Eulerian Formulation of the Torque and Drag Problem

Nonlinear Dynamics and Control of Deep Drilling Systems

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FREEDOM TO RESEARCH

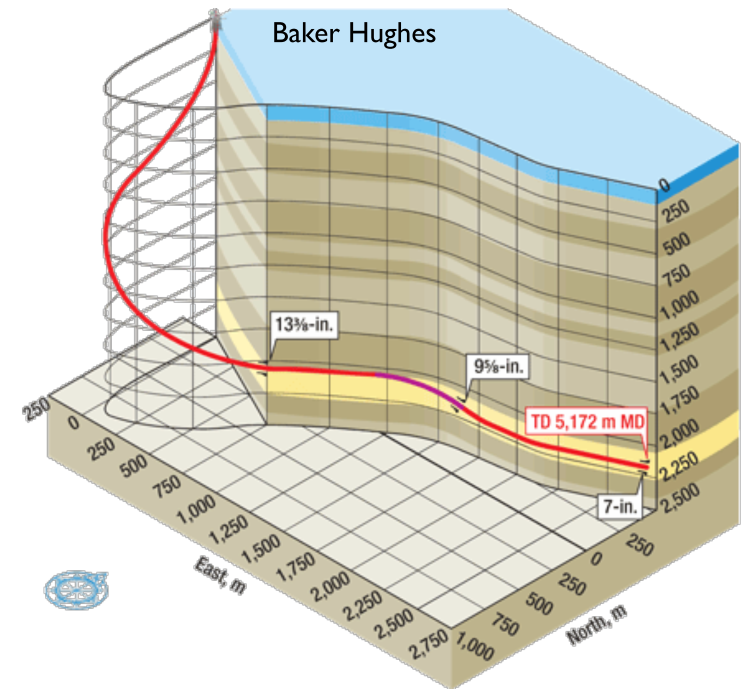
Université de Liège

Friday May 30 2014

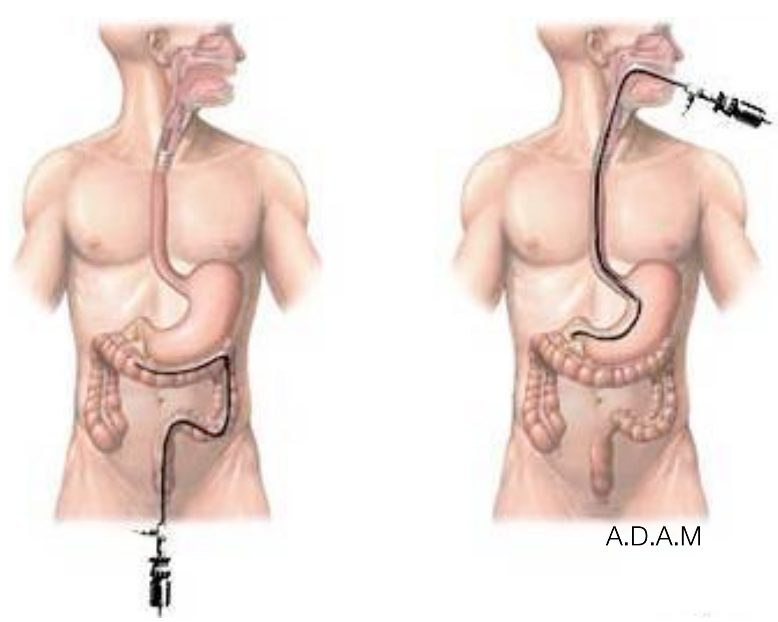
Constrained Rod - Inside

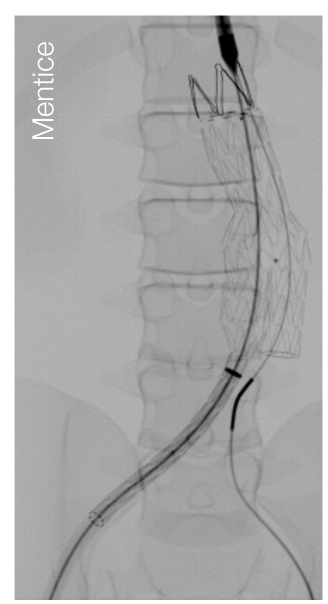
- Engineering applications
  - Petroleum, mining, gas, geothermal, etc.

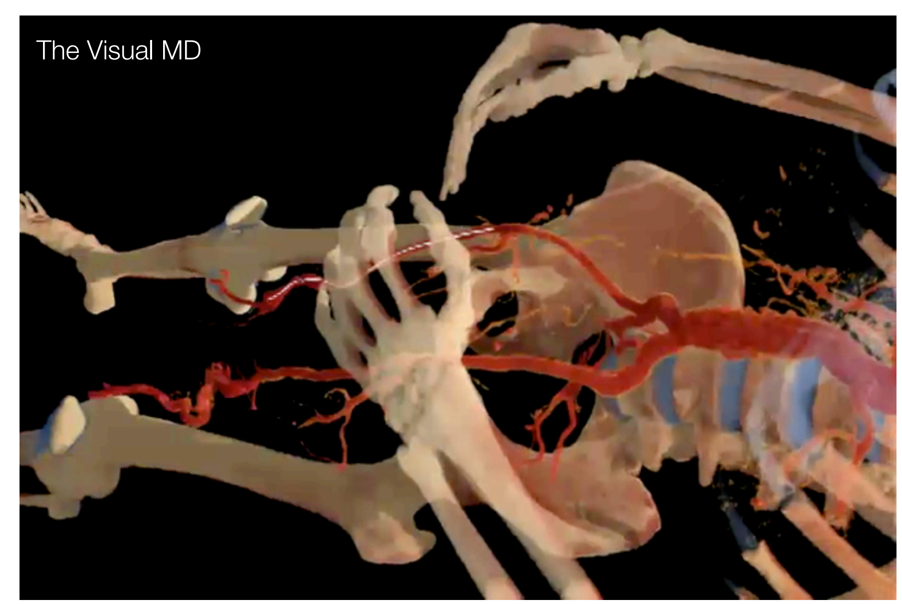
Drillstring length
~ 5 km



- Medical applications
  - Endoscopic examination of internal organs
  - Endovascular procedures







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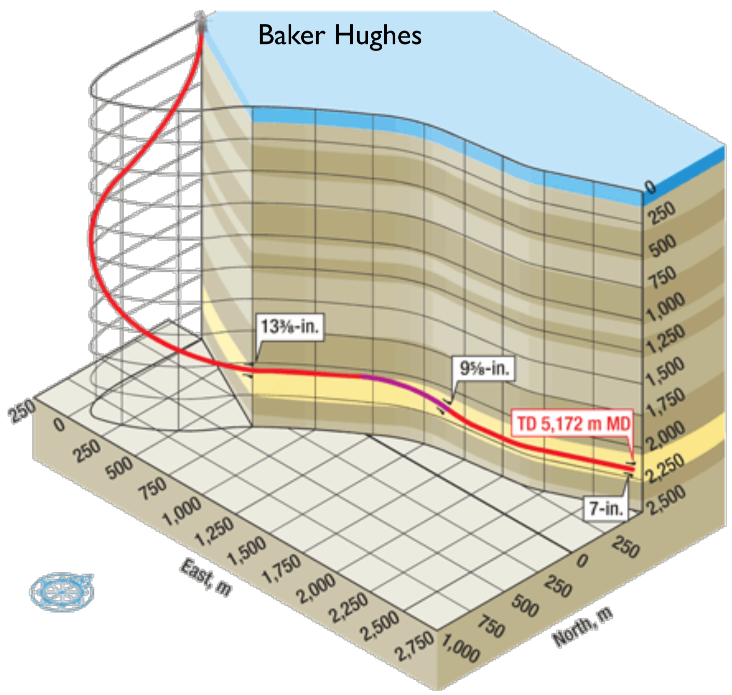

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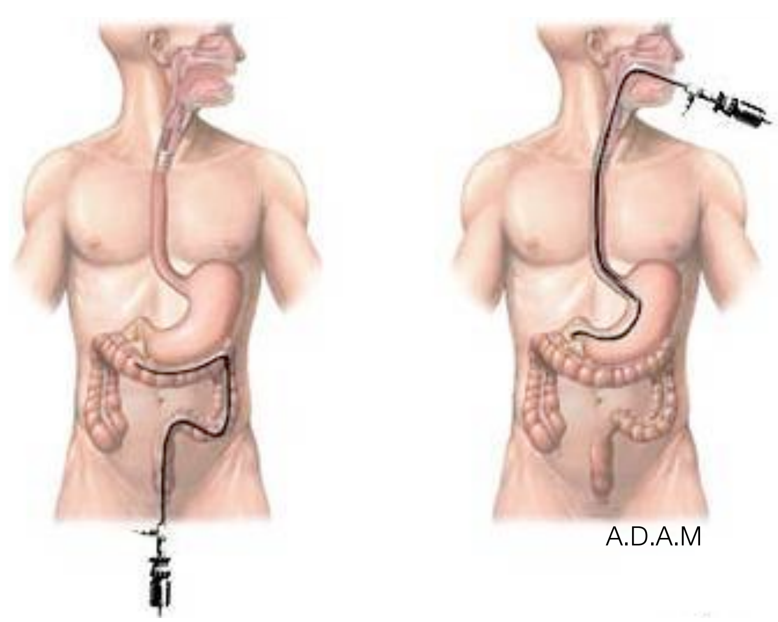
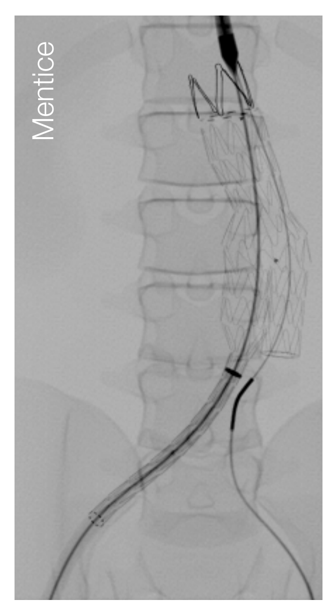
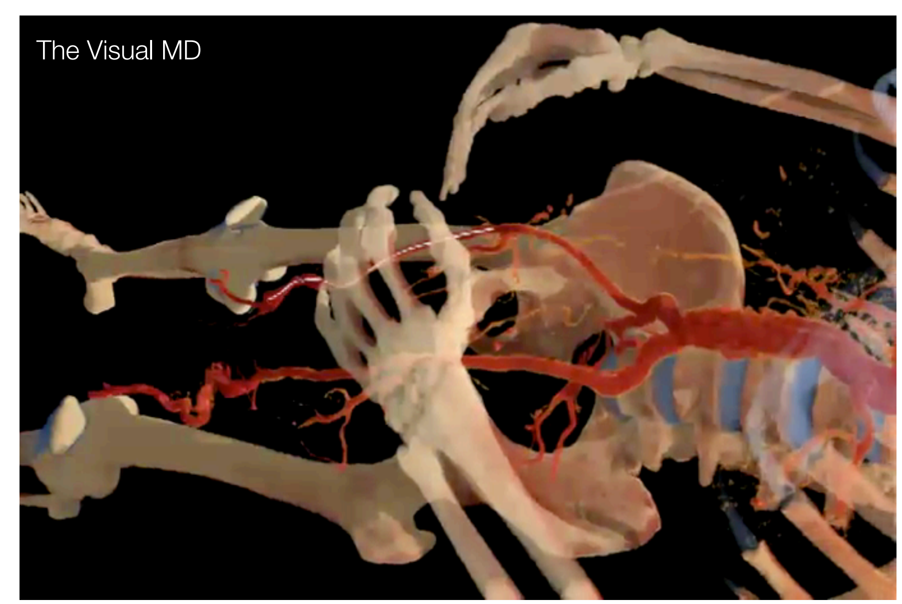


