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Olivier Machiels Project Engineer[a], Michel Pirotton (IAHR Member), Professor[b], Archambeau Pierre Research Associate[c], Benjamin Dewals (IAHR Member), Assistant Professor[d] & Sébastien Erpicum (IAHR Member), Research Associate[e]

[a] Arcadis Belgium, B-4000 Liège, Belgium

[b] Hydraulics in Environmental and Civil Engineering (HECE), University of Liege (ULg), B-4000 Liège, Belgium Email:

[c] Hydraulics in Environmental and Civil Engineering (HECE), University of Liege (ULg), B-4000 Liège, Belgium Email:

[d] Hydraulics in Environmental and Civil Engineering (HECE), University of Liege (ULg), B-4000 Liège, Belgium Email:

[e] Hydraulics in Environmental and Civil Engineering (HECE), University of Liege (ULg), B-4000 Liège, Belgium Email:

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Research paper

Experimental parametric study and design of Piano Key Weirs

OLIVIER MACHIELS, Project Engineer, *Arcadis Belgium, B-4000 Liège, Belgium*
Email: o.machiels@arcadisbelgium.be (author for correspondence)

MICHEL PIROTTON (IAHR Member), Professor, *Hydraulics in Environmental and Civil Engineering (HECE), University of Liege (ULg), B-4000 Liège, Belgium*
Email: michel.pirotton@ulg.ac.be

ARCHAMBEAU PIERRE, Research Associate, *Hydraulics in Environmental and Civil Engineering (HECE), University of Liege (ULg), B-4000 Liège, Belgium*
Email: pierre.archambeau@ulg.ac.be

BENJAMIN DEWALS (IAHR Member), Assistant Professor, *Hydraulics in Environmental and Civil Engineering (HECE), University of Liege (ULg), B-4000 Liège, Belgium*
Email: b.dewals@ulg.ac.be

SÉBASTIEN ERPICUM (IAHR Member), Research Associate, *Hydraulics in Environmental and Civil Engineering (HECE), University of Liege (ULg), B-4000 Liège, Belgium*
Email: s.erpicum@ulg.ac.be

ABSTRACT
Piano Key Weirs are an effective solution for dam rehabilitation as well as new dam projects with a high level of hydraulic constraints. In order to improve the efficiency of their design, an experimental study of the influence of the main geometric parameters has been performed. Thirty one configurations were tested for a wide range of discharges. The results of the study show the influence of the weir height, the keys widths and the overhangs lengths on the discharge capacity and flow characteristics. Based on hydraulic considerations, optimum values of the main geometric ratios are provided. An analytical formulation is developed to predict the discharge capacity of the weir as a function of its geometry. It shows an accuracy of 10% compared to the experimental results of this study and from other sources.

Keywords: Control structure; discharge capacity; hydraulic models; hydraulic structure; spillway

1 Introduction

The Piano Key Weir (PKW) is a particular shape of labyrinth weir involving up- and downstream overhangs to reduce the base length requirement (Fig. 1). For a given upstream head, the discharge capacity may be up to four times higher with a PKW than with an ogee-crested weirs of the same width (Ouamane and Lempérière 2006) and 10% higher than with a labyrinth weirs of the same crest footprint (Anderson and Tullis 2011).

The complex geometry of PKW involves a large set of parameters. In order to unify the notations, a specific nomenclature has been developed (Pralong *et al.* 2011). The PKW-unit is defined as the basic structure of a PKW, composed of an inlet key, two side walls and two halves of outlet keys. The main geometric parameters of a PKW are the weir height P, the PKW-unit width W_u, the number of PKW-units N_u, the side crest length B, the inlet and outlet keys widths W_i and W_o, the up- and downstream overhangs lengths B_o and B_i, and the wall thickness T_s (Fig. 1). Basic geometry of a PKW, called type A, includes up- and downstream overhangs. When the downstream or the upstream overhang is omitted, the PKW is of type B or C, respectively. A PKW without overhangs, i.e. a rectangular labyrinth weir with sloped floors, is called type D.

A number of investigations have been performed to identify the optimal values of geometric parameters influencing the PKW efficiency. Ouamane and Lempérière (2006) show that the L/W ratio is the main parameter controlling the discharge capacity. This is confirmed by Leite Ribeiro *et al.* (2012a).

