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MULTISCALE ASPECTS IN MODELING PERCUSSIVE DRILLING: FROM  
WAVE PROPAGATION TO RIGID BODY DYNAMICS

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<sup>1</sup> **1 Introduction**

<sup>2</sup> In down-the-hole (DTH) percussive drilling, penetration is mainly achieved via  
<sup>3</sup> the repeated impulsive loading of a bit through the impacts of a piston on the  
<sup>4</sup> bit shank adapter, the generated stress pulses leading to the indentation, the  
<sup>5</sup> crushing and the chipping of the rock in contact with the bit buttons [1]. Most  
<sup>6</sup> suitable to drilling medium to hard rock formations where it outperforms con-  
<sup>7</sup> ventional drilling technologies, DTH percussive drilling finds a widespread usage  
<sup>8</sup> in the Earth Resources Industry, for both shallow and deep drilling.

<sup>9</sup> Due to the complexity of the process, its scientific understanding lags well  
<sup>10</sup> behind the knowledge body acquired in the field. Analytical and numerical  
<sup>11</sup> models have well been proposed to assess its efficiency; see [1, 2], for instance.  
<sup>12</sup> They, however, enable this assessment on the basis of a single activation only,  
<sup>13</sup> which is a strong restriction. Integrated models capable of predicting the process  
<sup>14</sup> long-term dynamics at a reasonable computational cost are thus required to  
<sup>15</sup> increase the process understanding and drive its optimal utilization.

<sup>16</sup> This paper is concerned with the analysis of such a model [3] that builds  
<sup>17</sup> on the coupling of the axial motion of an elastic piston activated by a simplified  
<sup>18</sup> pressure law, interacting with an elastic bit that itself is in unilateral contact with  
<sup>19</sup> an interface model representative of the bit/rock interaction. Following a brief  
<sup>20</sup> presentation of the model, the analysis of its stationary response is conducted  
<sup>21</sup> for a reference configuration. Emphasis is put on the features relative to the  
<sup>22</sup> multiscale nature of the model, with a particular interest into the aspects related  
<sup>23</sup> to wave propagation and rigid body dynamics.

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## 2 Semi-discrete DTH percussive drilling model

2 The model proposed by the authors [3] relies on the interaction of three essential  
3 subprocesses: (i) a simplified pressure law defining the motion-dependent force  
4  $F_A$  driving the piston, (ii) a generalization of the standard single drilling cycle  
5 bilinear bit/rock interaction model to successive drilling cycles that defines the  
6 force on bit  $F_R$ , and (iii) an elastic representation of the piston and bit bodies  
7 considered as collinear cylinders of identical cross section. Figure 1 shows a  
8 representation of this simplified model.

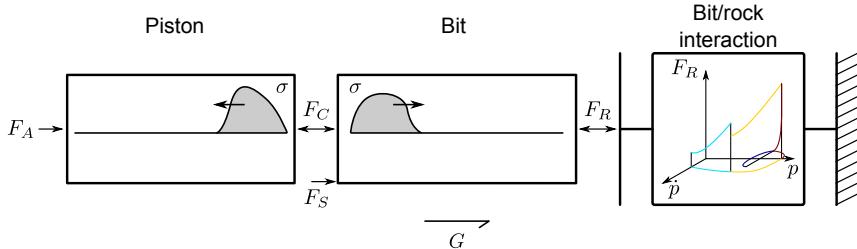


Figure 1: Simplified DTH model, from [3].

9 Given the piecewise definition of (i) and (ii), the authors have developed  
10 a numerical integration procedure that combines a spatial discretization using  
11 the finite element method with an event-driven integration strategy that fully  
12 exploits the piecewise linear character of the governing equations. In its dimen-  
13 sional form, the semi-discrete computational model can be summarized by the  
14 generic equation of motion

$$\mathbf{M}\ddot{\mathbf{v}} + \mathbf{C}_A\dot{\mathbf{v}} + (\mathbf{K} + \mathbf{K}_A)\mathbf{u} = \mathbf{f} + \mathbf{f}_A, \quad \mathbf{v} = \dot{\mathbf{u}}, \quad (1)$$

