The application of NSGA-II optimization method in designing control charts

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Outline:

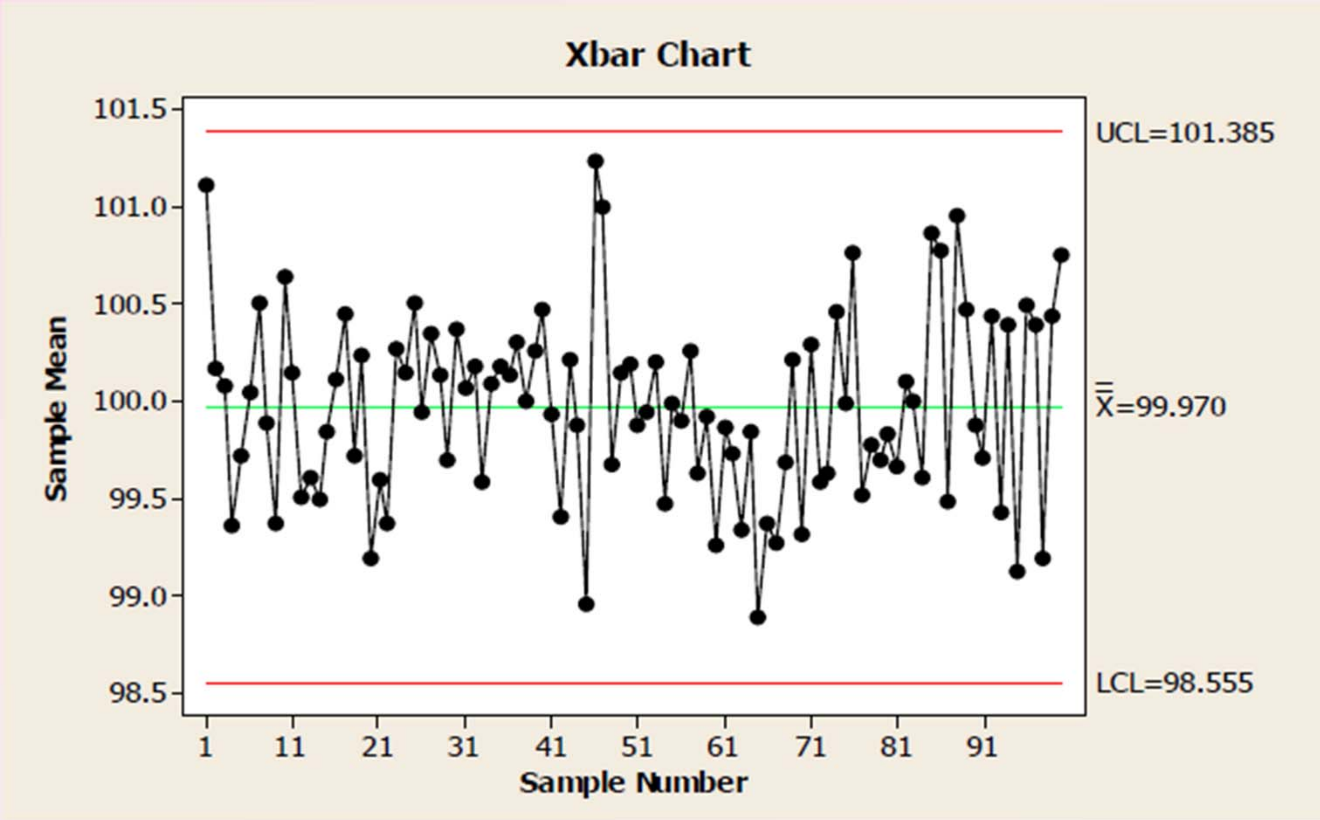
1. Brief description of control charts
2. Designing control charts
3. Multi-objective approach to designing control charts
4. NSGA-II optimization method
5. Our results
6. Conclusion

We first assume independent samples of size *n* >1 taken over time from a normal distribution. When process is stable, the mean is µ0 and the standard deviation σ0. In Phase II the ideal *X-bar* control chart limits would be the following:

$$LCL = \mu_0 - L\frac{\sigma_0}{\sqrt{n}} \qquad UCL = \mu_0 + L\frac{\sigma_0}{\sqrt{n}}$$

In practice, however, one must estimate µ0 and σ0 using *m* Phase I samples of size *n*.