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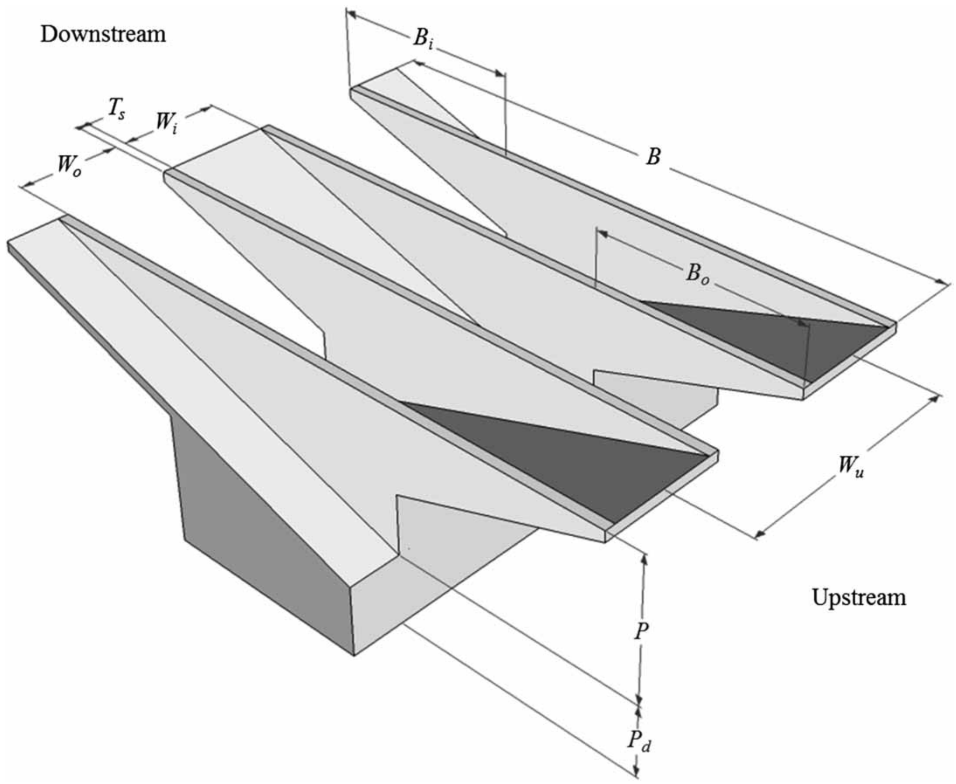

Figure 1 3D sketch of a type A PKW and main geometric parameters

Lempérière *et al.* (2011) state that a ratio L/W equal to 5 is a reasonable compromise between weir efficiency and structure complexity. Anderson and Tullis (2013) show that the optimal range of the ratio W_i/W_o should be in the range 1.25–1.5. Leite Ribeiro *et al.* (2012b) suggest a value of 1.6. Lempérière *et al.* (2011) state that a ratio B_o/B_i higher than one is more efficient.

Based on a detailed investigation of the flow features over a PKW (Machiels *et al.* 2011), the inlet key height P, the ratio W_i/W_o of the key widths and the overhangs positions ratio B_o/B_i have been identified as the main geometric parameters influencing the PKW hydraulic efficiency, for a given ratio L/W. Their influence on the inlet key cross section and thus on the flow velocity along the lateral crest explains the decrease in the PKW capacity observed for increasing heads (Machiels 2012).

This paper presents the results of experimental investigations carried out by the authors to quantify the influence of these three parameters and to find their optimal range for PKW design. From these results, an analytical formulation to predict the PKW discharge capacity is developed. It is compared to the two analytical formulations recently proposed by Leite Ribeiro *et al.* (2012a) and Kabiri-Samani and Javaheri (2012). Finally, design recommendations are also provided, accounting for the specific constraints of real-world projects.

2 Experimental set-up and method for the parametric study

A horizontal flume 7.2 m long, 1.2 m wide and 1.2 m high was used in this study (Machiels *et al.* 2011) (Fig. 2). Water was supplied by means of two pumps delivering up to 300 $\mathrm{l\,s^{-1}}$ into an upstream stilling basin. The upstream extremity of the flume was equipped with a metallic grid and a synthetic membrane ensuring uniform approach flow conditions. Acrylic plates on both sides of the flume enabled the observation of the flow patterns in the vicinity of the PKW model. The width of the flume was adjusted using specific convergent structures.

For each set of parameters, 2.5 units of a 0.3 m wide PKW model (made of PVC) were considered. This enables the observation of the flow in both half an inlet key and half an outlet key (on both sides of the model), as well as the measurement of the hydraulic parameters along the full keys in the central part of the model. This number of units is sufficient to provide representative results independent of the number of keys (Leite Ribeiro *et al.* 2012a). The PKW models were placed on a 0.2 m high support to avoid tailwater effects. A rating curve was derived for each geometry using an electromagnetic flowmeter to measure the upstream discharge (accuracy of 1 $\mathrm{l\,s^{-1}}$) and an ultrasonic probe for the upstream free surface elevation (accuracy of 0.5 mm). To estimate the kinetic term of the head, a uniform velocity distribution throughout the channel cross section was assumed at the location where the free surface elevation was measured. The distance between this location and the weir was chosen higher than three times the value of the head. In addition, measurements of the free surface level were performed along the full inlet key for several upstream heads, using ultrasonic sensors. For the same heads, extensive observations of the flow characteristics along the different part of the weir were also conducted for the different models (Machiels 2012).