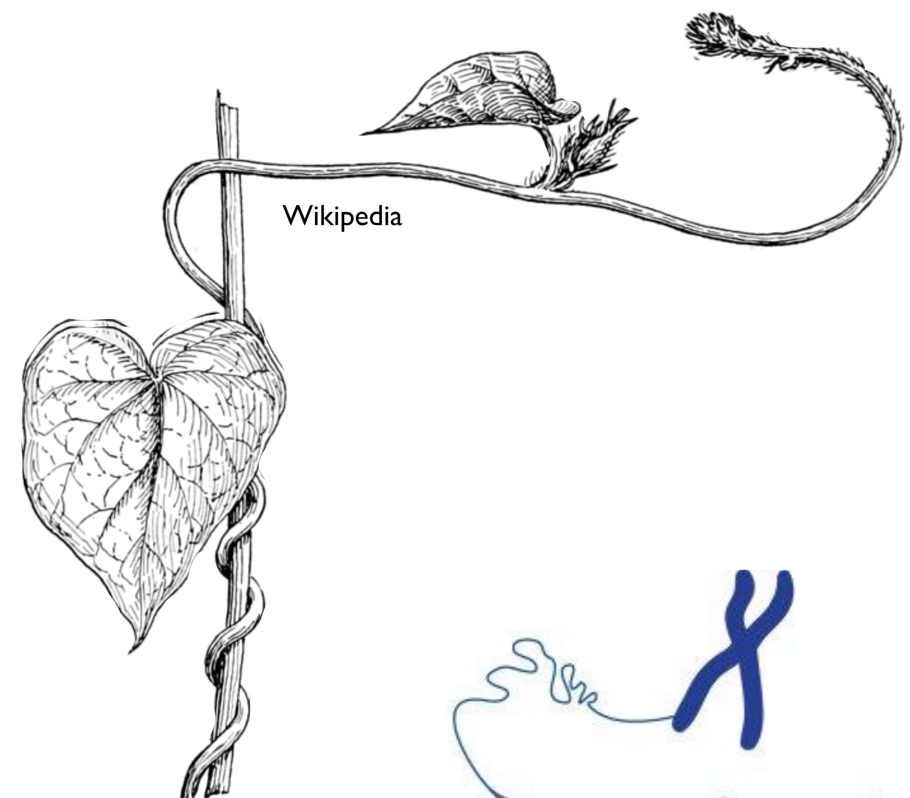




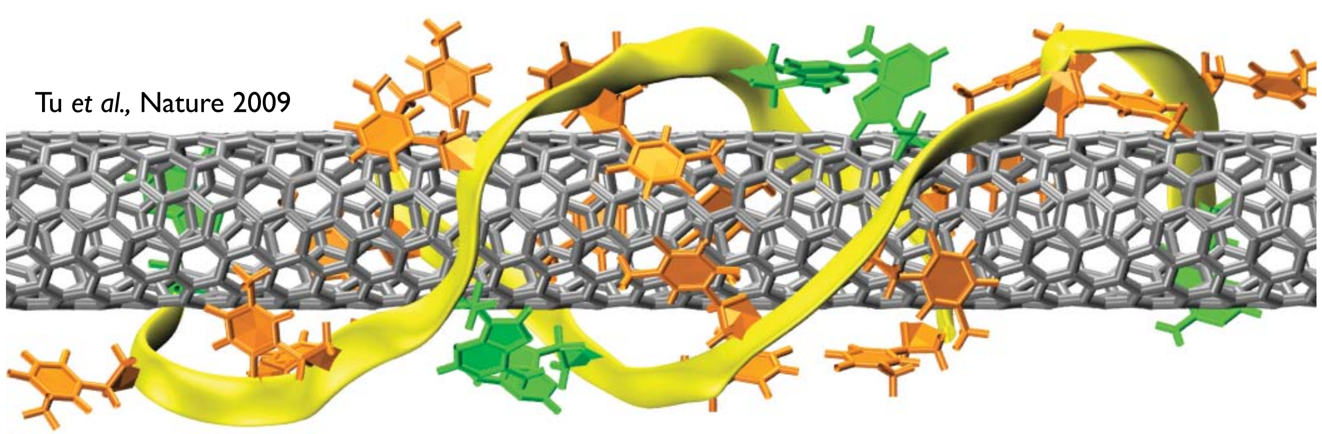
Constrained Rod - Outside

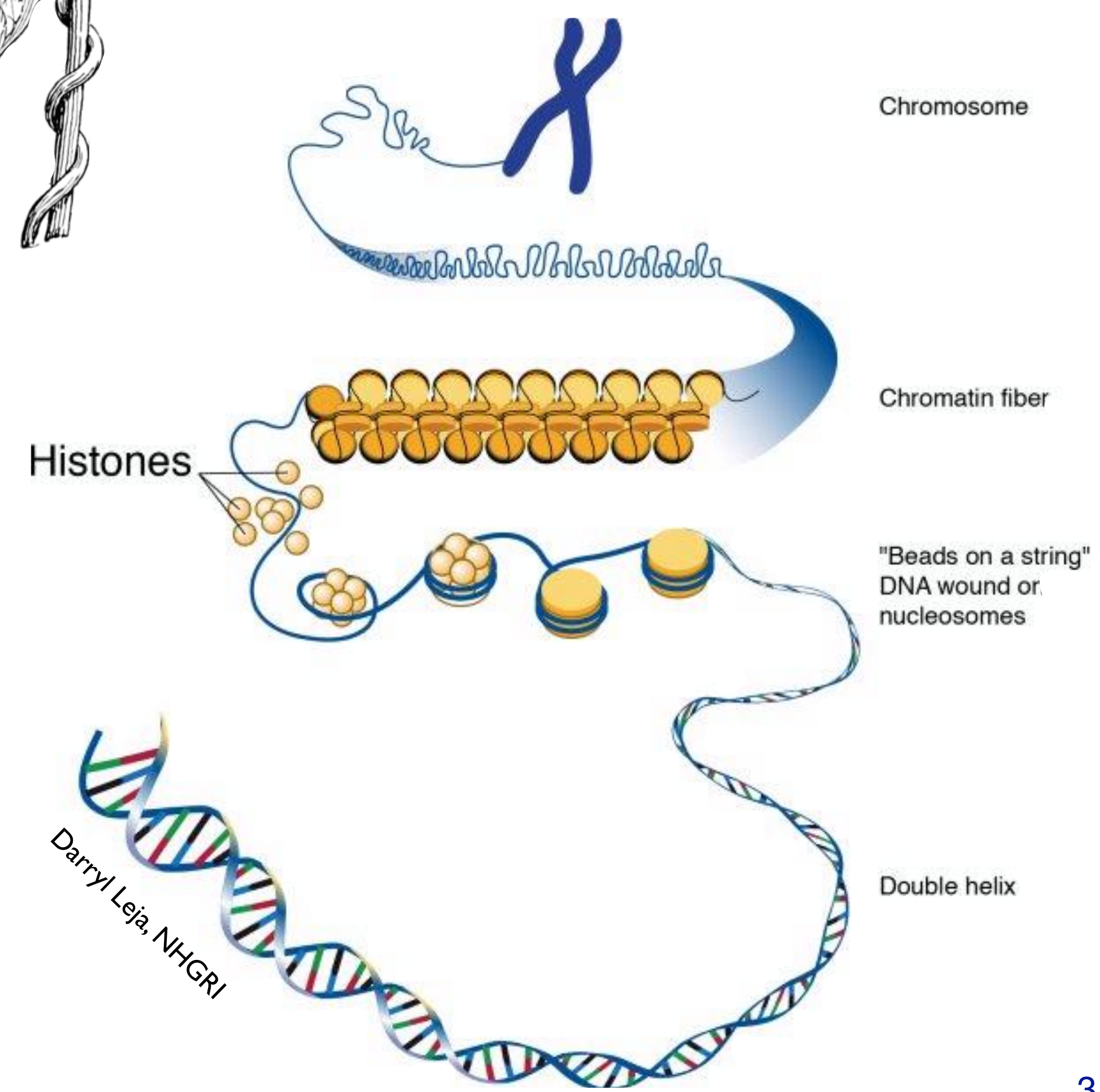
- Twining plants

1 sec. = 4 h



- DNA wrapping





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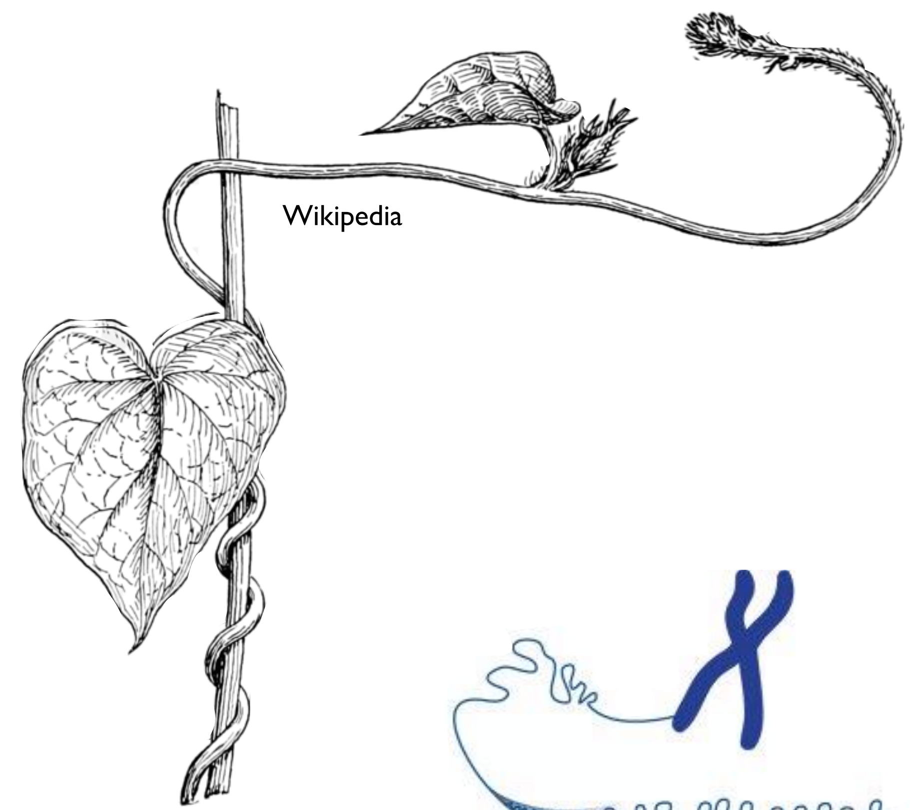

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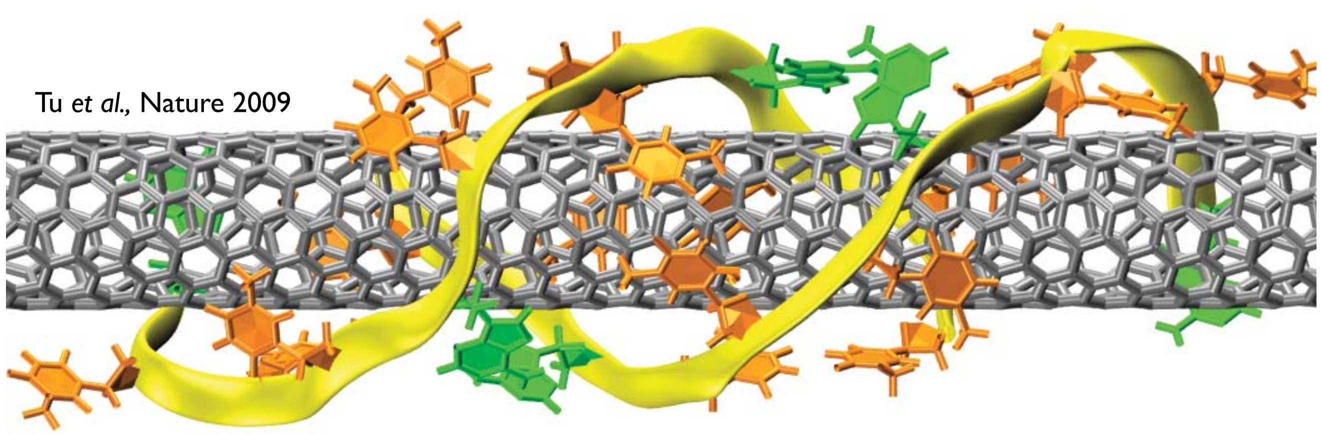
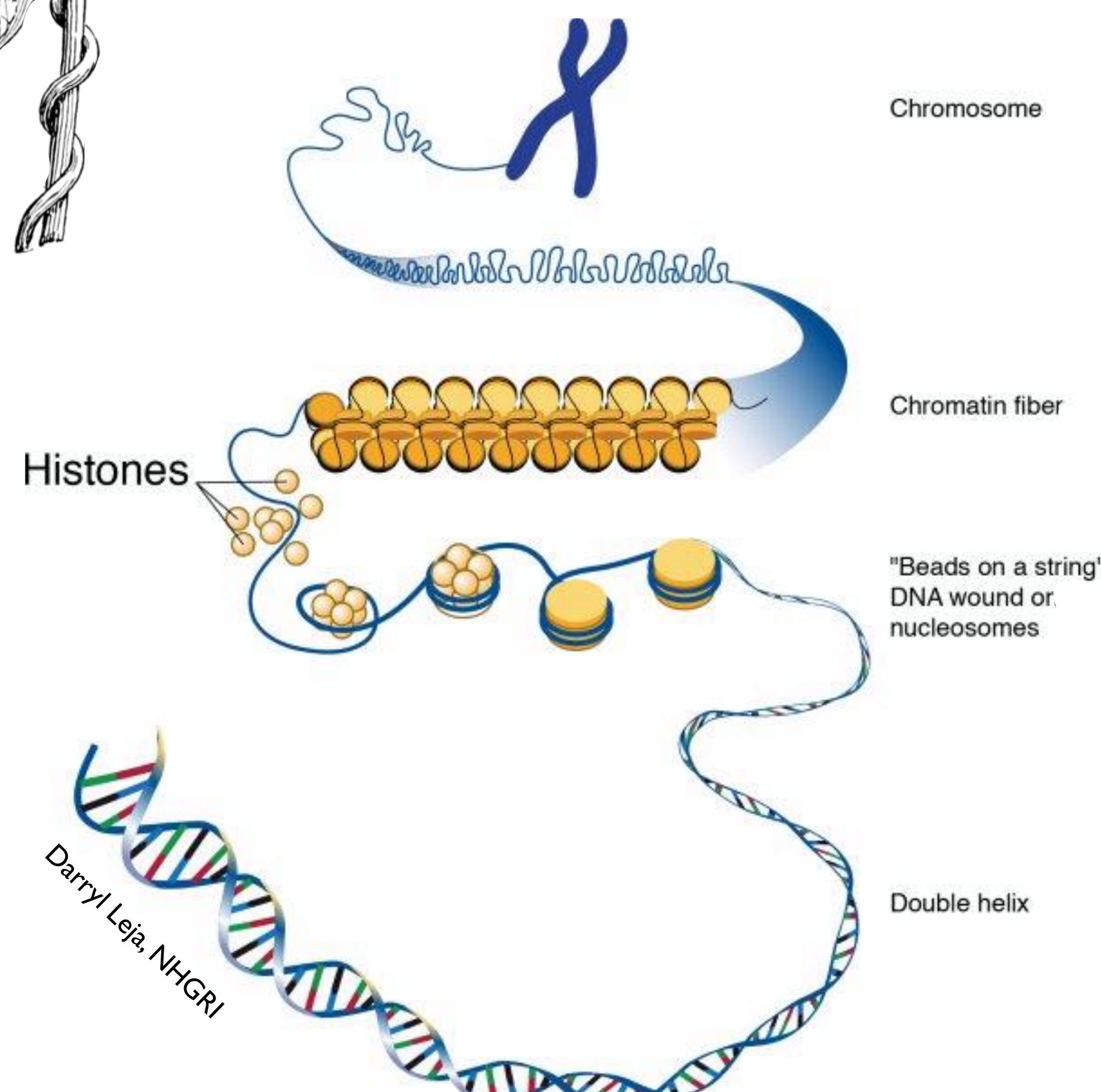
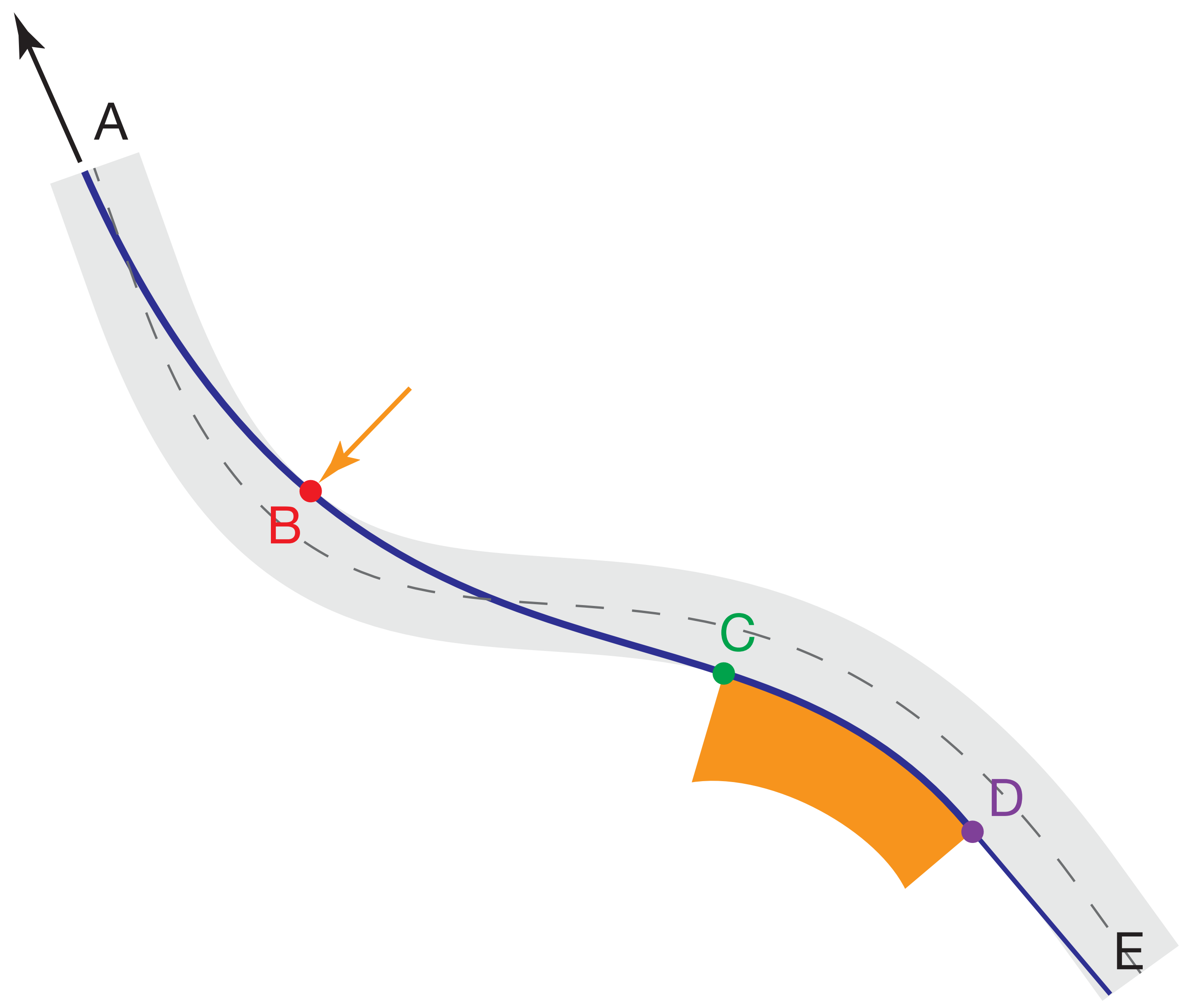




