

Cycles found in air temperature data and proxy series using the Morlet wavelet method

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The Continuous Wavelet transform

The wavelet analysis provides a two-dimensional unfolding of a one-dimensional signal by decomposing it into scale–time coefficients.

The continuous wavelet transform turns a signal s into a function W

$$W[s](t, a) = \int s(x) \bar{\psi}\left(\frac{x-t}{a}\right) \frac{dx}{a},$$

where $\bar{\psi}$ denotes the complex conjugate of the function ψ , a the scale and t the time.

Conditions on ψ

The function ψ must be integrable, square integrable and satisfy some admissibility condition. Such a function is called a wavelet. The Morlet wavelet is particularly well conditioned for frequency-based study. It satisfies the following equality

$$\hat{\psi}(\omega) = \exp\left(-\frac{(\omega - \Omega)^2}{2}\right) - \exp\left(-\frac{\omega^2 + \Omega^2}{2}\right),$$

where $\Omega > 5$ is called the central frequency.

Some Properties of the Wavelet Transform

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$$W[c(u + v)] = cW[s] + cW[v],$$
- the wavelet transform allows to handle noisy data,
- the wavelet transform is blind to polynomial behaviors (up to a degree n , depending on ψ): $W[s + P] = W[s]$, where P is a polynomial of degree $\leq n$.

Consequently, non-zero mean and linear tendencies do not affect the wavelet transform.

The Scale Spectrum

For wavelets such as the Morlet wavelet, we have

$$W[\cos(\omega_0 t)](t, a) = \exp(i\omega_0 t) \hat{\psi}(a\omega_0),$$

so that the frequency ω_0 is given by the maximum of $\hat{\psi}(a\omega_0)$:
 $a_\omega = \Omega/\omega_0$. Consequently, the unknown frequency ω_0 can be obtained through the maximum of $|W[\cos(\omega_0 t)]|$.

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Definition

The scale spectrum of a signal s is defined by

$$\Lambda(a) = E|W[s](t, a)|,$$

where E denotes the mean over the time t .

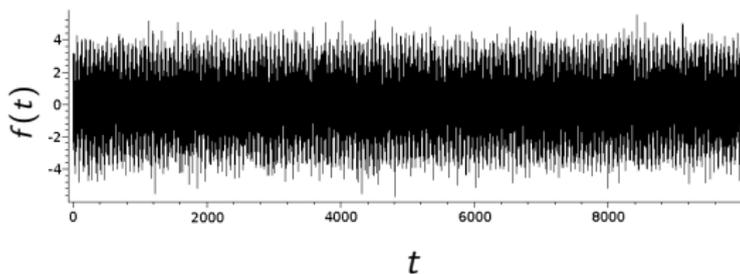
For What Purpose?

The scale spectrum has a very different meaning from the Fourier spectrum. When looking at a Fourier spectrum, the existence of energy at a given frequency means that a component of a sine wave persisted through the whole time span of the signal.

The presence of energy at the same frequency in the scale spectrum only means that, in the whole time span of the signal, there is a higher likelihood for such a wave to have appeared locally.