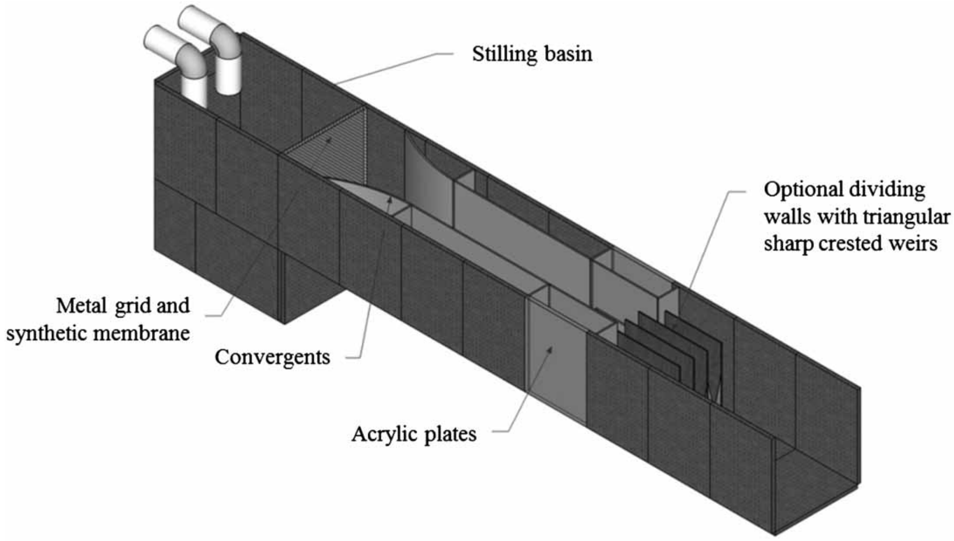

Figure 2 Experimental flume layout

In all the tests, the flow conditions lead to Weber numbers at the crest higher than 50, as suggested by Crookston and Tullis (2010) to limit scale effects on labyrinth weir-scale models. In these conditions, Reynolds numbers are higher than 10^4 and viscous effects are negligible according to Novak *et al.* (1990).

According to previous studies, the ratio between the developed length L of the crest and the width W of the weir is the dominant parameter governing the PKW hydraulic efficiency. This parameter also strongly influences the difficulty of the construction of the structure, and thus its cost. Indeed, it is directly linked to the side crest length B and thus to the overhang lengths. Considering a constant value of L/W equal to 5, referred by Lempérière *et al.* (2011) as the optimal balance between economic and hydraulic interests, the present study focuses sequentially on the ratios P/W_u, W_i/W_o and B_o/B_i.

First, seven models with varying heights ($P/W_u = 0.33$; 0.5; 0.67; 0.8; 1; 1.33; 2) were studied. The ratio W_i/W_o was 1.57, with symmetric overhangs of length equal to one-third of the side crest length B. Second, the influence of W_i/W_o was investigated considering two PKW height ratios ($P/W_u = 1.33$; 0.5) corresponding, respectively, to the hydraulic optimum design and to an economic optimum design, as obtained from the first step. For both weir heights, seven models with different ratios between inlet and outlet keys widths ($W_i/W_o = 0.46$; 0.64; 0.78; 1; 1.29; 1.57; 2.18) were tested. Overhangs were again symmetric with a length equal to one-third of the total side crest length. Finally, the influence of the overhangs position was studied considering again the two PKW heights. Five models with varied ratios between up- and downstream overhangs lengths ($B_o/B_i = 0$; 0.33; 1; 3; ∞) were tested. The W_i/W_o ratio was 1.57 and the base length was equal to one-third of the total side crest length. All tests were conducted considering flat topped crests.

Table 1 summarizes the geometric characteristics of the 31 considered models, depending on the ratio to be analysed. The models were systematically tested for specific discharges ranging from 0.013 to 0.4 m^2s^{-1}, leading to H/P ratios in the range of 0.06 to 3.2.

Table 1 Geometric characteristics of the PKW configurations considered to analyse the effect of the ratios P/W_u, W_i/W_o and B_o/B_i, respectively

Ratio analysed	P/W_u	W_i/W_o	B_o/B_i
W (m)	0.75	0.75	0.75
L (m)	3.75	3.75	3.75
P (m)	0.10; 0.15; 0.20; 0.24; 0.30; 0.40; 0.60	0.15; 0.40	0.15; 0.40
W_u (m)	0.3	0.3	0.3
W_i (m)	0.165	0.085; 0.105; 0.118; 0.135; 0.152; 0.165; 0.185	0.165
W_o (m)	0.105	0.185; 0.165; 0.152; 0.135; 0.118; 0.105; 0.085	0.105
T_s (m)	0.015	0.015	0.015
B (m)	0.6	0.6	0.6
B_o (m)	0.2	0.2	0.4; 0.3; 0.2; 0.1; 0
B_i (m)	0.2	0.2	0; 0.1; 0.2; 0.3; 0.4

3 Results

3.1 *PKW height*

For low PKW heights ($P/W_u < 1$), the discharge capacity increases with the weir height, whatever the upstream head