Segmentation Strategy (Chen & Li 2007, Denoël 2008)

- Division of the problem in rod segments bounded by two contacts
  - Solve the sequence of problems
  - Check the validity of the contact pattern



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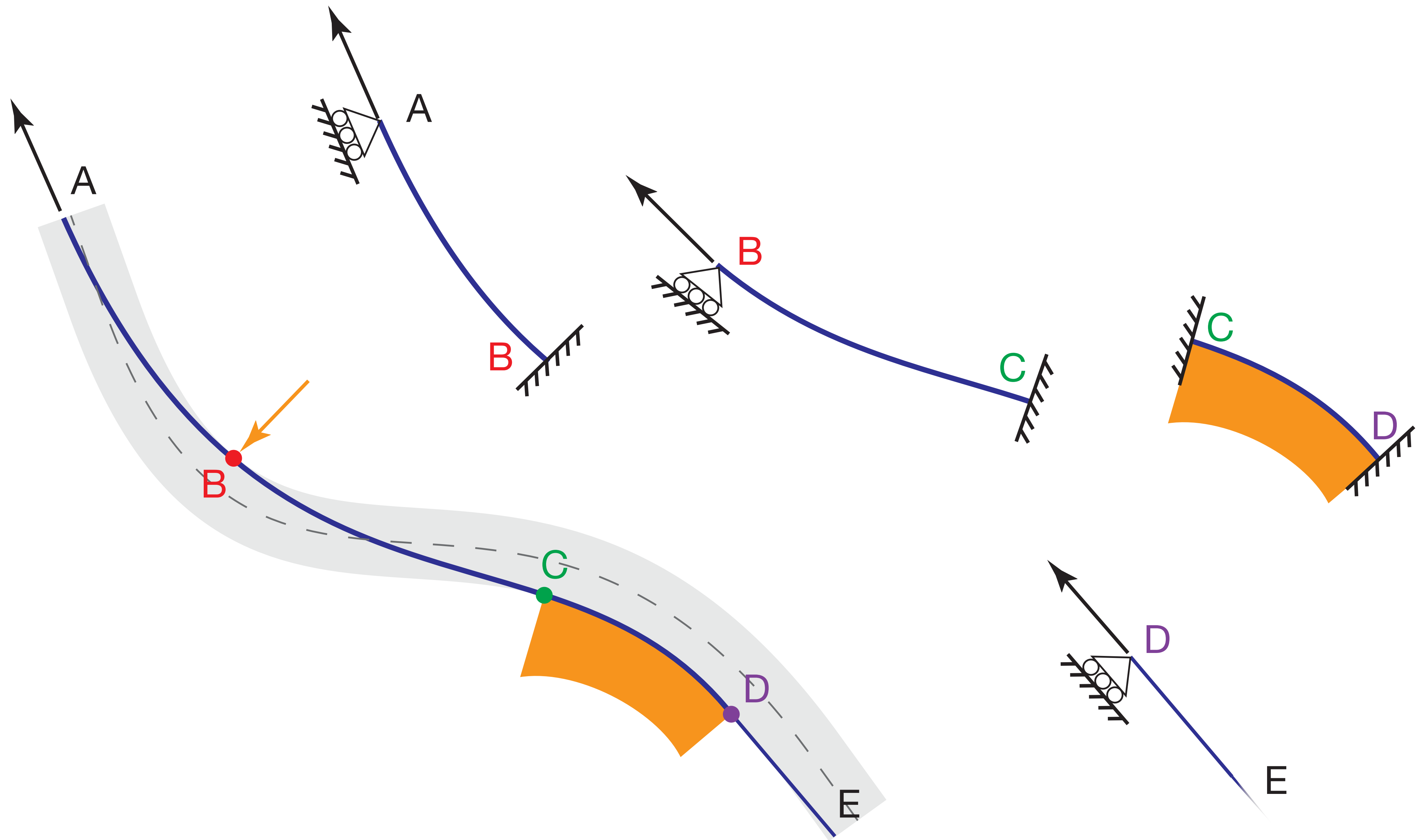
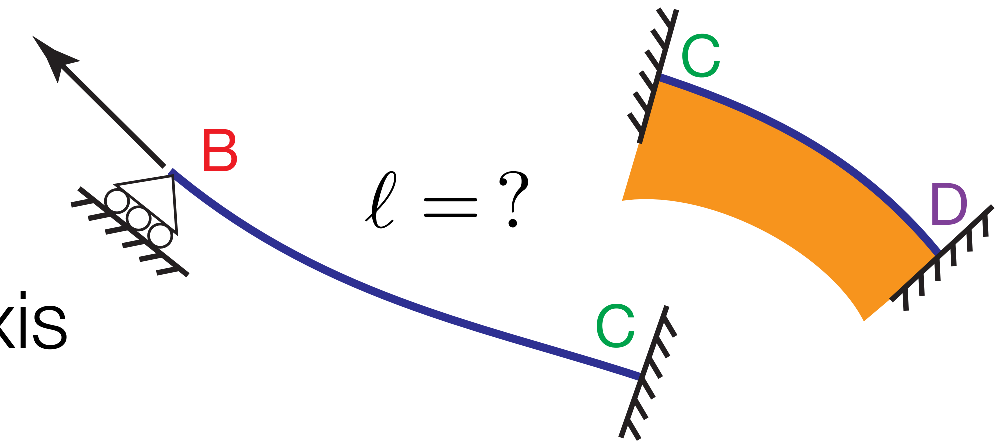
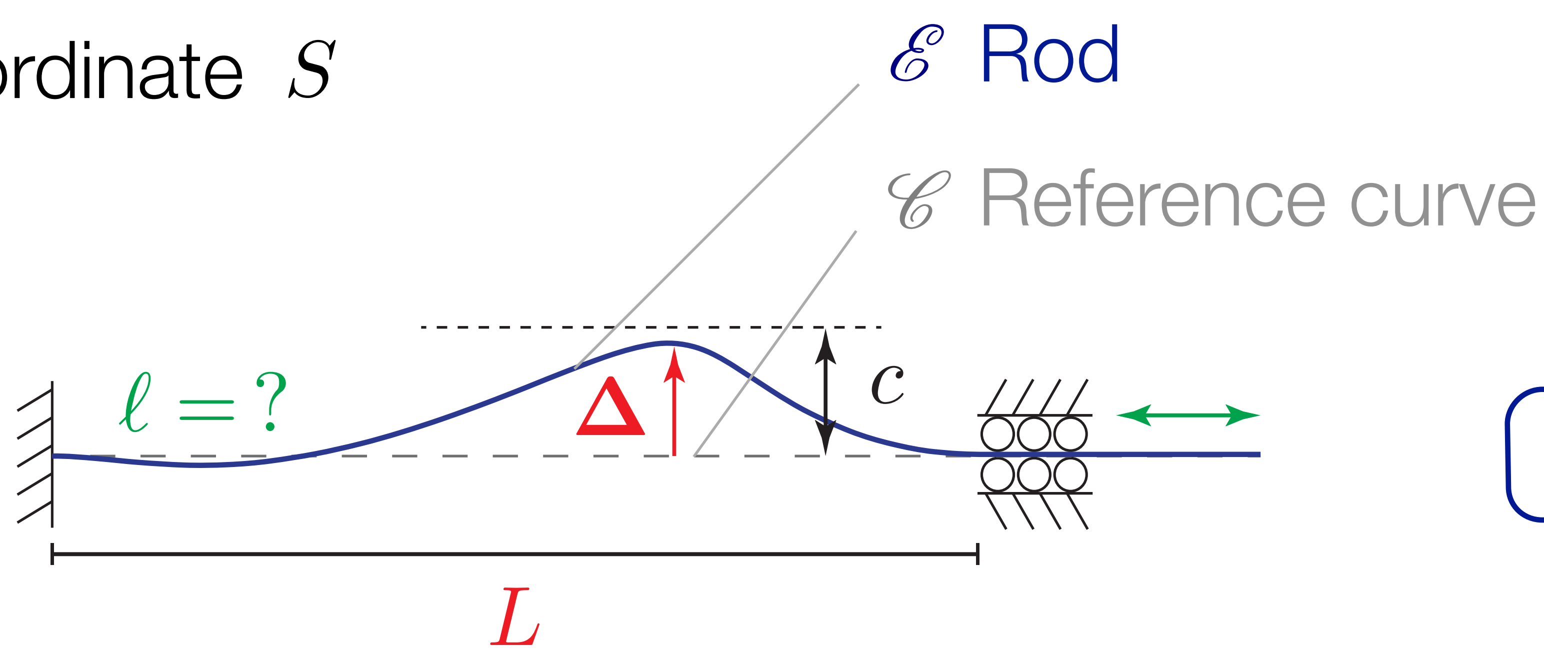


Lagrangian vs. Eulerian

- Segmentation drawbacks
  - Initially unknown domain
  - Evaluation of the distance rod/conduit axis

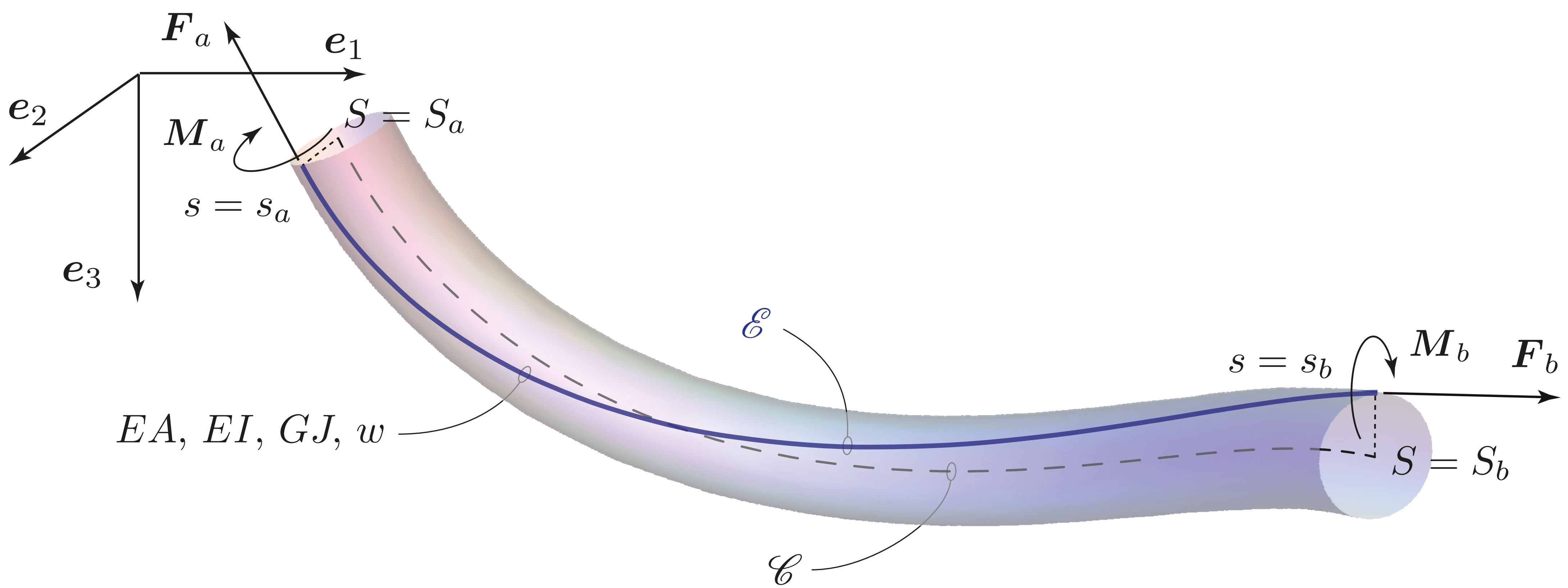


- Eulerian formulation (Denoël & Detournay, 2011)
  - Rod *relative* deflection $\boldsymbol{\Delta}(S)$
  - Problem length L
  - Coordinate S



Self-feeding

Canonical Problem

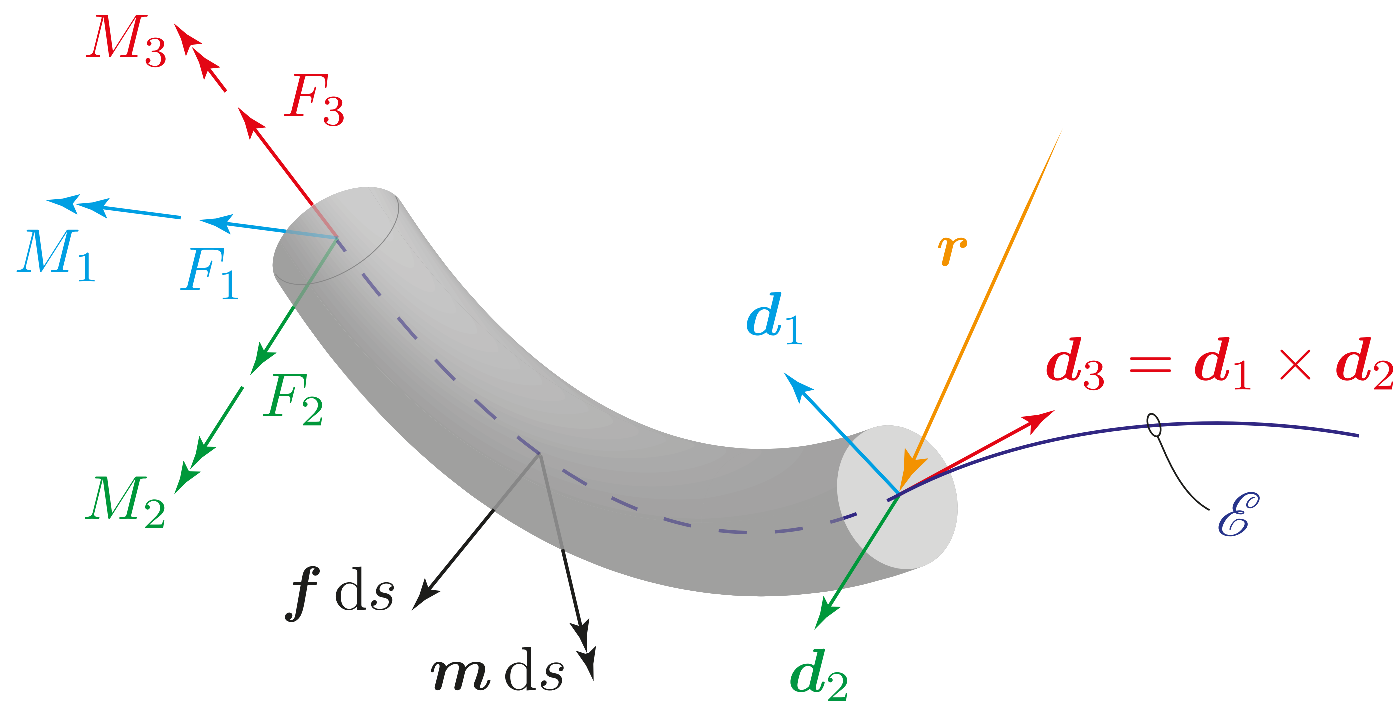


- Rod configuration between contacts ($s \in [s_a, s_b]$)
  - Known extremities positions and inclinations $x_j\left(s_{a,b}\right), x_j'\left(s_{a,b}\right)$
  - Known axial force $\boldsymbol{F}_b$ and torque $\boldsymbol{M}_b$

    ↪ Boundary value problem
- Unknowns
  - Rod length $\ell = s_b - s_a$, axial force $\boldsymbol{F}_a$ and torque $\boldsymbol{M}_a$

Lagrangian Formulation (Antman, 2005)