15 where fields  $\mathbf{u}, \mathbf{v}$  denote the nodal displacements and velocities,  $\mathbf{K}, \mathbf{M}$  are the  
16 constant stiffness and mass matrices resulting from the finite element discretiza-  
17 tion of the piston and bit bodies using linear 1D finite elements, vector  $\mathbf{f}$  denotes  
18 the external dead loads (gravity  $G$  and feed force  $F_S$  applied to the bit), and  
19 an overhead dot denotes differentiation with respect to time.  $A$ -subscripted  
20 variables are piecewise defined in accordance with the status of the bit/rock in-  
21 teraction model, the pressure law and the contact at the piston/bit interface that  
22 is handled using the penalty method. They can be expressed as

$$\begin{aligned} \mathbf{K}_A &= \beta_R^1 K_R \mathbf{w}_1 \mathbf{w}_1^T + \beta_C K_C \mathbf{w}_2 \mathbf{w}_2^T, & \mathbf{C}_A &= \beta_R^2 C_R \mathbf{w}_1 \mathbf{w}_1^T, \\ \mathbf{f}_A &= \alpha F_0 \mathbf{M} \mathbf{1}_1 + \beta_R^1 K_R u_R \mathbf{w}_1 + \beta_C K_C g_0 \mathbf{w}_2, \end{aligned} \quad (2)$$

23 where  $\mathbf{w}_1, \mathbf{w}_2$  are signed localization vectors that enter the definitions of the  
24 penetration while drilling (an affine transformation of the bit displacement at

1 the bit/rock interface so that the force on bit remains continuous at the start of  
 2 a new drilling cycle) and of the gap function at the piston/bit interface,  $F_0$  is the  
 3 reference force of the pressure law,  $\mathbf{1}_1$  is a localization vector with unit entries  
 4 at degrees of freedom corresponding to the piston and zeros at other entries,  $K_C$   
 5 is the numerical contact stiffness,  $g_0$  the initial gap at the piston/bit contact  
 6 interface, and  $K_R, C_R$  are the stiffness and viscosity parameters relative to the  
 7 bit/rock interaction law. Other parameters are state-dependent and defined as  
 8 follows

$$\begin{aligned} \beta_R^1 &:= \begin{cases} 0 & \text{if FF,} \\ 1 & \text{if DFC, FC,} \\ \gamma & \text{if BC, SS,} \end{cases} \quad \beta_R^2 := \begin{cases} 1 & \text{if DFC,} \\ 0 & \text{otherwise,} \end{cases} \quad u_R := \begin{cases} 0 & \text{if FF} \\ u_\ell & \text{if DFC, FC,} \\ u_u & \text{if BC, SS,} \end{cases} \\ \beta_C &:= \begin{cases} 0 & g := \mathbf{w}_2^T \mathbf{u} + g_0 > 0, \\ 1 & g \leq 0, \end{cases} \quad \alpha := \begin{cases} 1 & \text{if } (v_r, \dot{v}_r) \in (-\infty, -D_1] \times \mathbb{R} \\ & \cup (-D_1, 0] \times \mathbb{R}^+, \\ -1 & \text{otherwise,} \end{cases} \end{aligned} \quad (3)$$

9 with variables  $v_r, \dot{v}_r$  denoting the average piston motion relative to that of the  
 10 bit;  $D_1$  is the second parameter of the pressure law that governs the switch of  
 11 the pressure force direction;  $u_\ell, u_u$  denote the positions of the rock surface corre-  
 12 sponding to the lower and upper positions along the drilling cycle. Acronyms FF,  
 13 DFC, FC, BC, SS refer to the modes of the bit/rock interaction, namely free flight,  
 14 dissipative forward contact, forward contact, backward contact and standstill.  
 15 For a complete definition of the model, the equation of motion is supplemented  
 16 by the mode transition conditions that correspond to the zeros of the event func-  
 17 tions. These functions trigger the switching from one mode to another whenever  
 18 they cross zero. For the bit/rock interaction law, they read