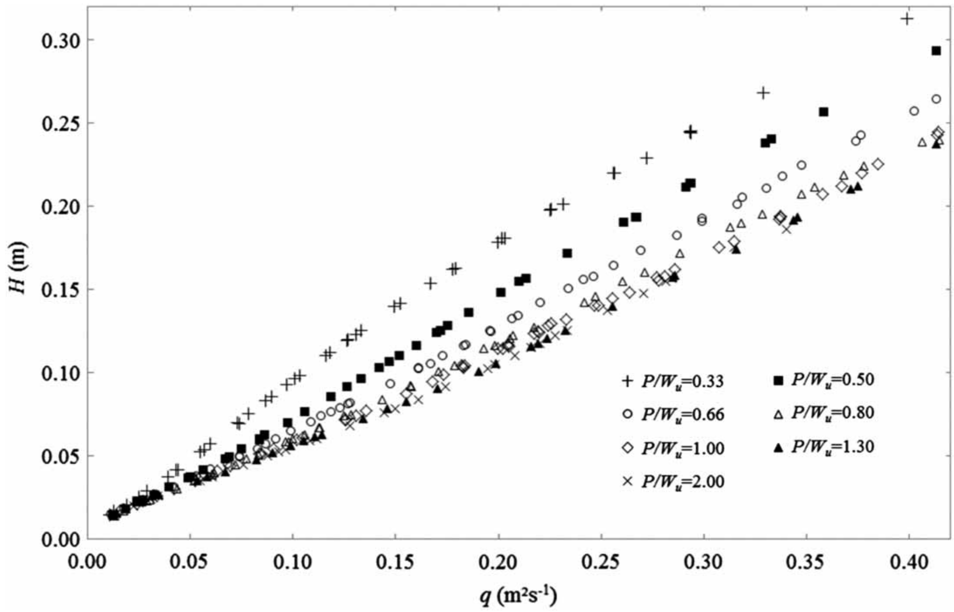

Figure 3 Effect of the PKW height on the rating curve

H (Fig. 3). However, for high PKW heights ($P/W_u > 1$), the discharge capacity becomes virtually independent of the weir height. In the range of tested P/W_u ratios, the best value of P/W_u is 1.3. A former study used parapet walls to highlight the relative influence of the weir height and of the key slope (Machiels *et al.* 2012). Based on 14 scale models, it revealed that the weir efficiency is mainly governed by its height and not by the key slope.

The detailed observation of the flow characteristics (Machiels 2012) shows that for high weirs ($P/W_u > 1.3$), the outlet keys are able to evacuate the flow released from the inlet keys for the whole range of upstream heads. The weir efficiency is thus controlled by the hydraulic capacity of the inlet keys. As long as no significant head losses occur along the weir, a quasi-horizontal free surface is observed along the inlet key, as a result of relatively constant flow velocities. The variation of the inlet key section is suitable to counterbalance the decrease in the discharge along the lateral crest. Under such conditions, the PKW efficiency is mainly a function of the upstream head and the weir height has almost no influence. For very low weir heights ($P/W_u < 0.5$), the outlet key slope is insufficient to lead to supercritical flow conditions in the whole range of considered heads, despite the fact that the tailwater level remains below the outlet key toe. The capacity of the outlet key controls thus the upstream crest efficiency. Furthermore, as the outlet free surface level increases, it exceeds the crest level along a significant section of the side wall. For the highest heads, the whole lateral crest is submerged by the outlet flow. This reduces significantly the side crest discharge. Finally, the decrease in the side crest discharge (due to the outlet flow) as well as the decrease in the inlet key slope and thus also in the inlet cross section, contribute both to an increase in the flow velocity and in the inlet key. This increase in the approach velocity decreases even more the lateral crest efficiency.

The PKW with a P/W_u ratio equal to 1.3 (Fig. 3) is the lowest geometry which shows the highest discharge capacity. It is thus the hydraulic optimum as regards the PKW height. However, a PKW designed based on this criterion would be relatively high and would thus not necessarily comply with other design and construction constraints (dam height, acceptable decrease in the reservoir level for dam rehabilitation works, etc.). Assuming that the total construction cost of a PKW is proportional to the concrete volume of the structure, the most economical geometry provides, for a given head, the highest discharge per cubic meter of concrete. In that case, the optimal P/W_u ratio value is close to 0.5 (Machiels 2012). Low weirs are usually better suited for implementation on existing dams where the depth of concrete demolition is a key factor.

In the next steps of the study, two PKW heights were considered: P/W_u equal to 1.3 as the hydraulic optimum and a ratio of 0.5 as a more economical design. In each case, we investigated how the keys width and the overhangs length may be chosen to optimize the weir efficiency.

Besides, an analytical approach to predict the discharge capacity of a PKW depending on its geometry has been developed based on the data presented above.

The discharge per PKW-unit width $q = Q_u/W_u$ can be estimated as the sum of three components (Eq. 1):

- the discharge per unit length of the upstream crest of the outlet key q_u, mainly controlled by the approaching flow depth, which is the sum of the upstream head H and the overall structure height P_T (sum of the PKW height P and the dam height P_d).
- the discharge per unit length of the downstream crest of the intlet key q_d, influenced by the inlet height P and the upstream head H,

- the discharge per unit length of the lateral crest q_s, controlled by the side wall height and the water depth varying along the inlet key.

$$q = q_u \frac{W_o}{W_u} + q_d \frac{W_i}{W_u} + q_s \frac{2B}{W_u} \tag{1}$$

The three specific discharges can be computed using standard weir equations related to the same upstream head H and considering specific formulations of the discharge coefficient. For up- and downstream crests, the discharge coefficients may be calculated using the SIA formulation for sharp-crested weir (Swiss Society of Engineering Architects 1926). In Eqs. 2 and 3, constant discharge coefficient terms have been modified to take into account the weir inclination over the vertical (Machiels 2012).

$$q_u = 0.374 \left(1 + \frac{1}{1000H + 1.6}\right) \times \left[1 + 0.5\left(\frac{H}{H + P_T}\right)^2\right] \sqrt{2gH^3} \tag{2}$$

$$q_d = 0.445 \left(1 + \frac{1}{1000H + 1.6}\right) \times \left[1 + 0.5\left(\frac{H}{H + P}\right)^2\right] \sqrt{2gH^3} \tag{3}$$

Considering the lateral crest, a correction term was added to take into account the flow inertia in the inlet key direction. Furthermore, considering the theory for lateral weir (Hager 1987), the head in the weir equation is the water depth over the crest level. Assuming a linear water depth variation from the upstream head H to the critical depth $0.67H$, respectively, at the up- and downstream extremities of the side crest, the discharge on the lateral crest can be calculated as follows:

$$q_s = 0.41 \left(1 + \frac{1}{833H + 1.6}\right) \left[1 + 0.5\left(\frac{0.833H}{0.833H + P_e}\right)^2\right] \times \left[\frac{P_e^\alpha + \beta}{(0.833H + P_e)^\alpha + \beta}\right] K_{W_i} K_{W_o} \sqrt{2gH^3} \tag{4}$$

where α and β are parameters related to the weir geometry, P_e is the mean side wall height

$$P_e = \frac{B_o}{B} P_T + \left(1 - \frac{B_o}{B}\right) \frac{P}{2} \tag{5}$$

and K_{W_i} and K_{W_o} are factors related to the keys width ratio.