- Rod definition
  - Centroid $\boldsymbol{r}(s) = x_k\, \boldsymbol{e}_k$
    ↪ Space curve $\mathscr{E}$
  - Directors $\{\boldsymbol{d}_k(s)\}$
    ↪ Section orientation



- Equilibrium

$$\frac{\mathrm{d}\boldsymbol{F}}{\mathrm{d}s} + \boldsymbol{f} = 0$$

$$\frac{\mathrm{d}\boldsymbol{M}}{\mathrm{d}s} + \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}s} \times \boldsymbol{F} + \boldsymbol{m} = 0$$

- Kinematics

$$\frac{\mathrm{d}\boldsymbol{d}_k}{\mathrm{d}s} = \boldsymbol{u} \times \boldsymbol{d}_k$$

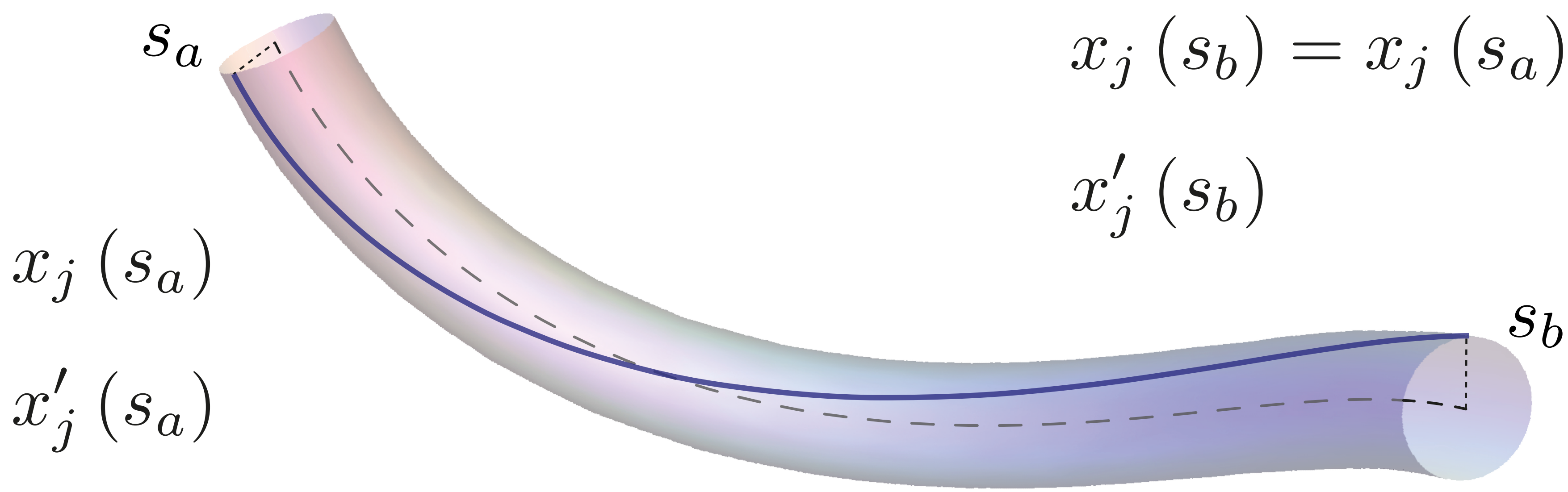
$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}s} = \alpha\, \boldsymbol{d}_3$$

- Constitutive equations

$$F_3 = EA\,(\alpha - 1) \qquad M_{1,2} = EI\, u_{1,2} \qquad M_3 = GJ\, u_3$$

Issues with Lagrangian Formulation (Chen & Li, 2007)

- Boundary conditions:

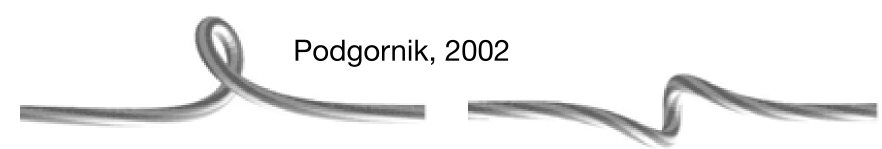


$$x_j(s_b) = x_j(s_a) + \int_{s_a}^{s_b} \boldsymbol{r}_{,s} \cdot \boldsymbol{e}_j \, \mathrm{d}s$$

$$x'_j(s_b)$$

↪ Integral constraints on the *unknown length* $\ell = s_b - s_a$ of the rod

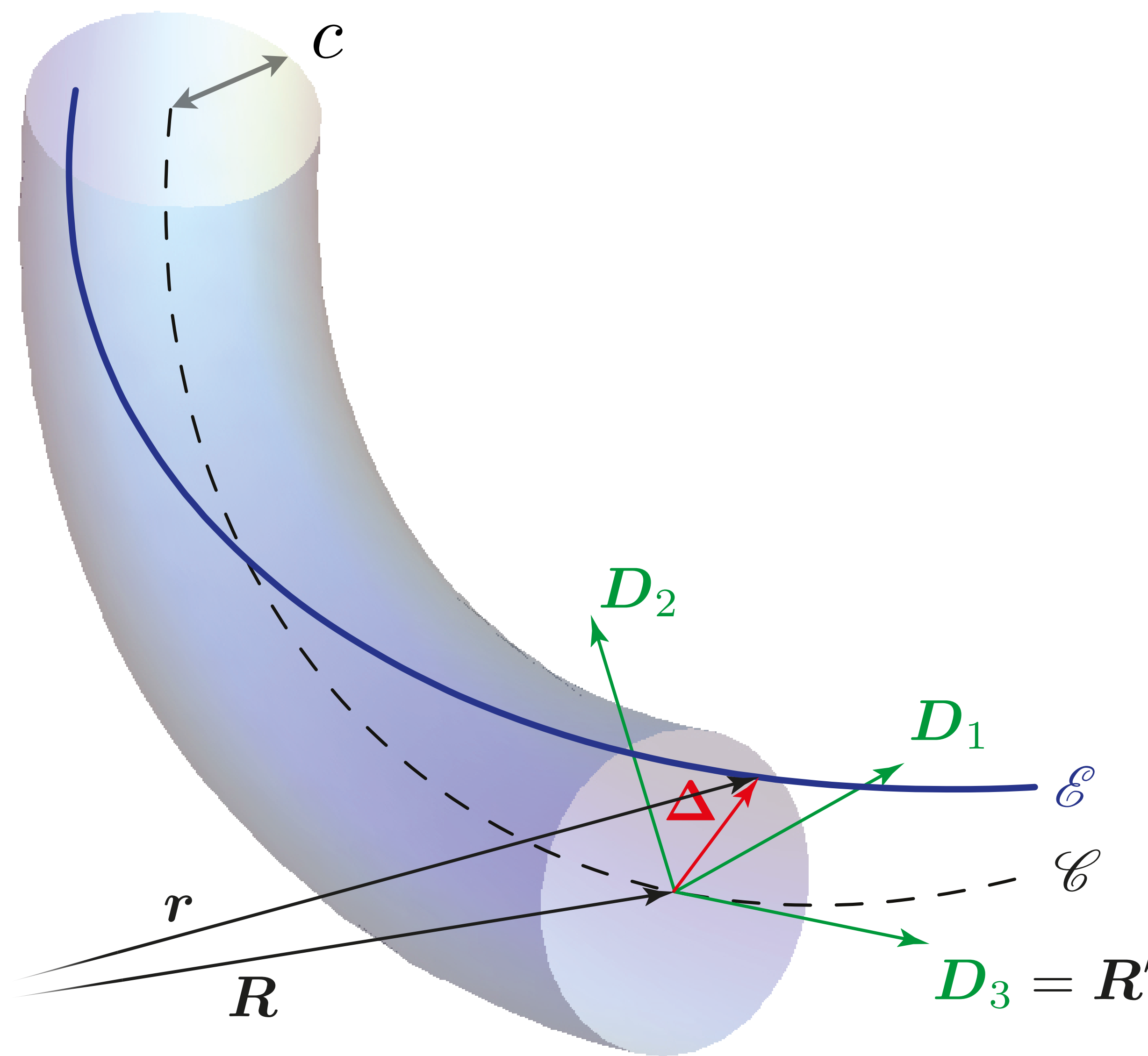
- Ill-conditioning of the governing equations when $EI/w\,\ell^3 \ll 1$
- Parasitic solutions with curling



- Contact detection: comparison of two curves parameterized by distinct curvilinear coordinates
  - Conduit axis: $X_j(S)$ (Eulerian coordinate)
  - Rod axis: $x_j(s)$ (Lagrangian coordinate)

Eulerian Formulation

- Orthonormal frame $\{\boldsymbol{D}_j(S)\}$ attached to the reference curve $\mathscr{C}$



- Eccentricity vector

$$\begin{cases} \boldsymbol{r}(s) = \boldsymbol{R}(S) + \boldsymbol{\Delta}(S) \\ \dfrac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}S} \cdot \boldsymbol{\Delta} = 0 \end{cases}$$

↪ Contact detection $\|\boldsymbol{\Delta}\| \leq c$

- Jacobian of the mapping

$$S(s) \longrightarrow \frac{\mathrm{d}\cdot}{\mathrm{d}s} = \boxed{\frac{\mathrm{d}S}{\mathrm{d}s}} \frac{\mathrm{d}\cdot}{\mathrm{d}S}$$

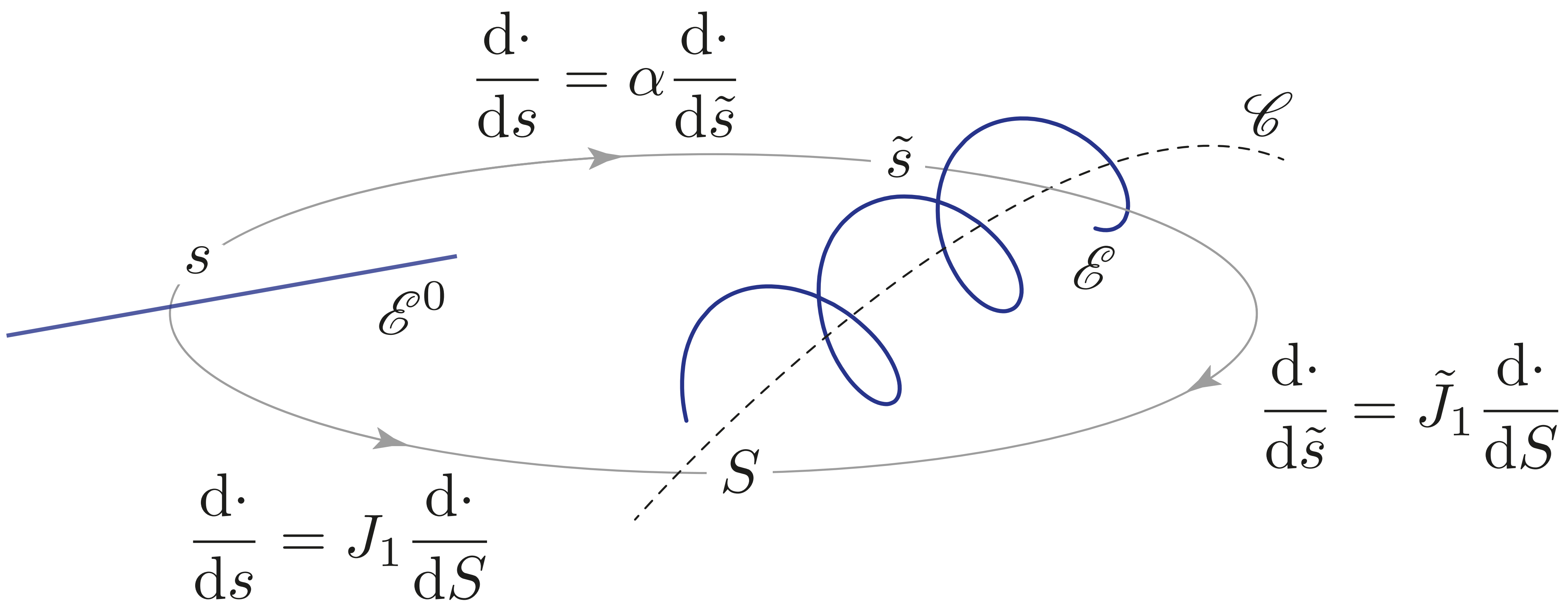
Mappings: 3 Curvilinear Coordinates

Lagrangian

Reference config.

Stretched

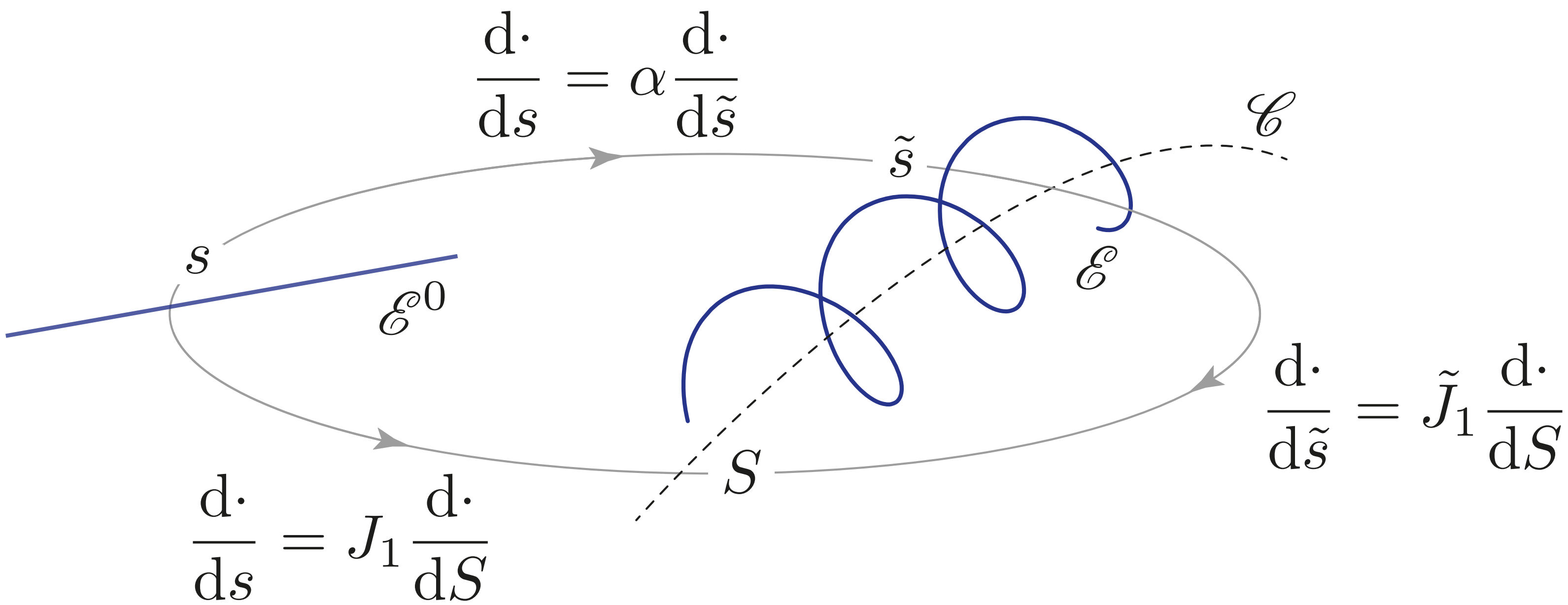
Deformed config.