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{m} \overline{X}_i}{m} \qquad \hat{\sigma}_1 = \frac{\overline{R}}{d_2(n)}, \qquad \hat{\sigma}_2 = \frac{\overline{S}}{c_4(n)},$$



The chart parameters: n, h, l

Different approaches in Designing Control Charts:

1- **Heuristic designs**:

Shewhart's design: k=3; n=3 , 4 or 5 and h at the ease of user, usually h=1.
Ishikawa's design: k=3; *n*=5 and *h*=8.
Figienbam's design: k=3; *n*=5, *h*=1
Juran's design: k=3; *n*=4 and *h*=7.

2- **Statistical designs**:

the parameter *k* is set according to the desired Type I error rate; The parameters *n* and *h* are selected such that maximize the power of the chart to detect a specific mean shift.

3- **Economic and Economic Statistical designs**:

Duncan (1956) was the first who determined the costs associated with the implementation of Xbar chart as a function of the chart parameters. Duncan found that the heuristic designs result in very large penalties when compared to EDs.

Woodall (1986) showed that Eds have poor statistical properties (large number of false alarms and poor power in detecting shifts).

Saniga (1989) introduced ESD by adding statistical constraints on optimal economic designs. His approach joints the benefits of the both statistical and economic designs.

The most two important objectives in Designing Control Charts:

The above mentioned designs are blind, The user doesn’t have a clear view of the tradeoff between different objects:

A) Economic Objective: Expected Cost

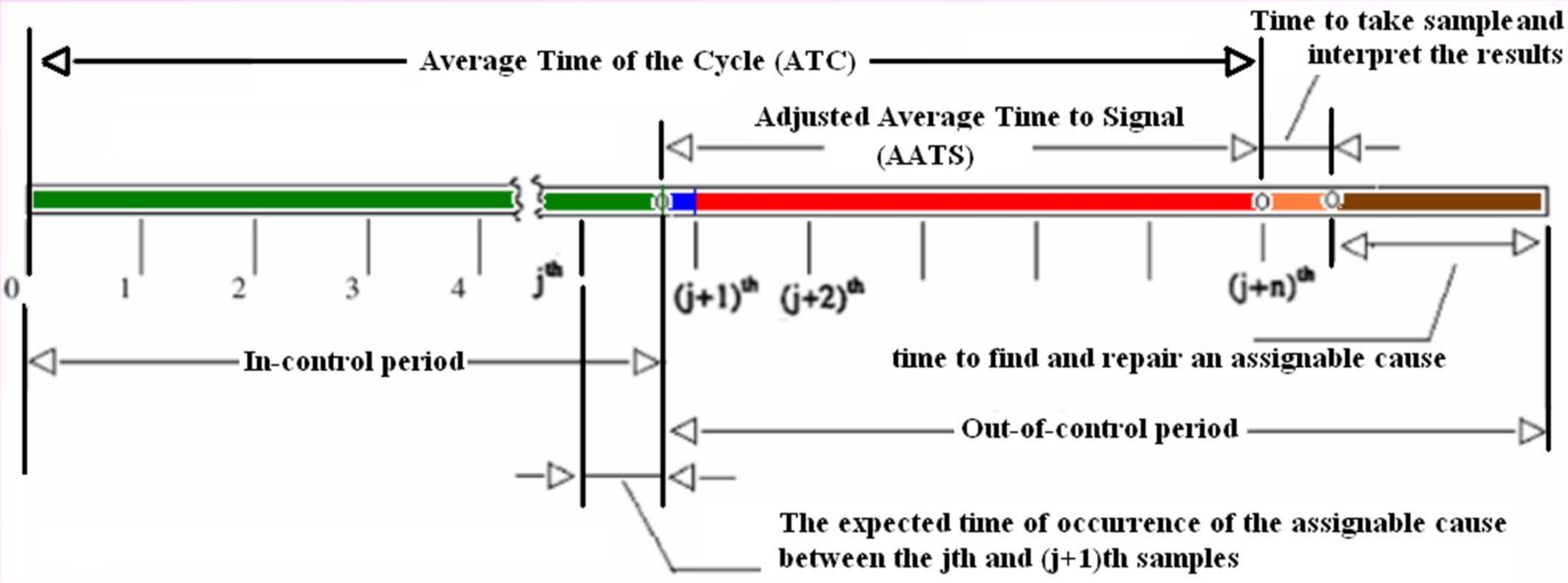
The Cost of Sampling
The cost of producing non-confirming products
The cost of investigating the process following false alarms
The cost of searching for assignable causes and then repairing the process

B) Statistical Objectives:

The false alarm rate = 1/alfa
The chart power   = AATS

There is a need to multi-objective methods which considers the above objectives simultaneously

A Quality Cycle



Important Objectives in Designing control charts (Minimizing Type):

A) Economic Objective: Average Cost per hour = E(c)/E(T)

B) Statistical Objectives: ARL1= 1/(1-beta) , ARL0= 1/alpha

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MULTIOBJECTIVE ECONOMIC DESIGN OF AN X CONTROL CHART

Celano G. and Fichera S.

$$Fm = E(L) * (W_1 + W_2 * \alpha - W_3 * (1 - \beta))$$

where W_1 W_2 W_3 are the coefficients which values ranging from 0.1 and 2.

