (2\vec{r}_1 - \vec{r}_2 - \vec{r}_3)/\sqrt{3}$. The results for baryons and mesons are exhibited in Table ??, together with the experimental values. The theoretical values lie at most ~ 50 MeV above the experiment. The baryons masses, obtained from the same Hamiltonian but in a more precise Fadeev type calculations [?] are also indicated.

For the pentaquarks, it is useful to consider the following system of internal Jacobi coordinates:

$$\begin{aligned}\vec{x} &= \vec{r}_1 - \vec{r}_2, \\ \vec{y} &= (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{3}, \\ \vec{z} &= (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_4)/\sqrt{6}, \\ \vec{t} &= (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 - 4\vec{r}_5)/\sqrt{10}.\end{aligned}\tag{8}$$

Then we assume a variational wave function of the form:

$$\Psi = \left(\frac{a}{\pi}\right)^{9/4} \left(\frac{b}{\pi}\right)^{3/4} \exp\left[-\frac{a}{2}(x^2 + y^2 + z^2) - \frac{b}{2}t^2\right].\tag{9}$$

The expectation value of the kinetic energy T involves the average of the inverse light masses defined as:

$$4/\mu_1 = \begin{cases} 1/m_a + 3/m_b & \text{for } q_a q_b^3, \\ 2/m_a + 2/m_b & \text{for } q_a^2 q_b^2, \end{cases}\tag{10}$$

for the light subsystem. With

$$\frac{1}{\mu_2} = \frac{1}{4\mu_1} + \frac{1}{m_Q},\tag{11}$$

the internal kinetic energy of the $q^4\bar{Q}$ system reads

$$T = \frac{3}{2} \left(\frac{3}{\mu_1} a + \frac{4}{5\mu_2} b \right).\tag{12}$$

For the calculation of the confinement potential energy, we first need to evaluate the colour operator matrix elements :

$$\langle O^C \rangle = \left\langle \sum_{i < j}^4 \frac{\lambda_i^c}{2} \cdot \frac{\lambda_j^c}{2} \right\rangle = 1/2(C_C - 4C_q), \quad (13)$$

where $C_q = 4/3$. For the colour state $[211]_C$ one has $C_C = 4/3$. Hence $\langle O^C \rangle_{[211]} = -2$. As there are 6 pairs in (??) one gets

$$\left\langle \frac{\lambda_i^c}{2} \cdot \frac{\lambda_j^c}{2} \right\rangle = -1/3, \quad i, j = q, \quad (14)$$

for each pair. For the antiquark the Casimir operator average is $C_{\bar{Q}} = 4/3$. Then for the light-heavy system, in a colour-singlet state with $C_C^0 = 0$, the colour average operator is

$$\left\langle \sum_{i=1}^4 \frac{\lambda_i^c}{2} \cdot \frac{\lambda_{\bar{Q}}^c}{2} \right\rangle = (C_C^0 - C_C - C_{\bar{Q}})/2 = -4/3, \quad (15)$$

from which each quark shares $1/4$. Hence

$$\left\langle \frac{\lambda_i^c}{2} \cdot \frac{\lambda_j^c}{2} \right\rangle = -1/3, \quad i = q, j = \bar{Q}. \quad (16)$$

This proves that the colour matrix element is $-1/3$ for all pairs. With the trial wave function (??) the space part of the confining interaction can be calculated exactly. Then the total confining energy is

$$\langle V_{\text{conf}} \rangle = \frac{C}{2} (6\langle r_{12} \rangle + 4\langle r_{45} \rangle) = \frac{2C}{\sqrt{\pi}} \left(\frac{3}{\sqrt{a}} + \sqrt{\frac{3}{2a} + \frac{5}{2b}} \right). \quad (17)$$