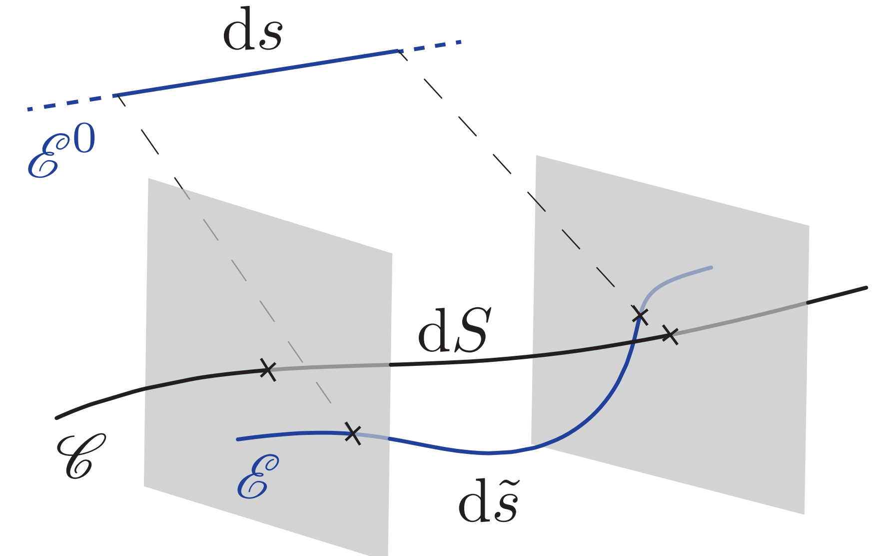


Eulerian

Reference curve

Jacobian of the Mapping



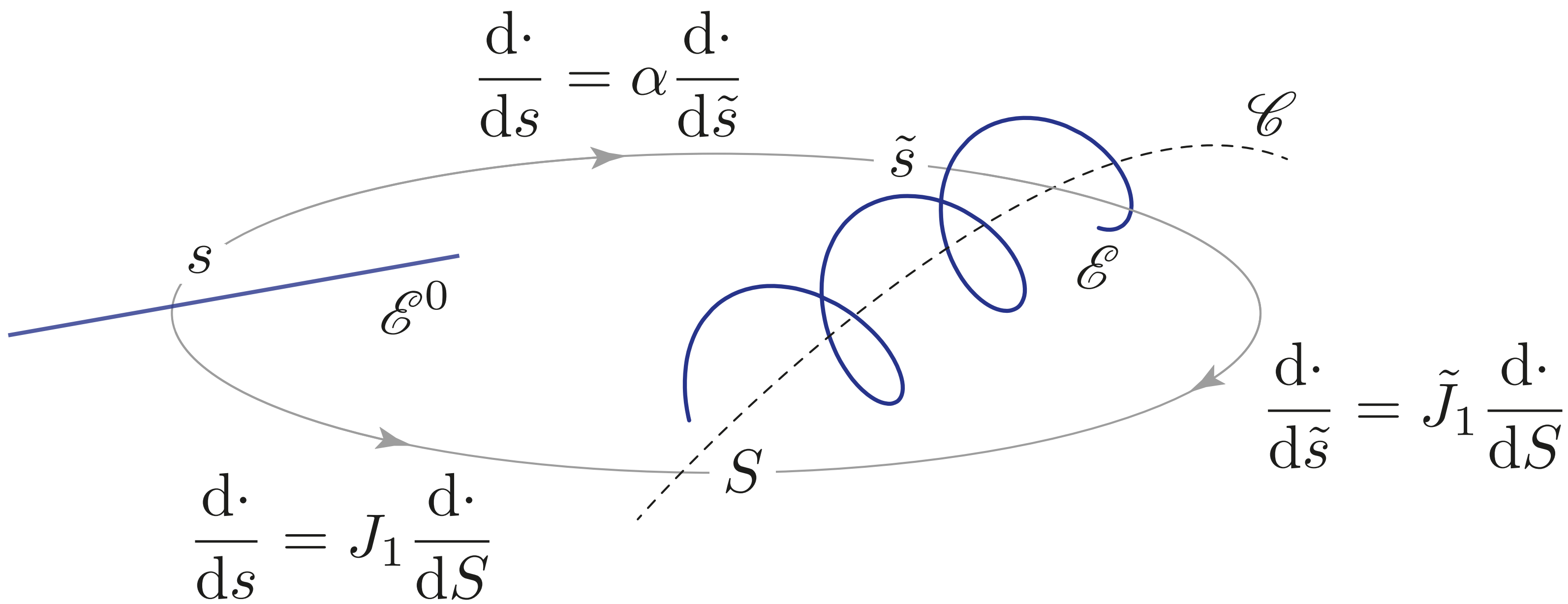


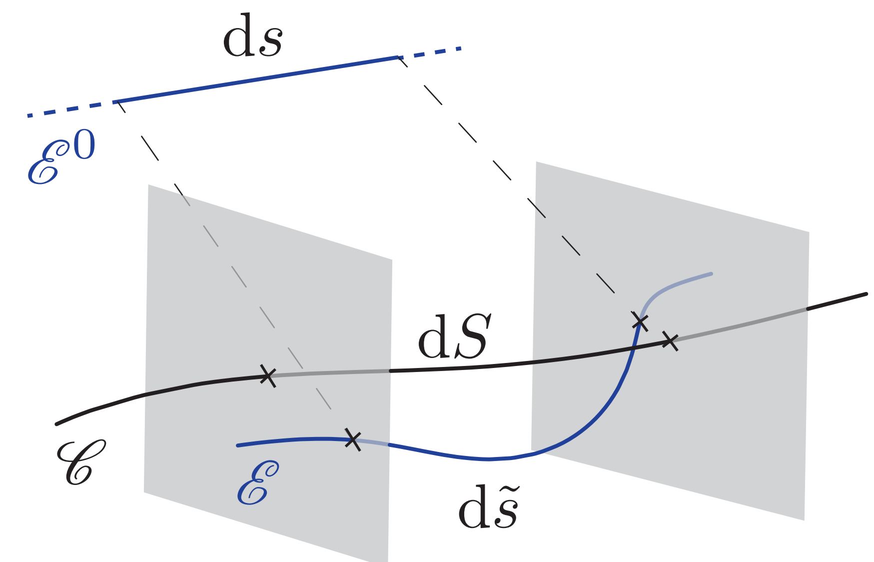
$$\left.\begin{aligned} \boldsymbol{r}\left(s\left(S\right)\right) &= \boldsymbol{R}\left(S\right) + \boldsymbol{\Delta}\left(S\right) \\ \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}s} &= \alpha\, \boldsymbol{d}_3 \end{aligned}\right\} \longrightarrow \quad \alpha\, \boldsymbol{d}_3 = \left(\boldsymbol{R}' + \boldsymbol{\Delta}'\right) \frac{\mathrm{d}S}{\mathrm{d}s}$$

↪ Drift between S and s:

- Eccentricity between the rod and the reference curve
- Stretch of the rod

Jacobian of the Mapping





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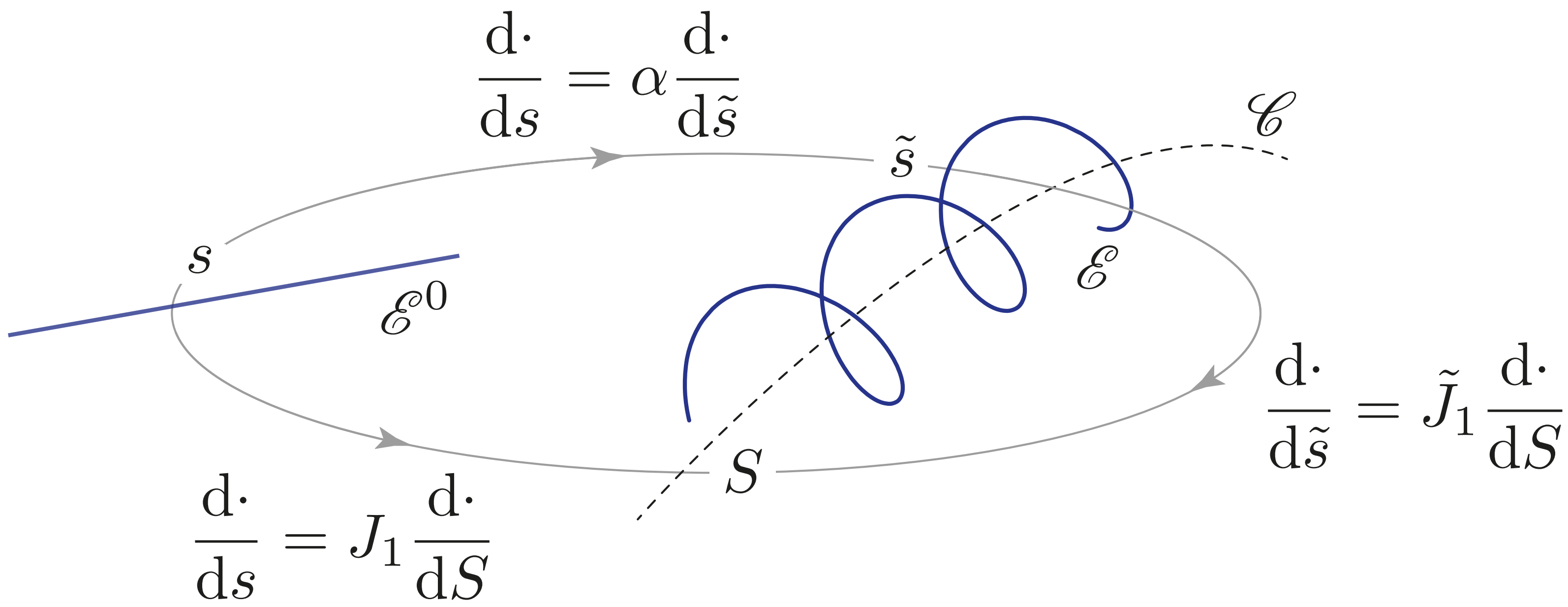

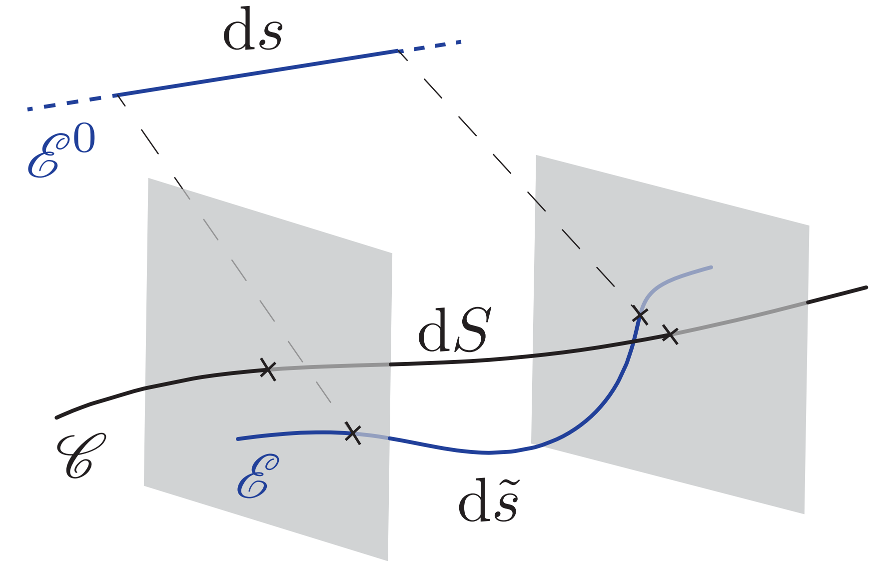

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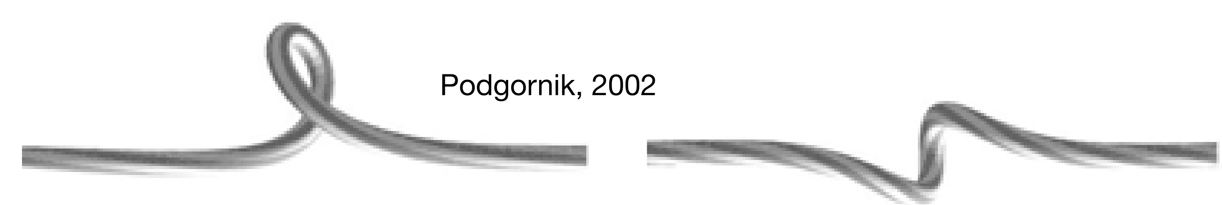


Podgornik, 2002

↪ Drift between S and s:

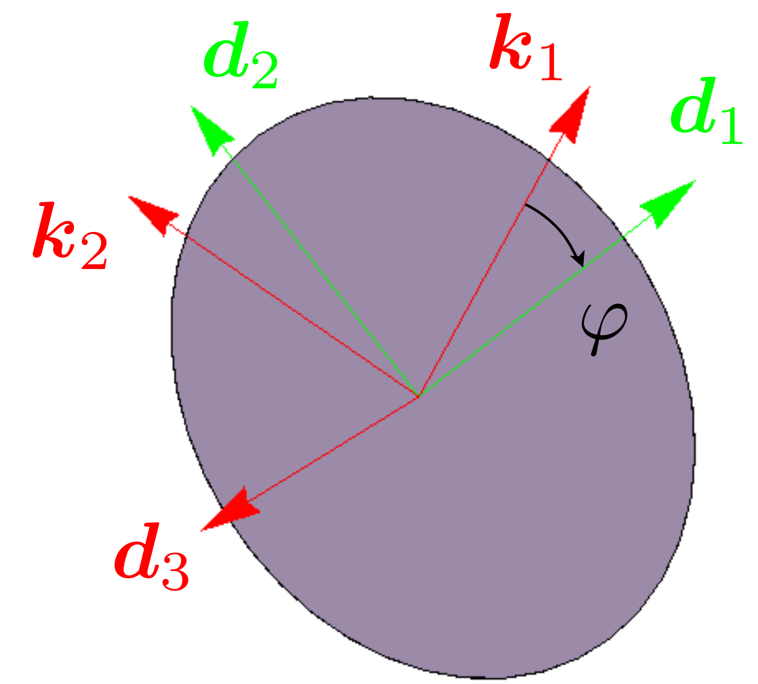
- Eccentricity between the rod and the reference curve
- Stretch of the rod

Rod Attitude

- Orientation of the rod directors $\{\boldsymbol{d}_j(s)\}$

$$\boldsymbol{d}_1(S) = \cos\varphi\,\boldsymbol{k}_1 - \sin\varphi\,\boldsymbol{k}_2$$
$$\boldsymbol{d}_2(S) = \sin\varphi\,\boldsymbol{k}_1 + \cos\varphi\,\boldsymbol{k}_2$$
$$\boldsymbol{d}_3(S) = J_1\left(\boldsymbol{D}_3 + \boldsymbol{\Delta}'\right)/\alpha$$

where $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are the images of $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ through the rotation mapping $\boldsymbol{D}_3$ on $\boldsymbol{d}_3$



- Strain variables

$$u_k = u_k\left(\alpha, \varphi, \boldsymbol{\Delta}, \boldsymbol{U}\right)$$

- Curvature and torsion

$$\kappa^2 = \tilde{J}_1^4\left(\mathcal{K}^2 + 2\,\boldsymbol{D}_3' \cdot \boldsymbol{\Delta}'' + \left\|\boldsymbol{\Delta}''\right\|^2\right) - \tilde{J}_1'^2$$

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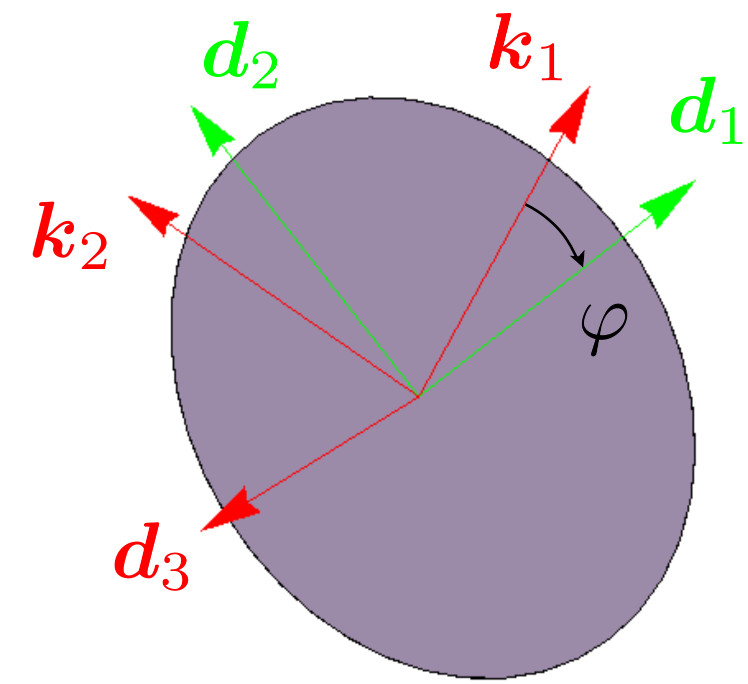