$W_1 = 1.3$, $W_2 = 1$, $W_3 = 0.5$.

IIE Transactions (1996) **28**, 467–474

Multiple-criteria optimal design of $\overline{X}$ control charts

ENRIQUE DEL CASTILLO[1], PATRICK MACKIN[2] and DOUGLAS C. MONTGOMERY[3]

$$\min f_1 = \alpha/\lambda h,$$

$$\min f_2 = \text{ATS} = h/P,$$

$$\min f_3 = (b + cn)\left[1/\lambda h + 1/P + \tfrac{1}{12}\lambda h - \tfrac{1}{2}\right],$$

Solution: Combining the individual objective functions into a single composite function.

Y= λ1*ANF+ λ 2*ATS + λ 3*(Expecteded_Cost)

λ_1	λ_2	λ_3	f_1^p	f_2^p	f_3^p	α	P	n	k	h	*Pref.*
0.373	0.369	0.257	0.998	0.960	0.938	0.0002	0.927	6.6	3.68	0.32	*
0.129	0.535	0.335	0.992	0.964	0.937	0.0009	0.929	5.7	3.32	0.30	
0.128	0.528	0.343	0.992	0.963	0.938	0.0009	0.929	5.7	3.31	0.30	
0.215	0.473	0.311	0.996	0.962	0.938	0.0005	0.928	6.1	3.48	0.31	

The problem lies in the defining of the weights or the decision-maker's preferences. In practice, it can be very difficult to precisely and accurately select these weights, even for someone familiar with the problem domain.

If the user likes multiple solutions, he must solve the problem multiple times with different weight.

Multiobjective optimization (**vector optimization**, **multicriteria optimization**, **multiattribute optimization** or **Pareto optimization**) is concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously where optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives.

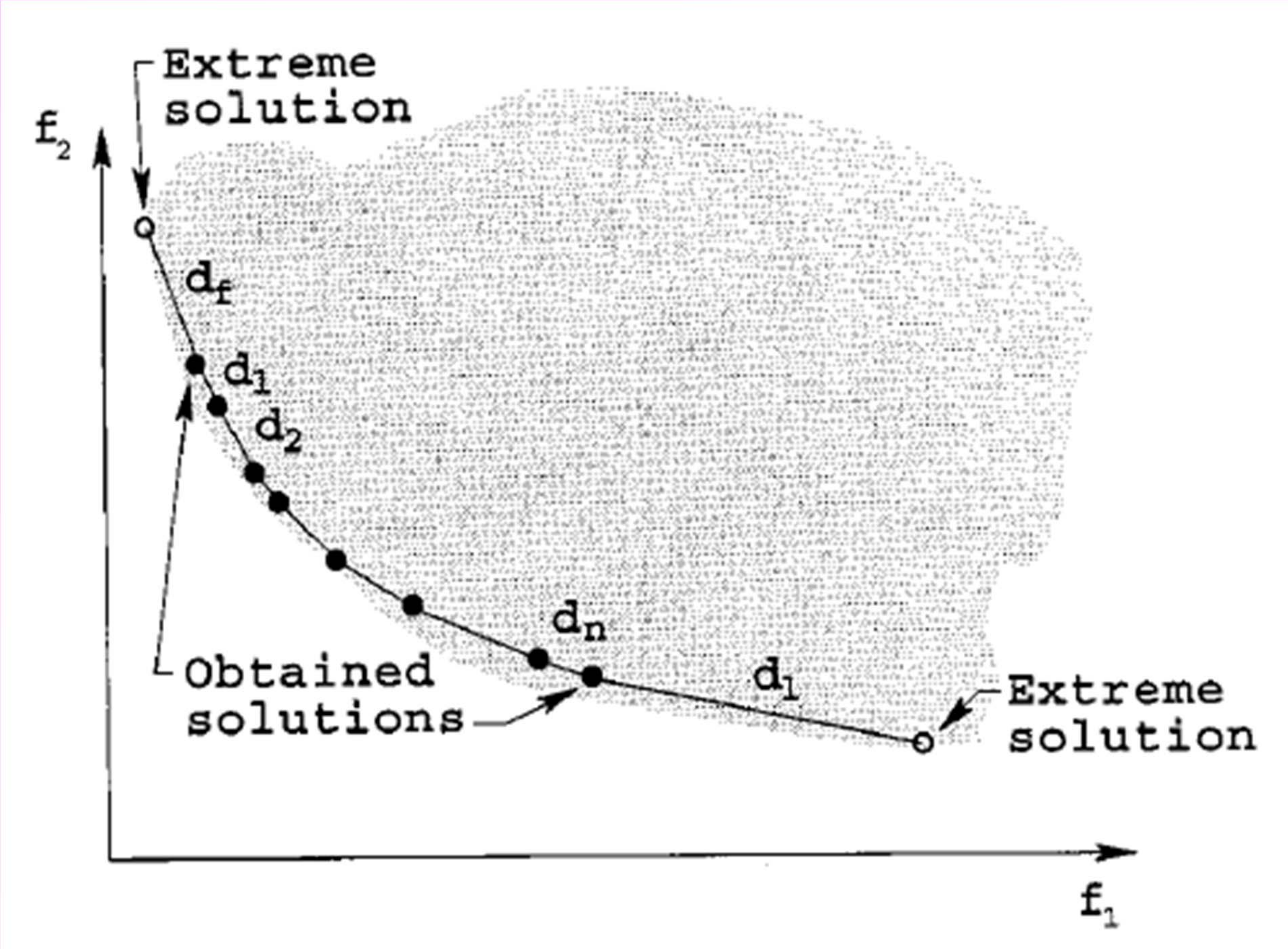
Minimizing Expected Loss per item/hour while maximizing the power of control charts, and minimizing Type I error rate is an example of multiobjective design of control charts.