$$\begin{array}{llll} \text{FF} \rightarrow \text{DFC} & : & Q_1 := \mathbf{w}_1^T \mathbf{u} - u_\ell, & \text{BC} \rightarrow \text{FF} \quad : \quad Q_5 := \mathbf{w}_1^T \mathbf{u} - u_u, \\ \text{DFC} \rightarrow \text{BC} & : & Q_2 := \mathbf{w}_1^T \mathbf{v}, & \text{BC} \rightarrow \text{DFC|SS} \quad : \quad Q_6 := \mathbf{w}_1^T \mathbf{v}, \\ \text{DFC} \rightarrow \text{FC} & : & Q_3 := \psi(\mathbf{w}_1^T \mathbf{v}) - W_R, & \text{SS} \rightarrow \text{DFC} \quad : \quad Q_7 := \mathbf{w}_1^T \mathbf{v} - v_s, \\ \text{FC} \rightarrow \text{BC} & : & Q_4 := \mathbf{w}_1^T \mathbf{v}, & \text{SS} \rightarrow \text{BC} \quad : \quad Q_8 := \mathbf{w}_1^T \mathbf{v} + v_s. \end{array} \quad (4)$$

19 Parameter  $v_s$  is an arbitrarily chosen velocity threshold (small); function  $\psi$  is an  
 20 evaluation of the work done by the viscous component of the bit/rock interaction  
 21 force; the energy barrier is denoted by  $W_R$ . Only the event functions relative  
 22 to transitions from the current mode must be considered for event-detection.  
 23 Further illustration of the possible transitions is given in Figure 2, on the basis  
 24 of the penetration while drilling. Transition to the standstill mode only takes  
 25 place if the penetration achieved over the last drilling cycle,  $\Delta p$  is below the

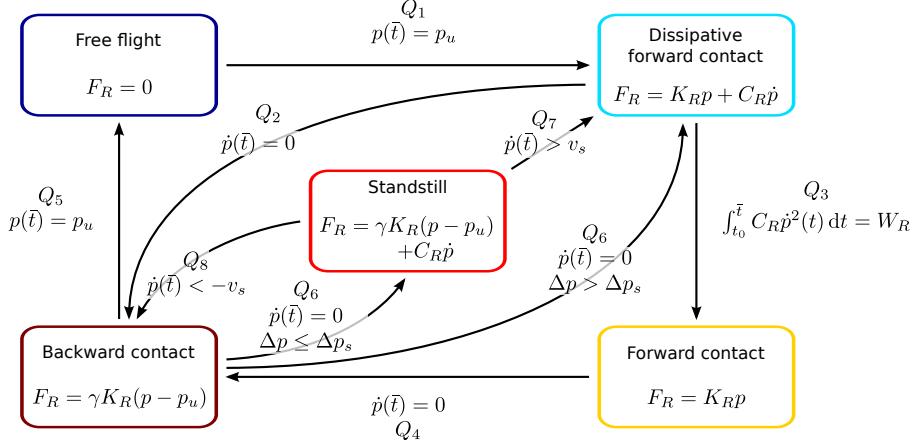


Figure 2: Definition of the bit/rock interaction law, on the basis of the penetration while drilling.

1 arbitrarily small threshold  $\Delta p_s$ .

2 Aside from the event functions related to the bit/rock interaction, two functions must be defined to handle the switches of the pressure law and one to detect 3 the occurrence of contact at the piston/bit interface

$$Q_9 := v_r, \quad Q_{10} := v_r + D_1, \quad Q_{11} := \mathbf{w}_2^T \mathbf{u} + g_0. \quad (5)$$

5 Event functions  $Q_9$  and  $Q_{10}$  are mutually exclusive and, thus, not simultaneously 6 active.

### 7 3 The interplay of slow and fast dynamics

8 The scaling analysis of the model reveals that the motion of the system is ruled 9 by the six timescales

$$\begin{aligned} T_1 &:= \frac{L_p}{c_0}, & T_2 &:= \frac{L_b}{c_0}, & T_3 &:= \sqrt{\frac{M_p D_1}{F_0}}, \\ T_4 &:= \sqrt{\frac{M_b}{K_R}}, & T_5 &:= \frac{W_R}{F_0 D_1} \frac{M_p}{C_R}, & T_6 &:= \sqrt{\frac{M_p}{K_R}}, \end{aligned} \quad (6)$$

10 where the piston and bit lengths and masses are denoted by  $L_p, L_b$  and  $M_p, M_b$ , 11 and the wave propagation speed in the material by  $c_0$ . For a typical drilling 12 system [3], these value (in milliseconds)

$$\begin{aligned} T_1 &= \mathcal{O}(10^{-2}), & T_2 &= \mathcal{O}(10^{-2}), & T_3 &= \mathcal{O}(10^1), \\ T_4 &= \mathcal{O}(10^{-1}), & T_5 &= \mathcal{O}(10^{-3}), & T_6 &= \mathcal{O}(10^{-3}), \end{aligned} \quad (7)$$