The matrix elements of the spin-flavour operators entering the GBE interaction (??) have been calculated by using the fractional parentage technique described in Ref. [?]. Accordingly, we first determine the explicit form of the spin and flavour states of a given symmetry $[f]_S$ and $[f]_F$ respectively. Then we rewrite each wave function as a sum of products of the wave function of the first two and of the last two particles. These are all either symmetric or antisymmetric two-body states, so that Eqs. (3.3) of Ref. [?] can be straightforwardly applied. In Table ??, we exhibit $\langle V_\chi \rangle$ of Eq. (??) for the cases of interest. The upper index uu or us in the orbital two-body matrix element V designates the nature of the quark

masses in the product $m_i m_j$ in Eq. (??). With the wave function (??), V can be explicitly calculated as:

$$V_{\mu}^{ij} = \frac{g^2}{4\pi} \frac{1}{12 m_i m_j} \frac{4 a^{3/2}}{\pi^{1/2}} \left\{ \frac{\mu^2}{2a} \exp[-r_0(\mu + a r_0)] - \frac{\alpha^3}{2 \exp(\alpha^2 r_0^2) (a + \alpha^2)^{3/2}} \right. \\ \left. \left(\frac{4\alpha^2 r_0}{\sqrt{(a + \alpha^2)} \sqrt{\pi}} + \left(2 + \frac{4\alpha^4 r_0^2}{a + \alpha^2} \right) \exp\left[\frac{4\alpha^2 r_0}{a + \alpha^2}\right] \operatorname{Erfc}\left[\frac{-\alpha^2 r_0}{\sqrt{(a + \alpha^2)}}\right] \right) \right. \\ \left. - \exp\left[\mu^2/(4a)\right] \operatorname{Erfc}\left[\frac{\mu + 2a r_0}{2\sqrt{a}}\right] \mu^3 a^{-3/2}/4 \right\} \quad (18)$$

In Table ??, we present the energy E of each $q^4 \bar{Q}$ system under discussion, obtained by minimizing $\langle H \rangle$ with respect to a and b . Compared to the lowest threshold mass E_T given in the last column of Table ??, one can see that the pentaquark energy lies several hundred MeV above the corresponding threshold, in all cases. One can also notice that the lowering of $E - E_T$ when replacing the quark c by b is small, of the order of 10 MeV.

Among the systems studied here, the pentaquarks with $S = -2$ appear to be the most “favourable”, i.e., have the lowest $E - E_T$, although the value of E of a $uuds\bar{Q}$ system is lower than that of a $udss\bar{Q}$ system, due to a larger GBE attraction, as discussed above. The presence of two strange quarks, instead of one, has a negligible effect. Thus the lowering of $E - E_T$ for $udss\bar{Q}$ is essentially due to an increase in E_T when passing from the $N + s\bar{Q}$ threshold to the $\Lambda + s\bar{Q}$ threshold.

In this first simple application of the chiral constituent quark model [?,?,?,?] to the pentaquark spectrum, we did not find any deeply bound state. We however believe that the present results are not completely discouraging for the reasons which follow. The energy of strange pentaquarks is located above the threshold only roughly 1/2 from the amount found for hexaquarks with one heavy flavoured quark [?]. The values found for E depend on the approximations involved. A lowering of E can be achieved by an increase of the basis vectors. A possibility would be to consider orbital states of type $[22]_O$ containing two quanta of excitation, like in the nucleon-nucleon case [?]. The lowest allowed value of V_{χ} , Eq.(??), is $-16 C_{\chi}$ as for $[4]_O[22]_S[211]_F$ of Table ?. Therefore an interference of the $[4]_O$ state considered here with the $[22]_O$ state is possible.

Moreover the quark-antiquark interaction was entirely neglected here. Assuming that the instanton-induced interaction is very important for mesons (see e.g. [?]), one could expect that an additional attractive contribution of this type would contribute towards stability of pentaquarks with respect to the meson–baryon threshold. As mentioned in the introduction, the GBE interaction through its Yukawa potential tail can also generate a medium- and long-range interaction between clusters. The structure of Eq. (??) suggests that, in a frozen configuration, a strong repulsion at short distances implies a long-range attraction. Hence our conclusion on the non-existence of deeply-bound compact pentaquarks encourages pursuing theoretical investigation of deuteron-like baryon-meson systems, based on a dynamical approach of a few-body system, as for example the resonating group method.