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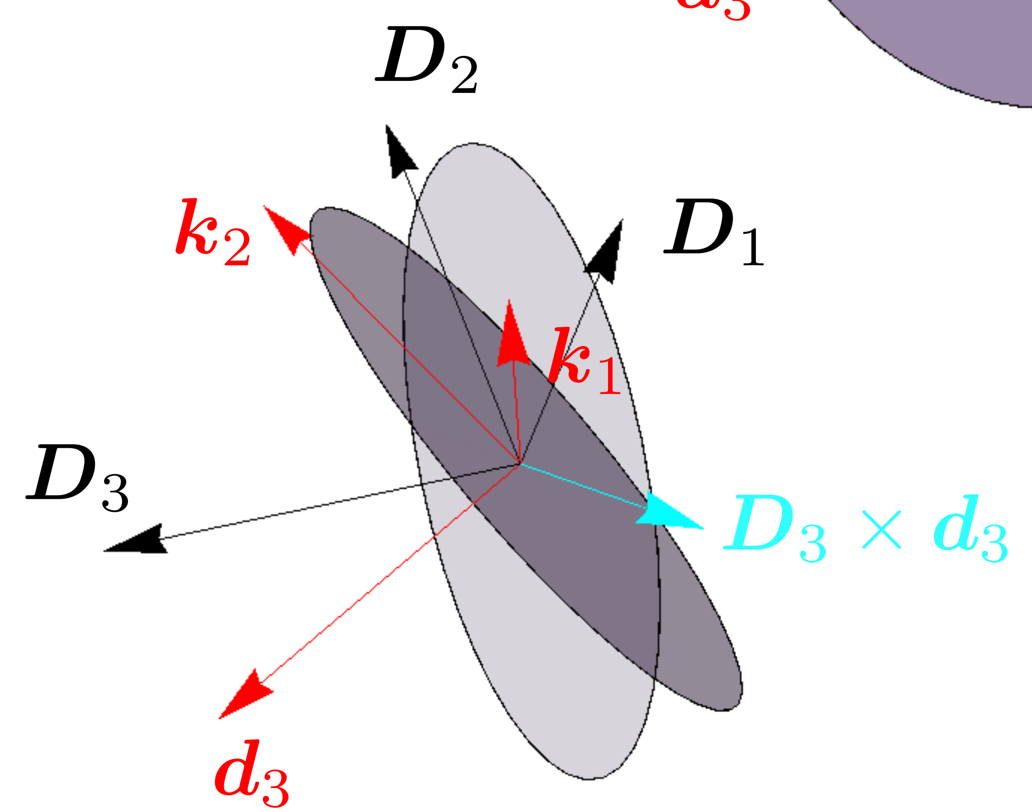

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Numerical Implementation

- Mixed order nonlinear BVP

$$J_1\,F_j' + G_j\left[\alpha,\varphi,\boldsymbol{\Delta},\boldsymbol{F},\boldsymbol{U}\right] + f_j = 0$$
$$J_1\,\alpha' + G_\alpha\left[\alpha,\varphi,\boldsymbol{\Delta},\boldsymbol{F},\boldsymbol{U}\right] + f_3 = 0$$
$$J_1^3\,\Delta_j''' + H_j\left[\alpha,\varphi,\boldsymbol{\Delta},\boldsymbol{F},\boldsymbol{U}\right] + m_j = 0$$
$$J_1^2\,\varphi'' + H_\varphi\left[\alpha,\varphi,\boldsymbol{\Delta},\boldsymbol{U}\right] + m_3 = 0$$

$$B_i\left[S_i;\alpha,\varphi,\boldsymbol{\Delta},\boldsymbol{F},\boldsymbol{U}\right] = 0, \qquad S_i \in \left[S_a, S_b\right],\ i = 1,\ldots,11$$

- Numerical solution: collocation method (Ascher et al., 1979)

$$\boldsymbol{\Delta}^* \in \mathcal{P}_{k+3,\pi} \cap C^2\left[S_a, S_b\right]$$
$$\varphi^* \in \mathcal{P}_{k+2,\pi} \cap C^1\left[S_a, S_b\right]$$
$$\boldsymbol{F}^* \in \mathcal{P}_{k+1,\pi} \cap C^0\left[S_a, S_b\right]$$

where $k \geq 3$ is the number of collocation points per subinterval and $\mathcal{P}_{n,\pi}$ is the set of all piecewise polynomial functions (*B*-splines) of order n

Continuous Contact

- Eccentricity vector $\boldsymbol{\Delta}(S)$
  - Magnitude $\|\boldsymbol{\Delta}\| = c$ (known)

$$\Delta_1(S) = c\cos\beta$$
$$\Delta_2(S) = c\sin\beta$$

  - Direction

$$\beta(S) = \arctan\frac{\Delta_2}{\Delta_1}$$ (unknown)

- Contact pressure $\boldsymbol{p}(S)$
  - Magnitude $\|\boldsymbol{p}\| = p$ (unknown)
  - Direction (no friction)

$$\boldsymbol{\Delta} \times \boldsymbol{p} = 0$$ (known)

- Modified BVP (differential algebraic system)

$$J_1 F_j' + G_j[\alpha, \varphi, \beta, \boldsymbol{F}, \boldsymbol{U}] + f_j + p_j = 0$$
$$J_1 \alpha' + G_\alpha[\alpha, \varphi, \beta, \boldsymbol{F}, \boldsymbol{U}] + f_3 + p_3 = 0$$
$$J_1^3 \Delta_j''' + H_j[\alpha, \varphi, \beta, \boldsymbol{F}, \boldsymbol{U}] + m_j = 0$$
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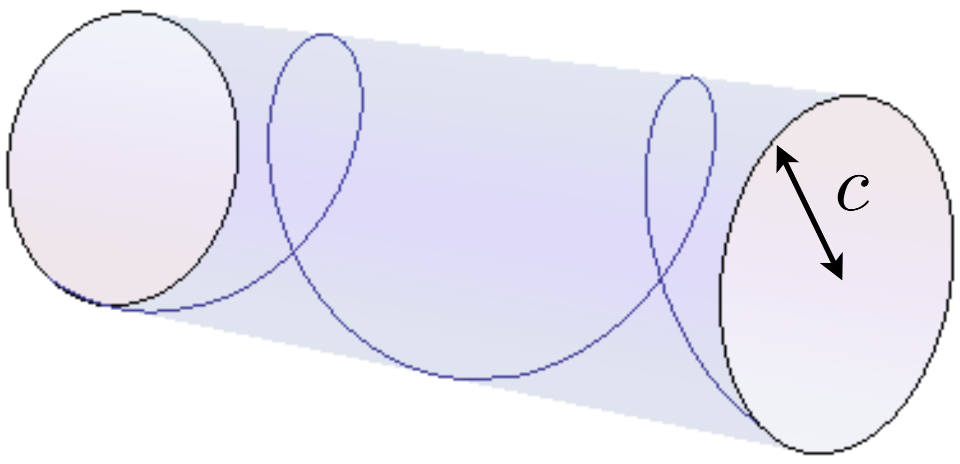
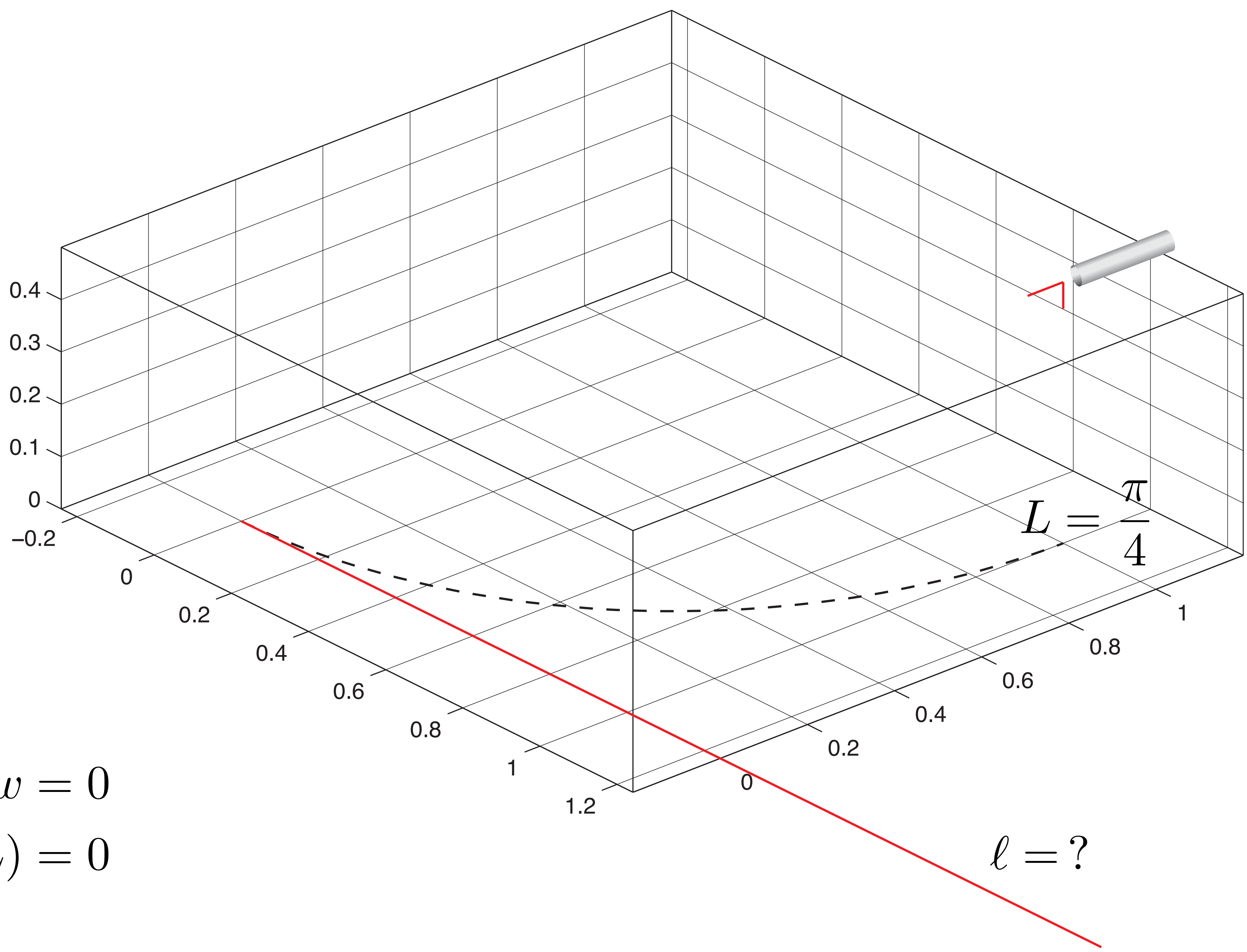