1 thus spanning about four orders of magnitude. While separation may take place  
2 between the extreme timescales, a coupling is, nevertheless, expected through the  
3 intermediate ones. Accordingly, the average motion of the system on the slow  
4 timescale, or rigid body motion, is expected to be driven by faster timescale pro-  
5 cesses such as the wave propagation in the mechanical system that, for instance,  
6 rules the transfer of momentum between the piston and the bit during the per-  
7 cussive activation. The average rate of penetration—the principal indicator of  
8 performance in drilling—must therefore be computed from full scale simulations.

9 Figure 3 displays the system stationary response for the reference configura-  
10 tion given in [3]; the rock surface displacement, as well as the average displace-  
11 ments of the piston and bit are given in plots a, the average velocities of the two  
12 bodies in plots b and the status of the bit/rock interaction law and of the contact  
13 at the piston/bit interface. The origin of time is set at the initiation of impact  
14 at the piston/bit interface. Right-column plots focus on the post-activation be-  
15 havior. Over the duration of a pressure cycle ( $T_3 \simeq 45$  ms), the slowest timescale  
16 of the model, several faster phenomena can be observed.

17 At the piston/bit contact interface, two successive collisions take place; see  
18 right-column plot c. According to the wave propagation theory, their duration  
19 ( $2T_1 \simeq 0.065$  ms) is controlled by the geometry of the system while the post-  
20 contact average velocities also depend on the initial velocities of the colliding  
21 bodies. The first collision corresponds to the percussive activation. During the  
22 impact, the piston acts as the driver and transfers linear momentum to the bit.  
23 The second collision results from the rebound of the bit after its interaction with  
24 the rock medium; the bit acts as the driver and returns momentum to the piston.  
25 Such a double impact sequence is a desired behavior as the energy returned by  
26 the rock contributes to increasing the piston impact frequency and to the overall  
27 performance of drilling.

28 Parallel to the interactions at the piston/bit interface, interactions at the  
29 bit/rock interface also evolve on a faster timescale than that of average motion.  
30 Two principal phases of motion can be observed. The first consists of a succession  
31 of DFC  $\rightarrow$  FC  $\rightarrow$  BC  $\rightarrow$  FF sequences during the post-activation period; the  
32 contact between the bit and the rock is repeatedly interrupted. The second phase  
33 follows with an alternation of DFC and BC cycles. Each drilling cycle consumes  
34 part of the bit energy, which conducts to the disappearance of free flight phases  
35 and the convergence of the bit to standstill; that is, a near equilibrium position.  
36 Drilling cycles have a longer duration than contact phases ( $T_4 \simeq 0.4$  ms).

37 These results clearly illustrate the interdependence between the long-term  
38 reponse of the bit—representative of drilling performance—and the faster pro-  
39 cesses that take place at the contact interfaces. For these fast processes drive the  
40 entire model response, their account in a full scale simulation is thus mandatory