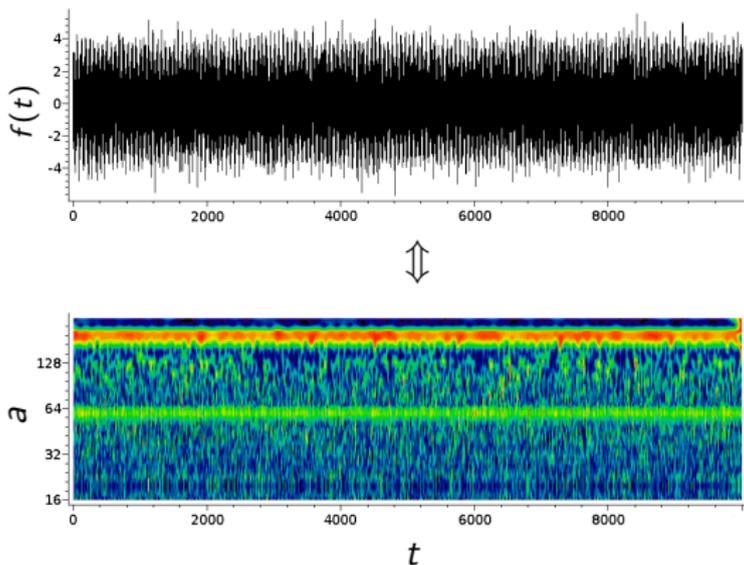
Example

A Visual Example



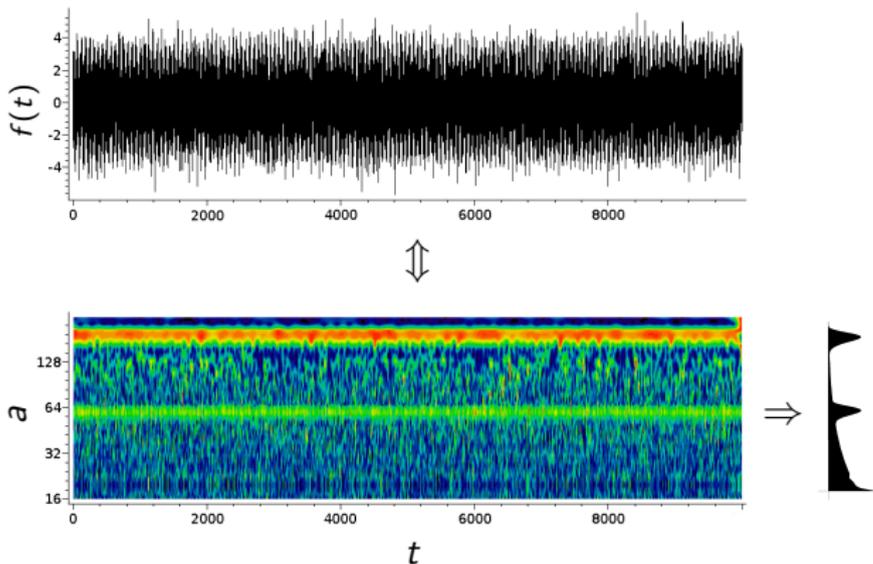
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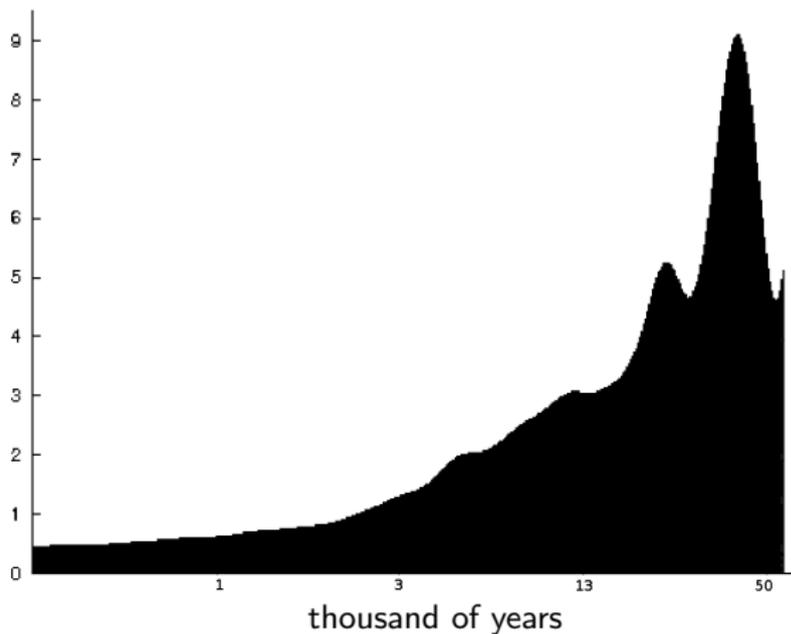
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Example

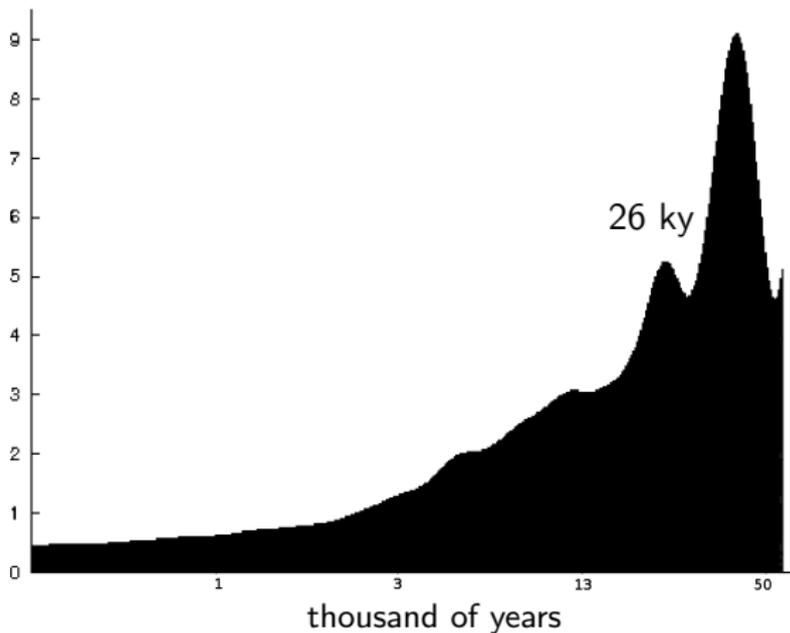
Known long cycles are observed in the spectrum