A least-square approach on the experimental results provides values of the parameters α and β related to the inlet key slope:

$$\alpha = \frac{0.7}{S_i^2} - \frac{3.58}{S_i} + 7.55 \tag{6}$$

$$\beta = 0.029\mathrm{e}^{-1.446/S_i} \tag{7}$$

3.2 *Keys widths*

The effect of a varying W_i/W_o ratio was studied using as a reference the symmetric geometry ($W_i/W_o = 1$) (Fig. 4).

For P/W_u equal to 1.3 (Fig. 4a) and relative heads lower than 0.4, a ratio W_i/W_o equal to 2.18 leads to an increase by up to 8% in the discharge efficiency compared to the reference geometry. However, for higher relative heads, the efficiency remains similar as in the reference geometry. W_i/W_o ratios of 1.29 and 1.57 provide a relatively constant gain in efficiency around 4%. W_i/W_o ratios lower than 1 systematically decrease the discharge capacity of the weir.

For P/W_u equal to 0.5 (Fig. 4b), only the ratios W_i/W_o equal to 1.29 and 1.57 resulted in an increase in the discharge efficiency compared to the symmetric configuration ($W_i/W_o = 1$). The gain in discharge reaches up to 6% for low heads and is limited to 1% for higher heads. W_i/W_o ratios lower than 1 and equal to 2.18 decrease the weir efficiency by up to 20%.

Again, the detailed observation of the flow characteristics (Machiels 2012) enables to explain the effect of the W_i/W_o ratio on the weir efficiency. For a high PKW height, the discharge capacity is mainly controlled by the flow velocity along the inlet key. The lower this velocity, the higher the side crest efficiency and thus the PKW efficiency. As the W_i/W_o ratio increases, the inlet cross section increases also and the flow velocity decreases. However, for the highest values of the W_i/W_o ratio, the outlet key is too narrow and significant interferences occur between opposite side nappes from adjacent side walls for the highest

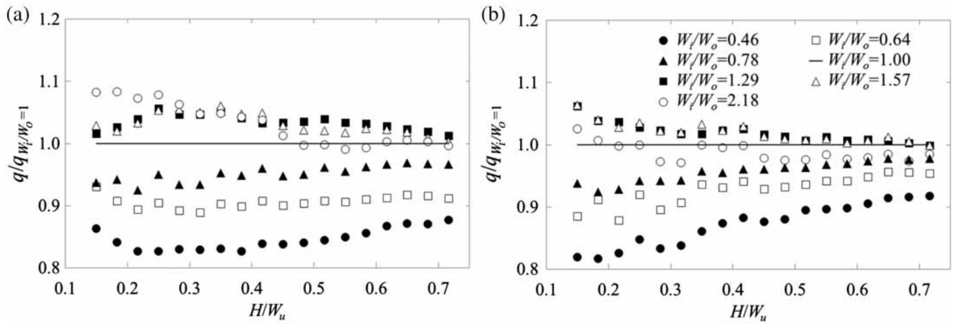

Figure 4 Relative effect of the keys widths ratio: (a) $P/W_u = 1.3$ and (b) $P/W_u = 0.5$

heads. For a low PKW height, the weir efficiency is governed by a combination of the flow acceleration in the inlet key due to the bottom slope and by the submergence of the side crest by the outlet flow. Since the increase in the inlet key width reduces the flow acceleration in the inlet key, increasing the W_i/W_o ratio increases the PKW efficiency. However, the relative decrease of the outlet key width increases the free surface level along the outlet key and thus increases the length of the submerged side crest.

The range of the ratio H/W_u considered up to now varies between 0.18 and 0.25 for dam rehabilitations in Europe (Vermeulen *et al.* 2011), and between 0.35 and 0.55 for dam rehabilitations as well as new dam projects in Asia (Das Singhal and Sharma 2011, Ho Ta Khanh *et al.* 2011). Regarding the results presented above for these ranges of H/W_u ratios, W_i/W_o ratios between 1.29 and 1.57 provide the optimal discharge capacity whatever the weir height. However, in dam engineering projects for which an economic design is of high importance (e.g. development projects) and when a large number of PKW-units is involved, the use of a symmetric geometry ($W_i/W_o = 1$) may remain the most relevant, as it facilitates the use of precast elements. Indeed, for low PKW heights, the gain in efficiency provided by non-symmetric key widths is only 2% of the gain resulting from the optimisation of the weir height.