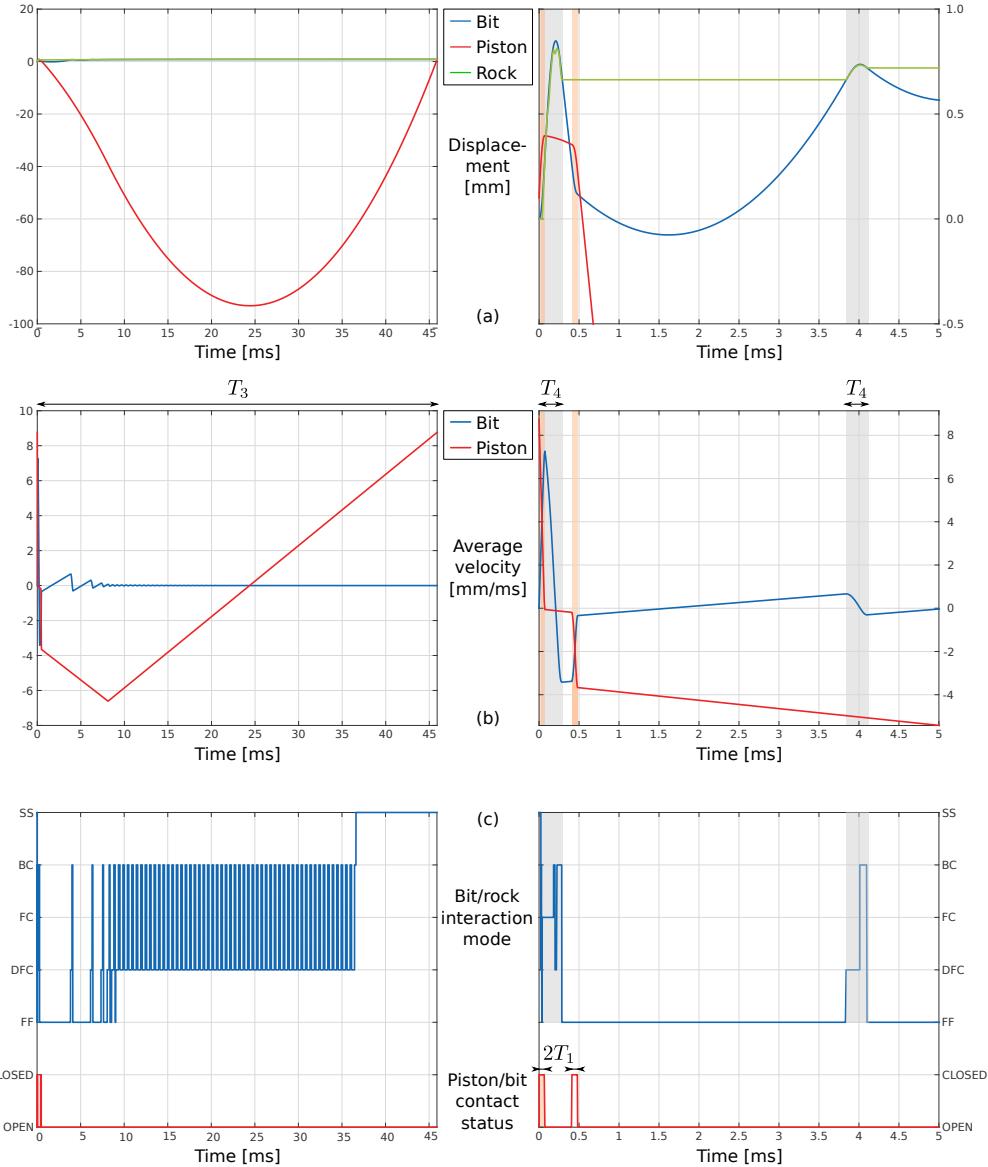


Figure 3: Stationary response of the system, for a reference configuration corresponding to a low-sized hammer; dissipative integration is used. See [3] for complete simulation details.

1 for a proper assessment of the system response. A model that accounts for these  
2 multiple scale phenomena is thus required for an adequate assessment of the  
3 rate of penetration in percussive drilling, on the basis of the model dynamical  
4 response rather than single activation considerations as is usually done in the  
5 literature.

## 6 **4 Summary**

7 This paper briefly introduces an integrated model of down-the-hole percussive  
8 drilling proposed by the authors [3] and analyzes its stationary response. The  
9 multiscale aspects of the model and their influence on the stationary response  
10 are considered. In particular, the analysis highlights the importance these fast  
11 processes play on the average response of the model, notably through the inter-  
12 actions taking place between the piston and the bit (percussive activation) and  
13 at the bit/rock interface (drilling). Their account in a model of the percussive  
14 drilling process is therefore of critical importance to assess its performance.

## 15 **References**

- 16 [1] Hustrulid, W.A. and Fairhurst, C.E. Theoretical and Experimental Study of  
17 Percussive Drilling of Rock - Part I - Theory of Percussive Drilling, *Int. J.*  
18 *Rock. Mech. Min.*, 8(4):311-333, 1971.
- 19 [2] Chiang, L.E. and Elías, D.A. A 3D FEM methodology for simulating the im-  
20 pact in rock-drilling hammers. *Int. J. Rock Mech. Min.*, 45(5):701711, 2008.
- 21 [3] Depouhon, A., Denoël, V. and Detournay, E. An integrated model of down-  
22 the-hole percussive drilling. *Int. J. Numer. Anal. Met.* In preparation.