EPICA dome

Example

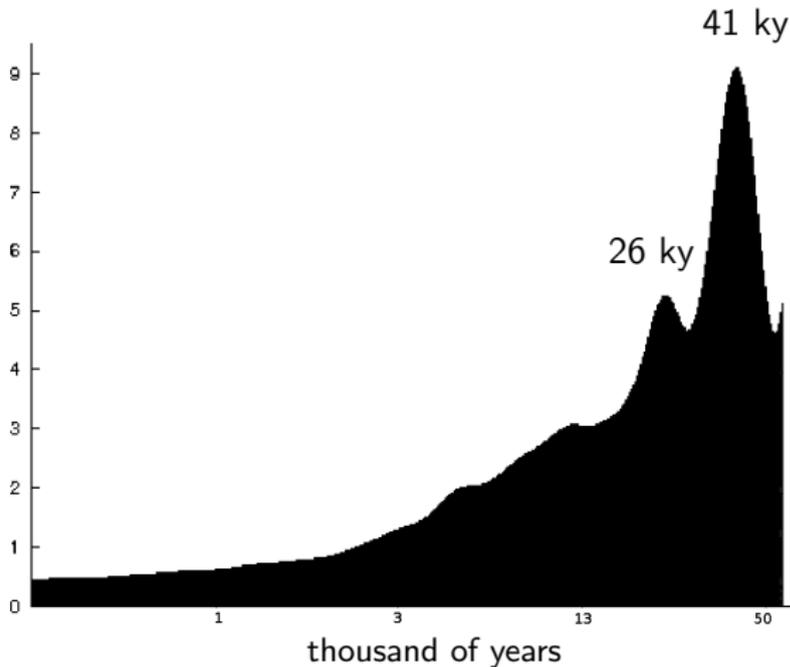
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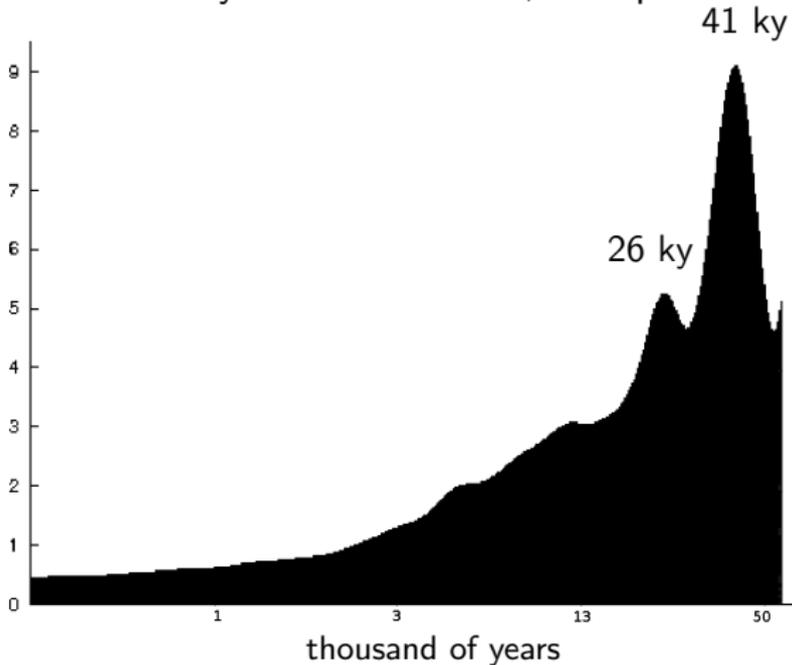
Known long cycles are observed in the spectrum



EPICA dome

Known long cycles are observed in the spectrum

Milankovitch cycles are detected, as expected



EPICA dome

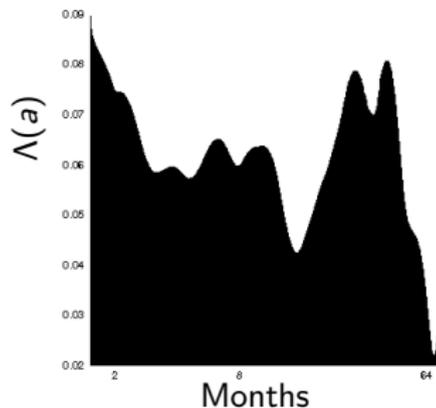
A weather station

The time series we are interested in...