Comparison

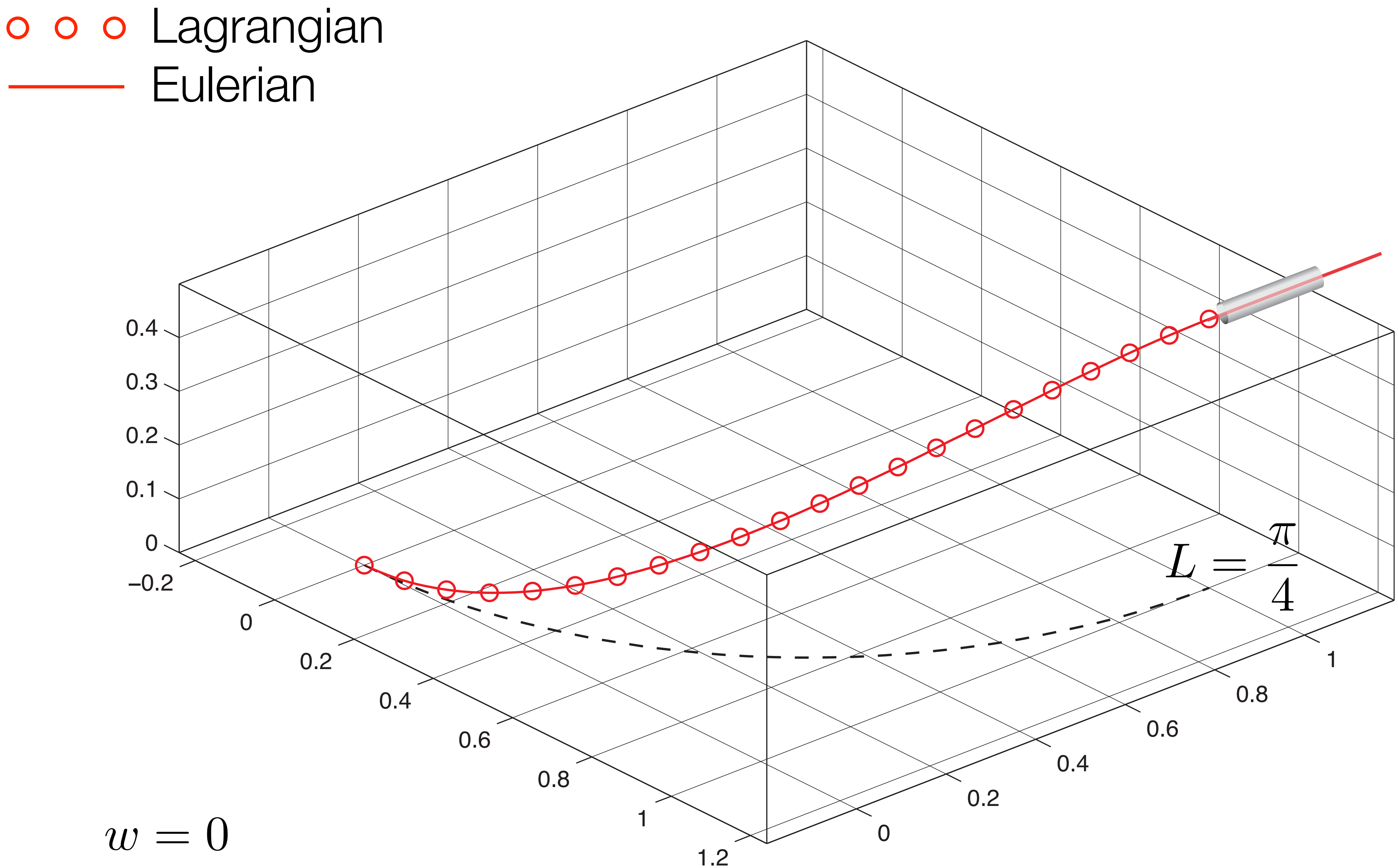


$$w = 0$$

$$F_3\left(L\right) = 0$$

$$\ell = ?$$

Comparison



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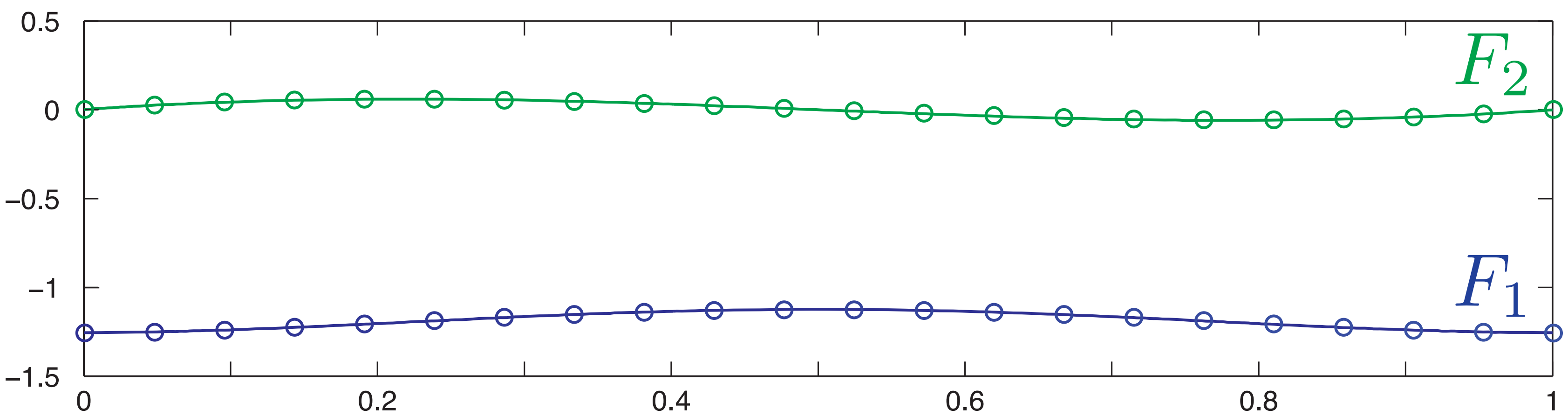
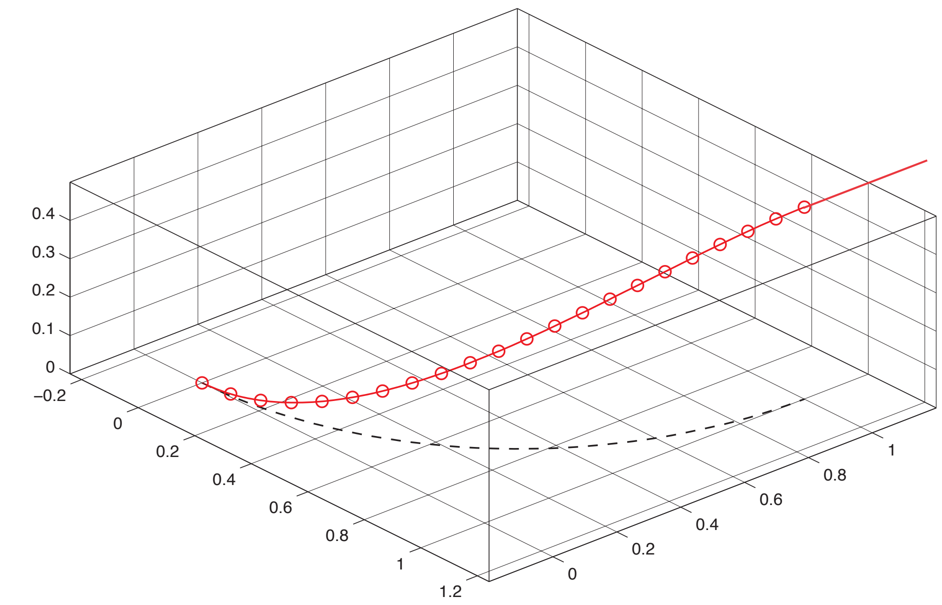
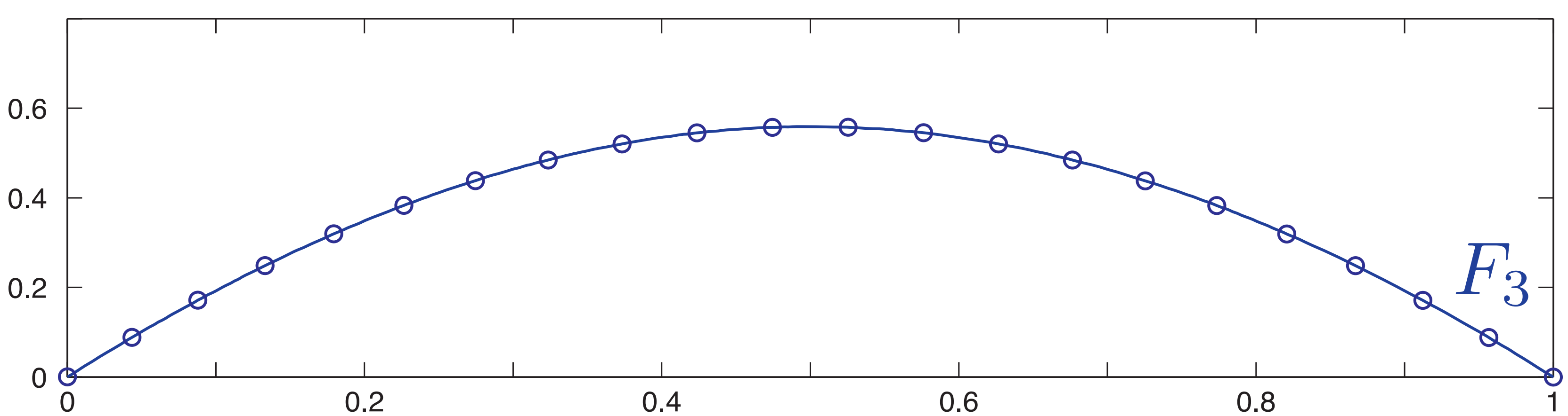
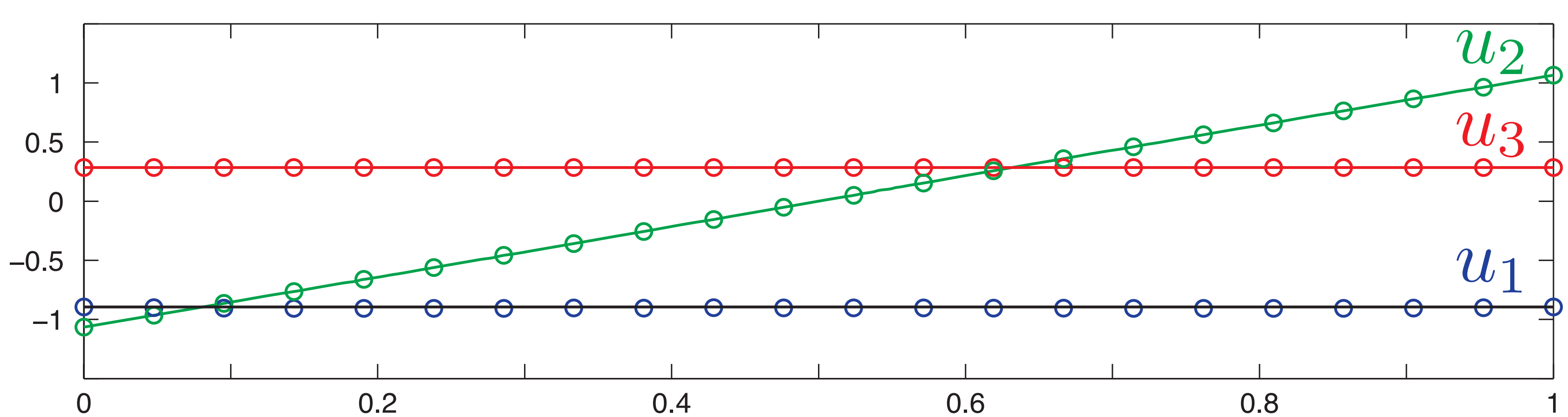


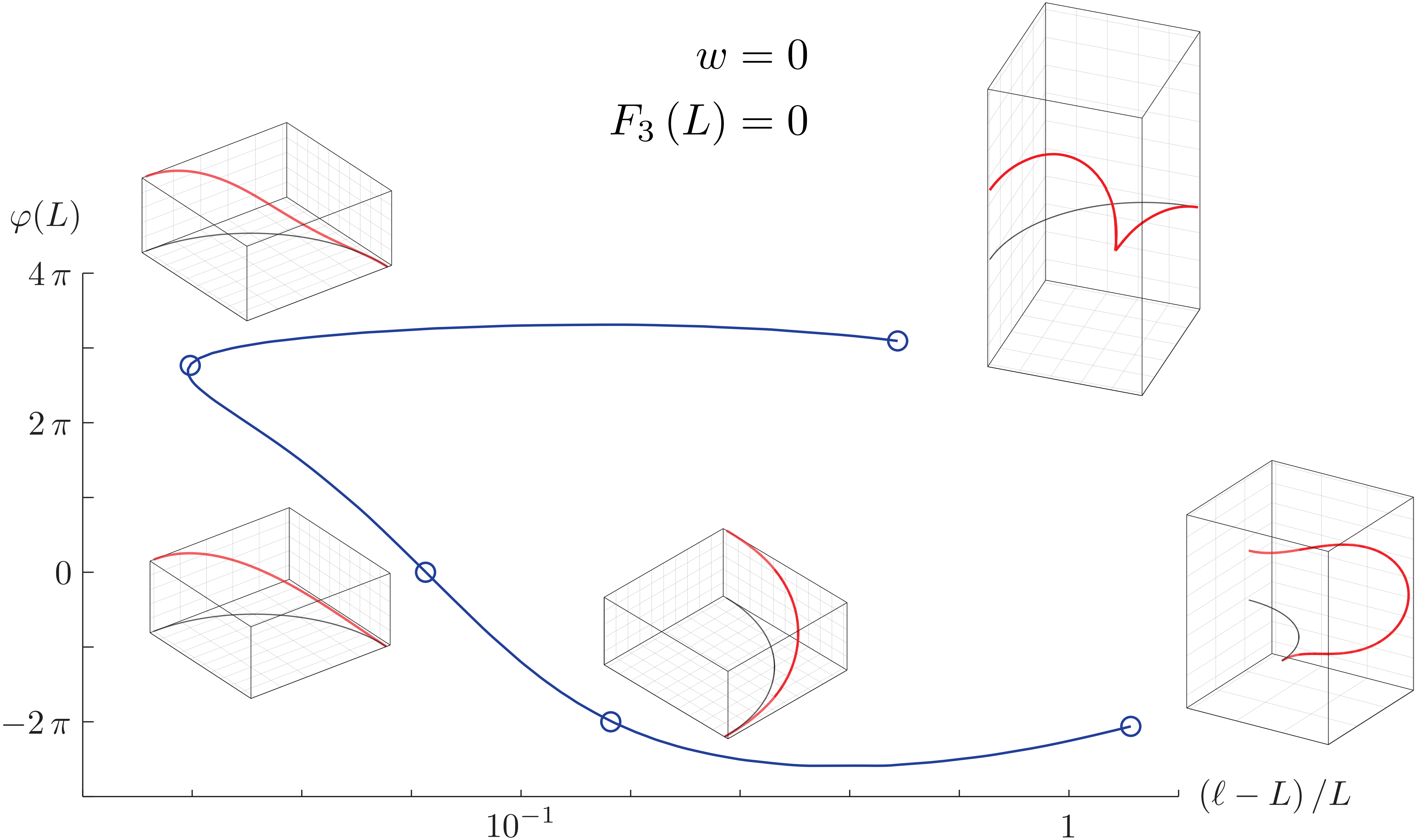




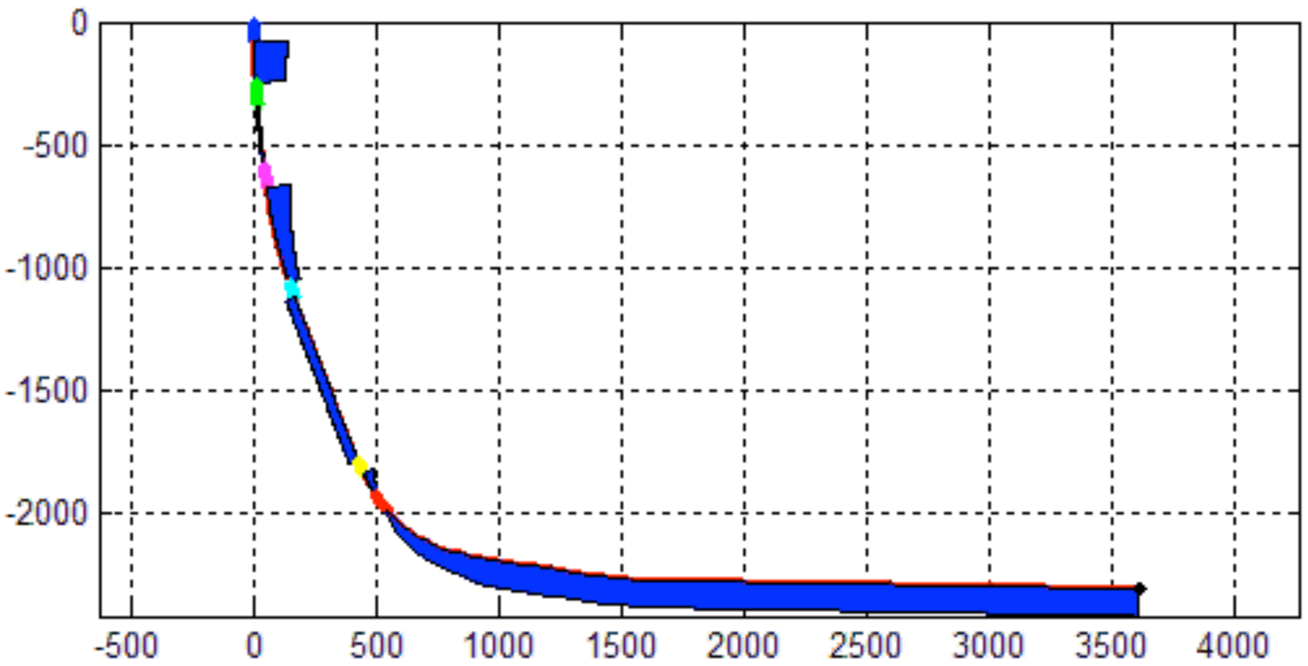


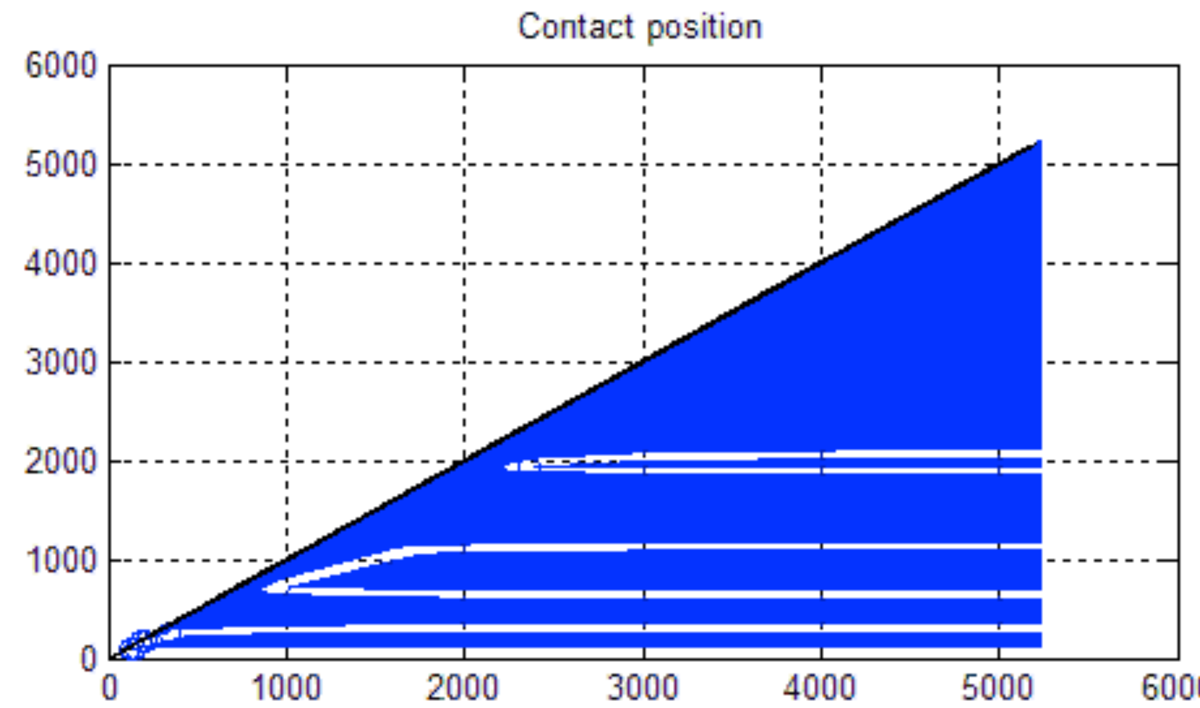
Computation Time	
Eulerian	~ 0.1 sec
Lagrangian	~ 5 sec