The influence of the inlet key width on the side crest discharge is included in the analytical formulation considering a factor K_{W_i} in Eq. 4, proportional to the square of the inlet key width. Indeed, according to Hager (1987), the discharge over a side weir (here the lateral crest) is a function of the square of the Froude number along the main channel (here the inlet key). As the Froude number is directly proportional to the inverse of the inlet key width, the following expression for K_{W_i} is proposed:

$$K_{W_i} = 1 - \frac{\gamma}{\gamma + W_i^2} \tag{8}$$

where γ is a parameter fitted on the experimental results obtained with $P/W_u = 1.3$ and $W_i/W_o \leq 1$ (Eq. 9). Indeed, these configurations are mainly controlled by the inlet key width and not the outlet key width.

$$\gamma = 0.0037\left(1 - \frac{W_i}{W_o}\right) \tag{9}$$

The outlet key width and slope also affect the effective length of the lateral crest. Therefore, a second correction factor K_{W_o} is introduced in Eq. 4. It is given as a function of the ratio between the upstream head and the outlet key width, which characterizes the combination of the outlet key submergence and the lateral flows interference. It is assumed that for H/W_o lower than a threshold value δ_1, the outlet key does not influence the flow over the lateral crest. For H/W_o higher than a second threshold value δ_2, the side crest is fully submerged by the outlet flow and the discharge over the lateral crest is zero. For intermediate values, Machiels (2012) found that the effective lateral crest length varies with the H/W_o ratio following Eq. 10.

$$\begin{aligned}
K_{W_o} &= 1 \quad \text{for } \frac{H}{W_o} \leq \delta_1 \\
K_{W_o} &= \frac{2}{(\delta_2-\delta_1)^3}\left(\frac{H}{W_o}\right)^3 - \frac{3(\delta_2+\delta_1)}{(\delta_2-\delta_1)^3}\left(\frac{H}{W_o}\right)^2 \\
&\quad + \frac{6\delta_2\delta_1}{(\delta_2-\delta_1)^3}\left(\frac{H}{W_o}\right) + \frac{\delta_2^2(\delta_2-3\delta_1)}{(\delta_2-\delta_1)^3} \quad \text{for } \delta_1 \leq \frac{H}{W_o} \leq \delta_2 \\
K_{W_o} &= 0 \quad \text{for } \delta_2 \leq \frac{H}{W_o}
\end{aligned} \tag{10}$$

The values of the two thresholds δ_1 and δ_2 are directly related to the outlet key slope, which governs the outlet flow regime. Their values are discussed in the following section, together with the influence of the overhangs lengths which define the key slopes for a given weir height.

3.3 *Overhangs lengths*

The most widespread design of PKW considers symmetric up- and downstream overhangs (type A PKW; $B_o/B_i = 1$). The influence of the B_o/B_i ratio was studied here, using as a reference the efficiency of the symmetric configuration (Fig. 5).

For $P/W_u = 1.3$ (Fig. 5a), a ratio B_o/B_i equal to 3 is leads to a 10% increase in the efficiency compared to the reference geometry, whatever the upstream head. For low heads, the model with only upstream overhangs (type B PKW; $B_o/B_i = \infty$) is the most efficient but this efficiency decreases rapidly with increasing head and, for $H/W_u > 0.3$, the gain becomes negligible compared to the reference model. B_o/B_i ratios lower than 1 always lead to a smaller efficiency than the reference model.

For $P/W_u = 0.5$ (Fig. 5b), all the tested B_o/B_i ratios lead to a decrease in the weir efficiency by up to 20% compared to the symmetric geometry.

The main influence of the overhangs position is to modify the bottom slope of the keys as well as their cross section. Considering the inlet for instance, the flow depth and width are less restricted along the upstream overhang than above the base length of the weir. Moving the overhangs towards upstream increases, thus, the inlet key cross section, which leads in turn to a decrease in the flow velocities along the inlet key. However, this also decreases the outlet key slope and cross section, causing a more rapid filling of the outlet as well as the submergence of the lateral crest by the outlet flow.

The optimal geometry results thus from a compromise between a decrease in the inlet flow velocity, in order to increase the lateral crest efficiency, and the saturation of the outlet key as a result of its slope and cross section reduction. For a high PKW height, the saturation of the outlet is delayed thanks to the steeper slope. The optimal geometry for the high PKW configuration corresponds thus to overhangs moved towards upstream

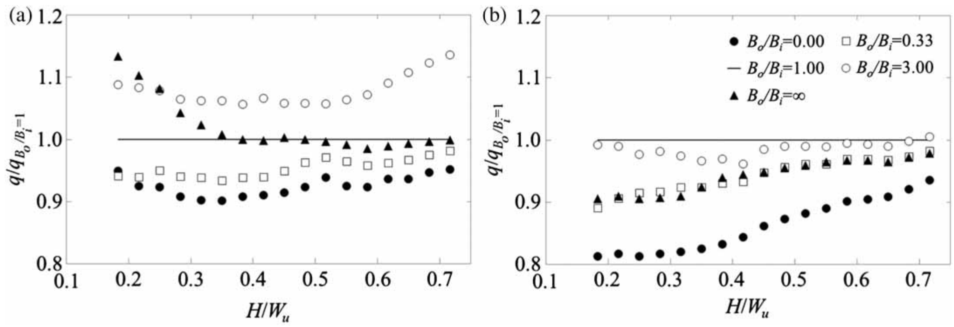

Figure 5 Relative effect of the overhangs lengths ratio: (a) $P/W_u = 1.3$ and (b) $P/W_u = 0.5$