Global temperature data

What About the Global Temperature Data?

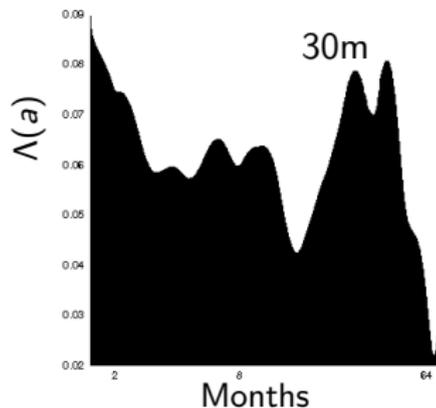


HadCRUT3, NH

1950–2007 monthly-sampled data

Global temperature data

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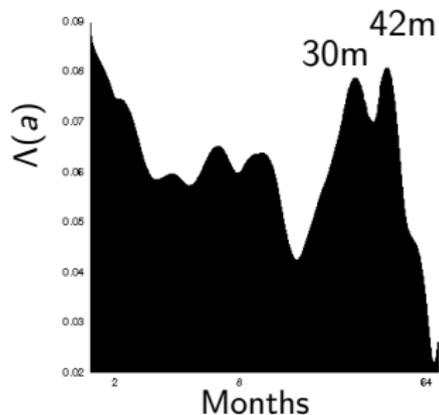


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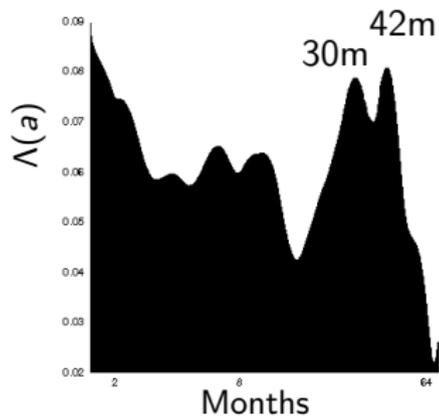


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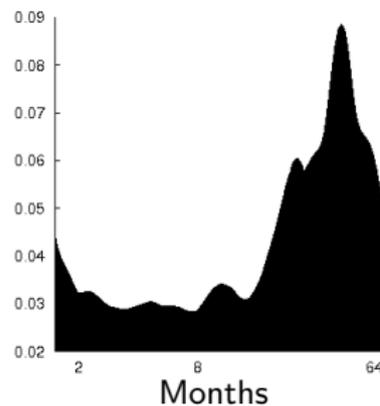
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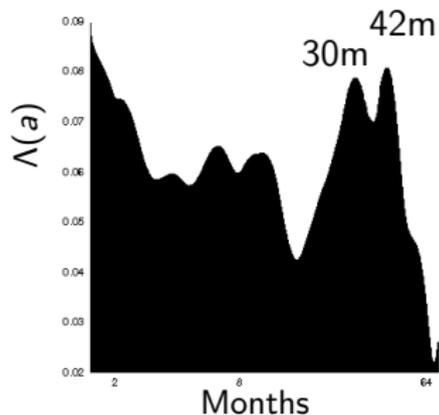


HadCRUT3, SH

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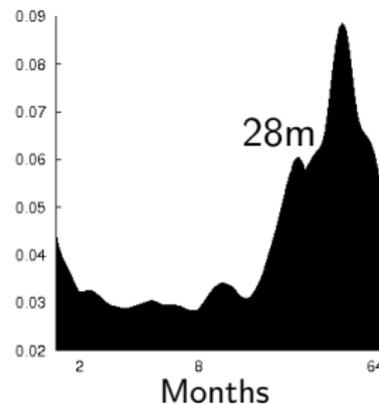
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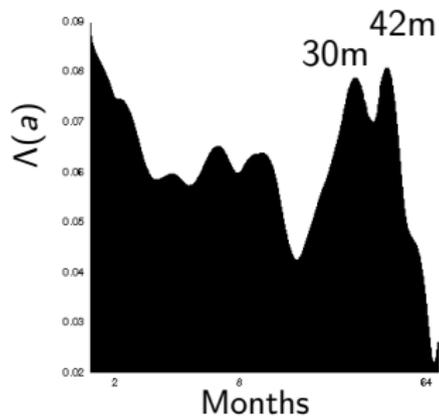