182 IEEE TRANSACTIONS ON EVOLUTIONARY COMPUTATION, VOL. 6, NO. 2, APRIL 2002

A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II

Kalyanmoy Deb, *Associate Member, IEEE*, Amrit Pratap, Sameer Agarwal, and T. Meyarivan

At each generation of size N, all chromosomes are compared together to determine the set of non)dominated solutions in each generations, All nondominated solutions get a fitness value based on the number of solutions they dominate and dominated solutions are assigned fitness worse than the worst fitness of any nondominated solution. This assignment of fitness makes sure that the search is directed toward the nondominated solutions.


Extreme
solution
f_2
d_f
d_1
d_2
d_n
Obtained
solutions
d_l
Extreme
solution
f_1

Min (E(A); AATS), subject to:

$\alpha \leq \alpha_u$

$\boldsymbol{L} \leq \boldsymbol{D} \leq \boldsymbol{U}$

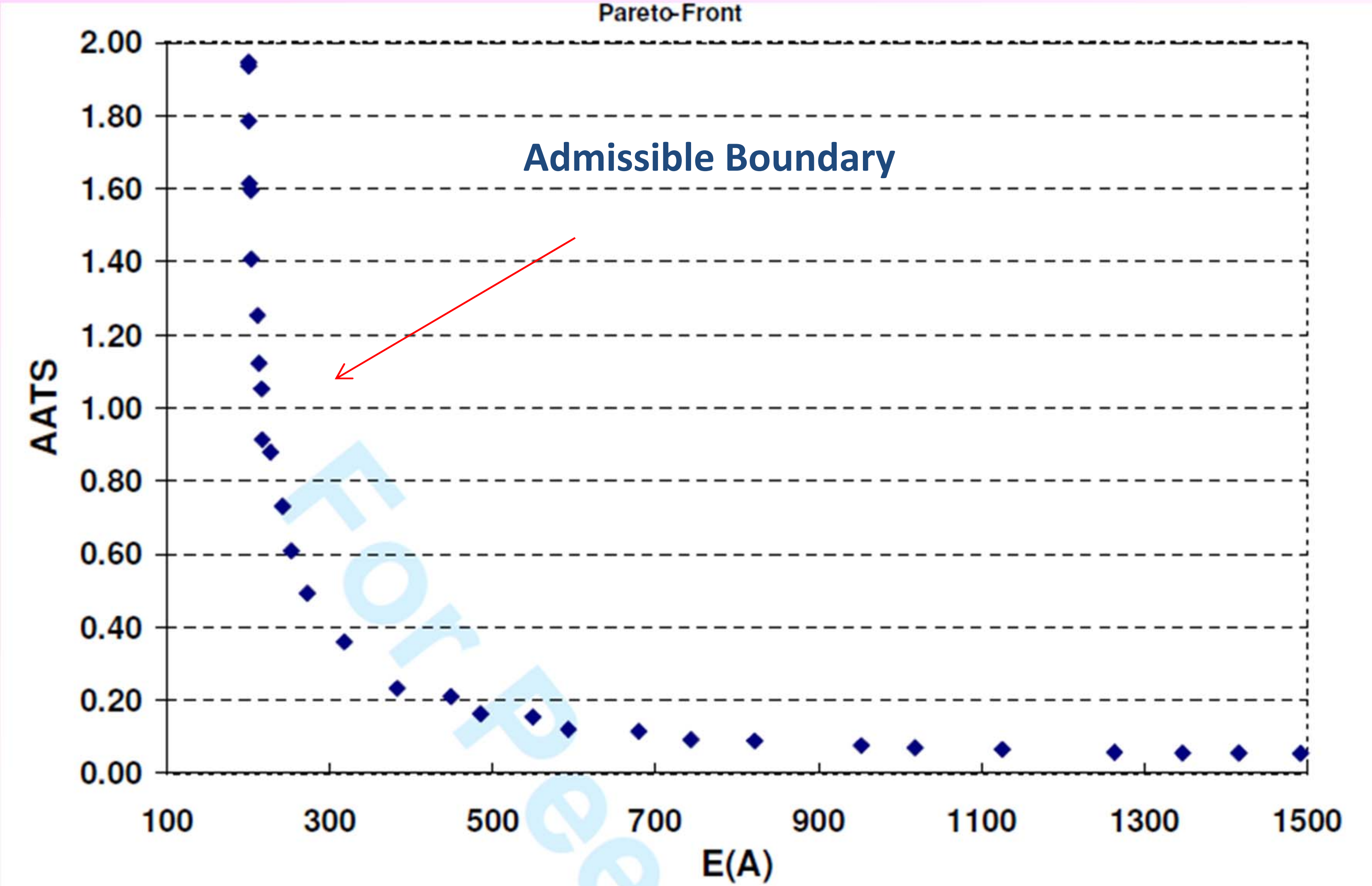


Table 2. The MOESD of joint Xbar and S^2 charts for Table 1

ID	*k*	*l*	*n*	*h*	E(A)	AATS	*α*
1	2.86	3.47	7	1.42	200.10	1.95	0.005
2	2.86	3.47	7	1.43	200.18	1.94	0.005
3	2.86	3.47	7	1.30	200.30	1.79	0.005
4	2.86	3.47	7	1.18	201.05	1.61	0.005
5	2.99	3.85	6	0.92	203.22	1.59	0.004
6	2.86	3.72	6	0.92	203.40	1.41	0.005
7	3.05	3.85	8	0.80	211.19	1.25	0.003
8	3.05	3.60	7	0.73	212.88	1.12	0.003
9	3.05	3.72	7	0.67	214.32	1.05	0.003
10	2.86	3.47	7	0.67	216.77	0.91	0.005
11	3.21	3.35	8	0.60	227.24	0.88	0.003

Faraz, A. and Saniga, E. Multiobjective Genetic Algorithm Approach to the Economic Statistical Design of Control Charts with an Application to *X*-bar and S^2 Charts. ***Quality and Reliability Engineering International***, 29:3 (2013) 407-415. DOI: 10.1002/qre.1390.

Papers can be provided upon request:

Seif, A., Faraz, A., Saniga, E. Economic Statistical Design of the VP X-bar Control Charts for Monitoring a Process under Non-normality, Revised and submitted to ***International Journal of Production Research,*** (2014).

Seif, A., Faraz, A., Sadeghifar, M. Economic Statistical Design of the VSI T^2 control chart- NSGA-II approach, The Hotelling's T^2 Control Chart with Variable Parameters: Markov Chain Approach, approach. Accepted in ***Journal of Statistical Computation and Simulation,*** (2014).

Faraz, A., Heuchenne, C., Saniga, E., and Costa, A.F.B. Double Objective Economic Statistical Design of the VP T^2 Control Chart: Wald's identity approach. ***Journal of Statistical Computation and Simulation***, (2013). DOI:10.1080/00949655.2013.784315

Faraz, A. and Saniga, E. Multiobjective Genetic Algorithm Approach to the Economic Statistical Design of Control Charts with an Application to X-bar and S^2 Charts. ***Quality and Reliability Engineering International***, 29:3 (2013) 407-415. DOI: 10.1002/qre.1390.

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Conclusion:
Ackoff (1967) noted that no system (or solution procedure) that is completely computerized, (i.e. completely determines a solution) can be as flexible and adaptive as a system or solution that requires a person to aid in determining the solution, which Ackoff calls a man-machine system.

In this research we address the control chart design problem in a way that users can have choices of designs and thus can tailor solutions to the temporal imperatives of the specific industrial situation.

Thanks for your patience and consideration

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