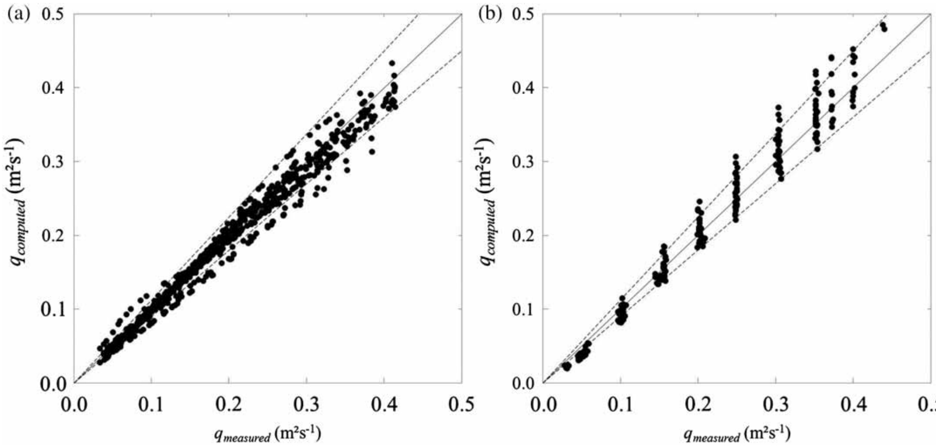

Figure 6 Comparison between specific discharges computed with the analytical formulation proposed in this paper and the experimental results from (a) the authors and (b) Le Doucen *et al.* (2009). Plain line = perfect agreement; dotted lines = ±10% error

($B_o/B_i = 3$ in our study), whereas the symmetric geometry is more efficient for the low weir configuration.

Type B PKW is more appropriate for the design of high PKW and considering a limited range of upstream heads. For structural considerations (self-balanced structure, the use of precast elements), most PKW prototypes use symmetric up- and downstream overhangs (type A geometry). However, at least one PKW with B_o/B_i equal to 3 has been built to date, at Malarce dam in France (Pinchard *et al.* 2011). No structural concern has been raised by the strongly non-symmetric profile of the structure.

The outlet key slope S_o controls the two limits δ_1 and δ_2 characterizing the side crest submergence in Eq. 10. Applying a least-square method to the results obtained for the 10 configurations with varied overhangs lengths, the limits δ_1 and δ_2 are computed as follows:

$$\delta_1 = -0.788 S_o^{-1.88} + 5 \tag{11}$$

$$\delta_2 = 0.236 S_o^{-1.94} + 5 \tag{12}$$

4 Discussion

The comparison of the final analytical formulation with the experimental results obtained on the 31 geometries considered in the present study (832 tests) is shown in Fig. 6a. For most configurations, the error does not exceed 10%. The coefficient of determination between the measured and computed values is $R^2 = 0.982$ for the whole data set. The analytical formulation has also been compared to the experimental results obtained by Le Doucen *et al.* (2009) at the Laboratory of Hydraulic Constructions of EPFL (Fig. 6b). Compared to the study presented in this paper, Le Doucen *et al.* (2009) tested PKW geometries with an increased range of L/W ratio (3–7) and different slopes in the inlet and the outlet keys ($P_i/P_o = 0.72$–1.4). For this data set with extended geometric parameters, the coefficient of determination between the measured and computed values with the formulation proposed in this paper is $R^2 = 0.975$.

Two formulations to predict the discharge capacity of a PKW geometry were published recently (Leite Ribeiro *et al.* 2012a, Kabiri-Samani and Javaheri 2012). The parameters range of the geometries considered by these authors to derive their formulations is summarized in Table 2. Compared to the parameters range used in the present study, Leite Ribeiro *et al.* (2012a) consider an extended range for L/W and P_i/P_o but a reduced one for P/W_u and B_o/B_i. Kabiri-Samani and Javaheri (2012) consider an extended range for L/W, but a more limited one for H/P, B/P and, especially, for B_i/P as well as B_o/P, as mentioned by Pfister *et al.* (2012). In addition, the present study considers flat topped crests, whereas Leite Ribeiro *et al.* (2012a) used half-circular rounded crests and Kabiri-Samani and Javaheri (2012) sharp crests. The comparison between the discharges predicted by the formulation proposed in the present study and those

Table 2 Parameters range used to derive the formulation proposed in the present study, in Leite Ribeiro *et al.* (2012a) and in Kabiri-Samani and Javaheri (2012)

	L/W	P/W_u	W_i/W_o	B_o/B_i	H/P	B/P	B_i/P and B_o/P	P_i/P_o
Present study	5.0	0.33 … 2.00	0.46 … 2.18	0 … ∞	0.06 … 3.20	1.0 … 6.0	0.00 … 2.67	1.00
Leite Ribeiro *et al.* (2012a)	3.0 … 7.0	0.29 … 0.65	0.50 … 2.00	1	0.10 … 2.80	1.5 … 4.6	0.25 … 4.00	0.72 … 1.40
Kabiri-Samani and Javaheri (2012)	2.5 … 7.0	?	0.33 … 1.22	0 … ∞	0.10 … 0.60	1.0 … 2.5	0.00 … 0.26	1.00