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TABLES

TABLE I. The allowed $[[f]_S[f]_F\rangle$ states of colour **3** for the q^4 subsystem of pentaquarks with strangeness 0, -1 , -2 and -3 . The spin S and isospin I are indicated in each case [?]. The quantity $\langle V_\chi \rangle$ is the corresponding expectation value of the operator (??) in units of C_χ .

$[[f]_S[f]_F\rangle$	S	I				$\langle V_\chi \rangle$
		$uudd\bar{Q}$	$uuds\bar{Q}$	$udss\bar{Q}$	$usss\bar{Q}$	
$[[22]_S[211]_F\rangle$	0	/	1/2	0	/	-16
$[[31]_S[211]_F\rangle$	1	/	1/2	0	/	-40/3
$[[31]_S[22]_F\rangle$	1	0	1/2	1	/	-28/3
$[[22]_S[31]_F\rangle$	0	1	1/2,3/2	0,1	1/2	-8
$[[4]_S[31]_F\rangle$	2	1	1/2,3/2	0,1	1/2	0
$[[31]_S[4]_F\rangle$	1	2	3/2	1	1/2	8/3

TABLE II. Masses of hadrons required to calculate the threshold energy $E_T = \text{baryon} + \text{meson}$. For D and B we indicate $M = (M + 3M^*)/4$. For the baryons, a comparison is shown with the Fadeev calculations of Ref. [?]

Hadron	Mass (GeV)		
	variational	Ref. [?]	experiment
N	0.970	0.939	0.939
Λ	1.165	1.136	1.116
Σ	1.235	1.180	1.192
Ξ	1.377	1.348	1.318
D	2.008		1.973
D_s	2.087		2.076
B	5.302		5.313
B_s	5.379		5.375

TABLE III. Energies of pentaquarks with $J = 1/2$ and $S = 0, -1, -2$ and -3 . In column 3, V_γ^{ab} ($\gamma = \pi, \eta, K$ or η') designates the quark-quark matrix elements of the interaction (??).

Pentaquark	I	$\langle V_\chi \rangle$ (Eq.(??))	Energy (GeV)	Lowest threshold	
				$B + M$	E_T (GeV)
$uudd\bar{c}$	0	$10V_\pi^{uu} - 2/3V_\eta^{uu} - 4/3V_{\eta'}^{uu}$	3.607	$N + \bar{D}$	2.978
$uudd\bar{b}$	0		6.889	$N + \bar{B}$	6.272
$uuds\bar{c}$	1/2	$7V_\pi^{uu} - 7/9V_\eta^{uu} - 14/9V_{\eta'}^{uu} +$	3.545	$N + \bar{D}_s$	3.057
$uuds\bar{b}$	1/2	$22/3V_K^{us} + 22/9V_\eta^{us} - 22/9V_{\eta'}^{us}$	6.827	$N + \bar{B}_s$	6.349
$udss\bar{c}$	0	$4V_\pi^{uu} - 4/9V_\eta^{uu} - 8/9V_{\eta'}^{uu} +$	3.630	$\Lambda + \bar{D}_s$	3.253
$udss\bar{b}$	0	$28/3V_K^{us} + 28/9V_\eta^{us} - 28/9V_{\eta'}^{us}$	6.911	$\Lambda + \bar{B}_s$	6.544
$uss\bar{c}$	1/2	$22/3V_K^{us} + 26/9V_\eta^{us} - 26/9V_{\eta'}^{us} -$	3.940	$\Xi + \bar{D}_s$	3.464
$uss\bar{b}$	1/2	$20/9V_\eta^{ss} - 10/9V_{\eta'}^{ss}$	7.223	$\Xi + \bar{B}_s$	6.756