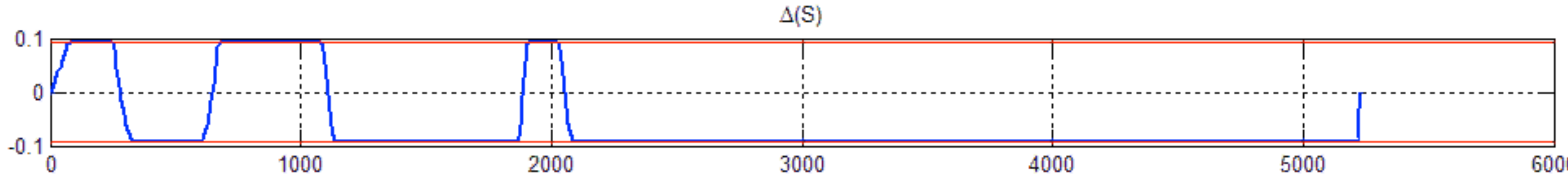
Self-feeding

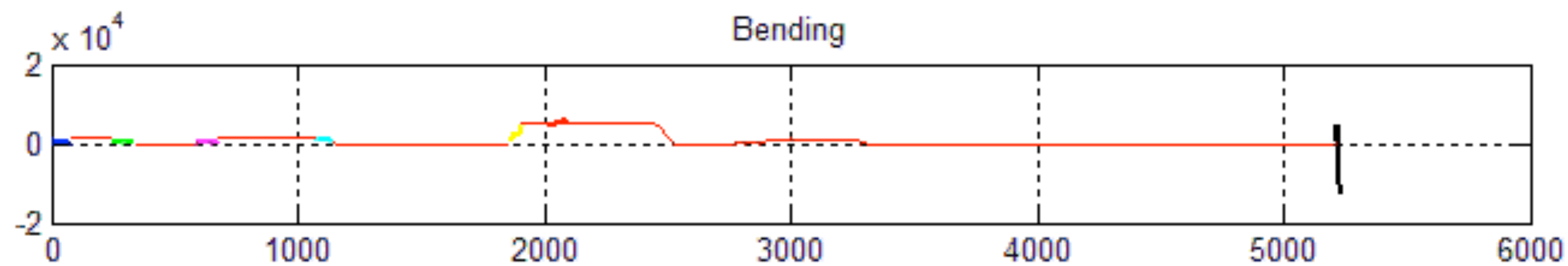


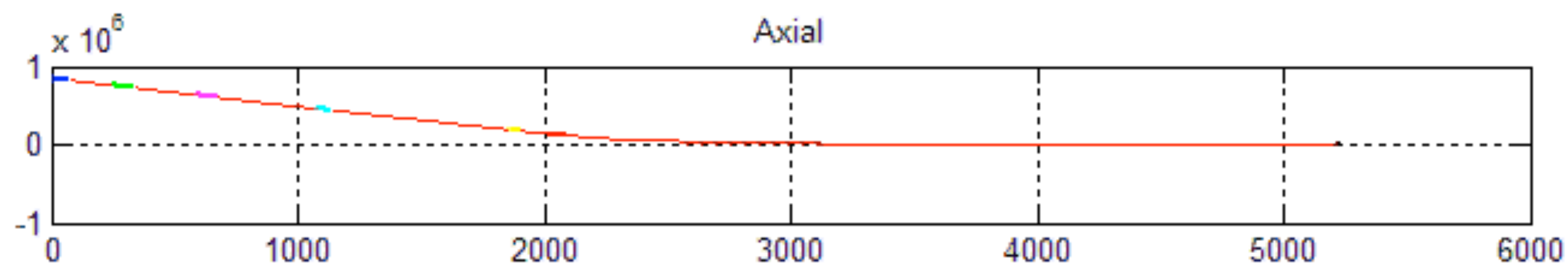
Application: Planar Configuration





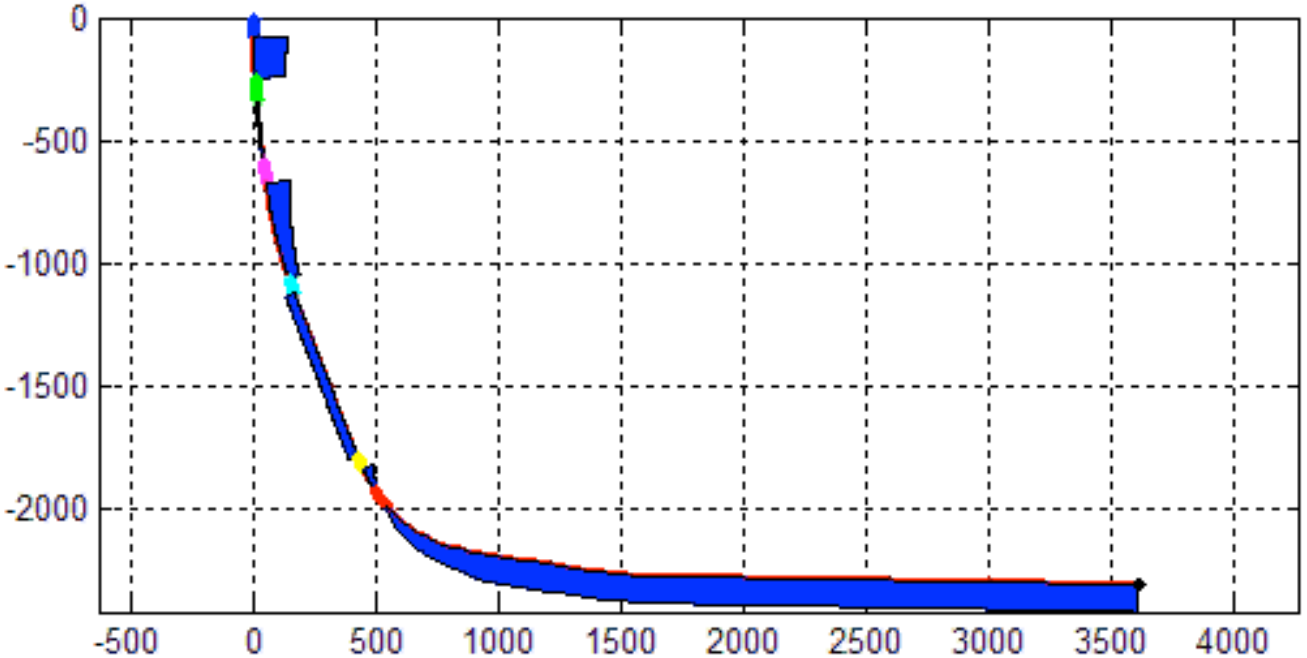


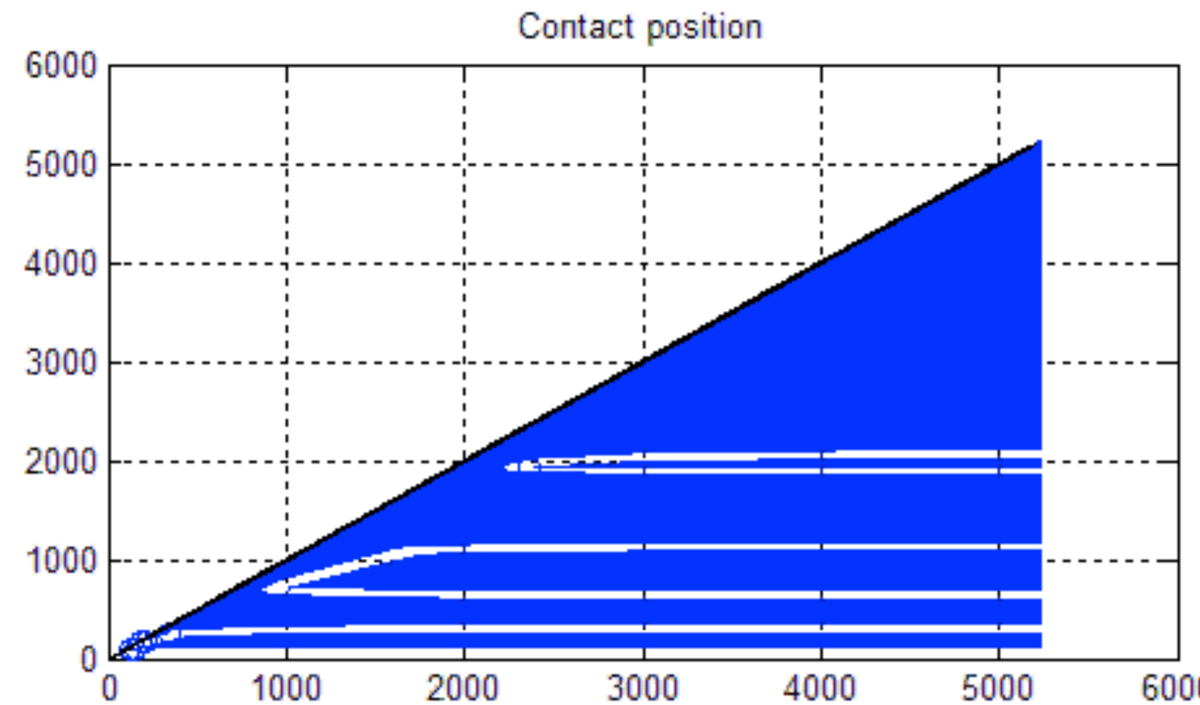


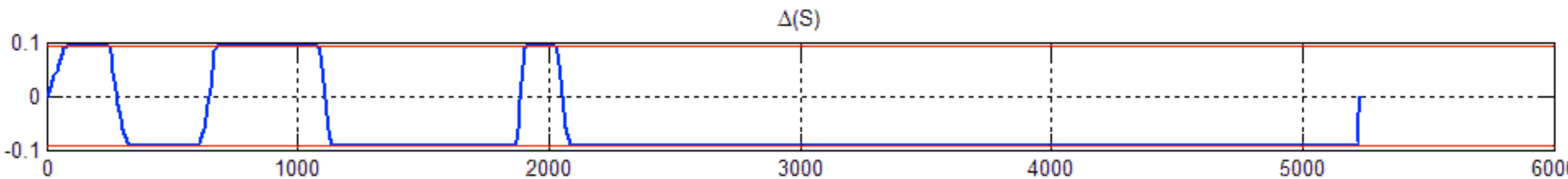


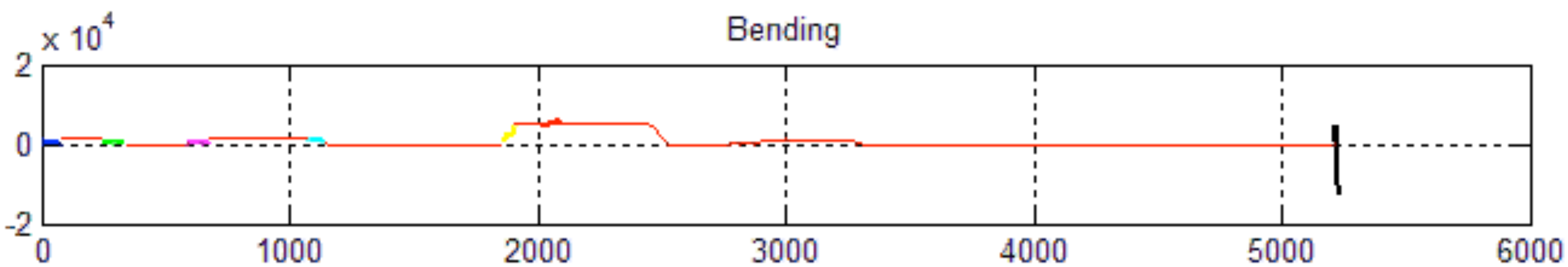
D	35.56 cm
OD	16.83 cm
ID	14.99 cm
EI	291.62 x 10
w	368.04 N/m

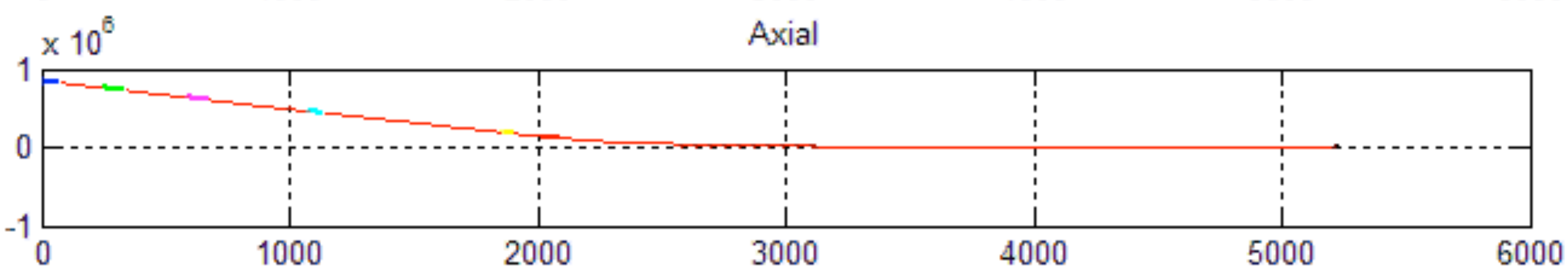
Application: Planar Configuration







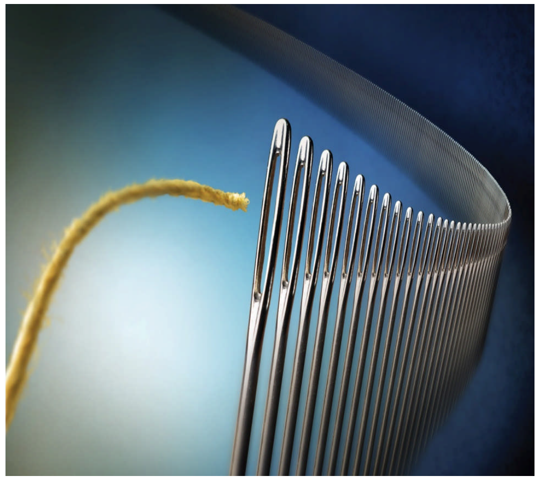




D	35.56 cm
OD	16.83 cm
ID	14.99 cm
EI	291.62 x 10
w	368.04 N/m

Conclusion

- 3-D reformulation of the problem within the Eulerian formalism
  - Introduction of the eccentricity vector $\boldsymbol{\Delta}\,(S)$
  - Description of the rod deformed configuration with respect to a reference curve
  - Fields seen as functions of the curvilinear coordinate associated to a reference curve
- Suppression of the integrals constraints (isoperimetric)
- Improvement of the governing equations conditioning
- Constrained problem
  - Simplification of the contact detection
  - Disregard parasitic solutions with curling
  - Applicable to the continuous contact problem



Thank you