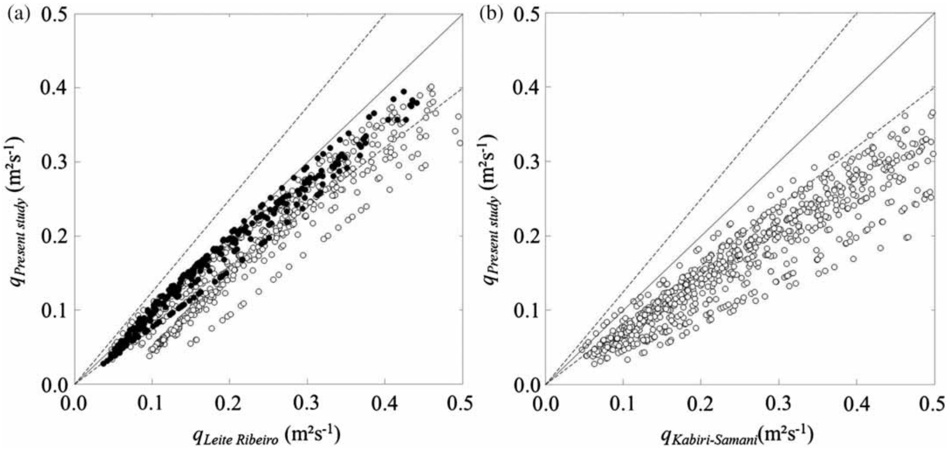

Figure 7 Comparison between specific discharges computed with the analytical formulation proposed in this paper and the analytical formulation from (a) Leite Ribeiro *et al.* (2012a) and (b) Kabiri-Samani and Javaheri (2012) using geometries of Table 1. Plain line = perfect agreement; dotted lines = ±20% error – Black dots correspond to geometries respecting the parameters range of both formulations; while white dots correspond to geometries outside the parameters range of previously published formulations

obtained using Leite Ribeiro *et al.* (2012a) or Kabiri-Samani and Javaheri (2012) formulation (Fig. 7) highlights the significant differences in PKW capacity estimation which may result from the application of a formulation outside its parameters range. Pfister and Schleiss (2013) show that the different crest shapes explain to a large extent the limited differences among the three formulations inside their common parameters range.

5 Conclusions

A systematic experimental analysis was performed to investigate the effect of the inlet key height P, the relative keys widths ratio W_i/W_o and the overhangs positions ratio B_i/B_o on the discharge capacity of a PKW with a L/W ratio equal to 5. Results show that a single optimal value of the three parameters does not exist whatever the upstream head. However, key points were highlighted to approach an optimal design.

From a hydraulic point of view, it is of primary importance to set the different geometric ratios in such a way to maximize the inlet cross section, as this section may be considered as the *engine* of the PKW. Indeed, increasing the inlet cross section decreases the flow velocity along the lateral crest, and thus it increases the efficiency of the lateral crest. The optimal value of the geometric parameters is reached when the release capacity of the outlet key starts to be affected. Indeed, the outlet key can be seen as a *brake* for the PKW. Too small outlet cross section and slope tend to increase the free surface level over the lateral crest elevation and thus to significantly hamper the weir efficiency.

A PKW design with a height ratio P/W_u equal to 1.3, a keys widths ratio W_i/W_o equal to 1.25 and an overhangs lengths ratio B_o/B_i equal to 3 was found to provide the highest discharge capacity when the L/W ratio is equal to 5.

However, the study also highlights the importance of technical and economic criteria in the definition of an optimal PKW design. Although a high PKW ($P/W_u = 1.3$) is more effective from a hydraulic point of view and should thus be preferred for new dam projects, lower weirs ($P/W_u \approx 0.5$) should be preferred for rehabilitation projects. For the later, W_i/W_o and B_o/B_i ratios equal to 1 are relevant values.

Using the experimental results, a process-oriented analytical formulation was developed to predict the discharge capacity of a PKW from its geometry. The comparison of the analytical discharge predictions to the experimental results shows a 10% accuracy. The formulation has been developed considering experiments carried out in a channel without tailwater level influence. Consequently, it is suited for top-of-dam applications. However, the formulation does not consider any abutments effects. Although less accurate than experimental models, such an analytical formulation constitutes a relevant tool for the design of new PKW solutions. Comparisons with previously published formulations show that significant differences in PKW capacity estimations may result from the application of a formulation outside its parameters range.

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Notation

B = side crest length (m)
B_i = downstream overhang length (m)
B_o = upstream overhang length (m)
H = water head (m)
K_{W_i} = side crest discharge corrective term due to inlet width influence (–)
K_{W_o} = side crest length corrective term due to outlet width and slope influence (–)
L = developed crest length (m)
N_u = number of PKW-units (–)
P = PKW height (m)
P_d = dam height (m)
P_e = mean lateral wall height (m)
P_i = inlet key height (m)
P_o = outlet key height (m)
P_T = total weir height (m)
q = specific discharge (m^2s^{-1})
q_d = downstream crest specific discharge (m^2s^{-1})
q_s = side crest specific discharge (m^2s^{-1})
q_u = upstream crest specific discharge (m^2s^{-1})
S_i = inlet key slope (–)
S_o = outlet key slope (–)
T_s = side wall thickness (m)
W = PKW width (m)
W_i = inlet key width (m)
W_o = outlet key width (m)
W_u = PKW-unit width (m)
α = lateral discharge parameter (–)
β = lateral discharge parameter (–)
δ_1 = lower limit of outlet key influence on the side crest discharge (–)
δ_2 = upper limit of outlet key influence on the side crest discharge (–)
γ = outlet width influence parameter (–)

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