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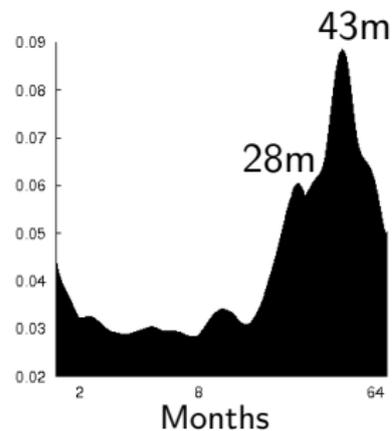
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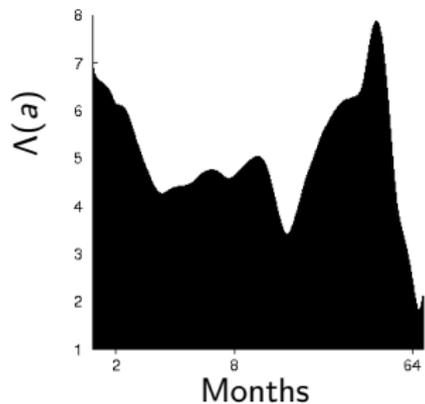


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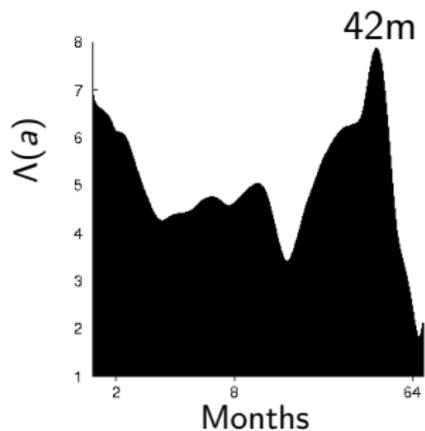


HadSST2

1950–2007 monthly-sampled data

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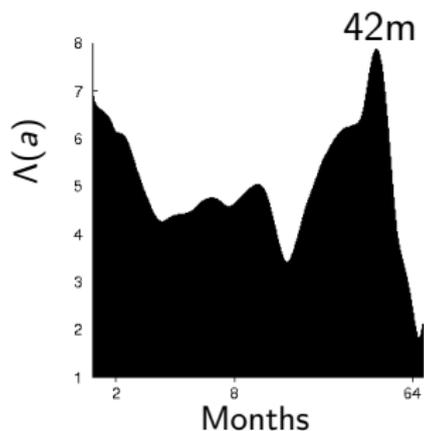


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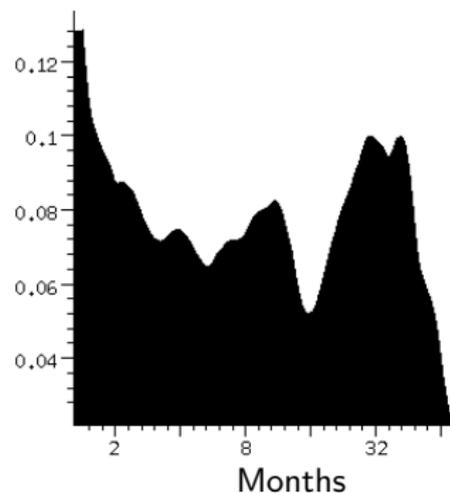
Global temperature data

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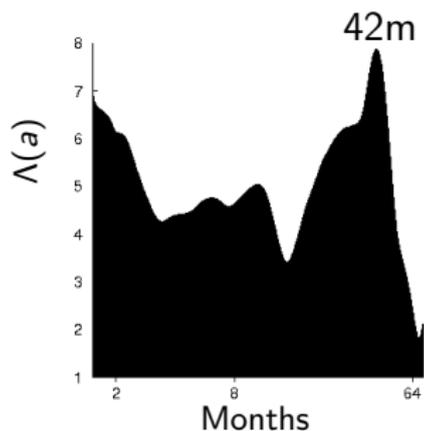


CRUTEM3gl

1950–2007 monthly-sampled data

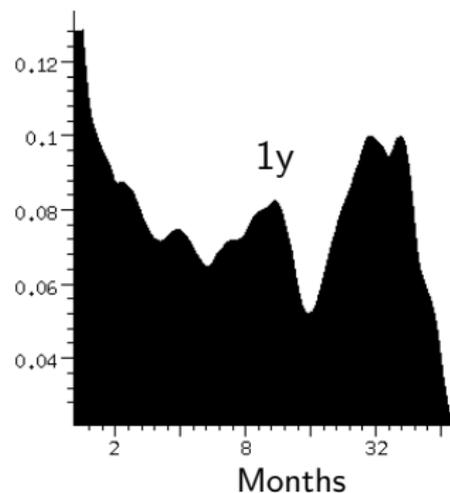
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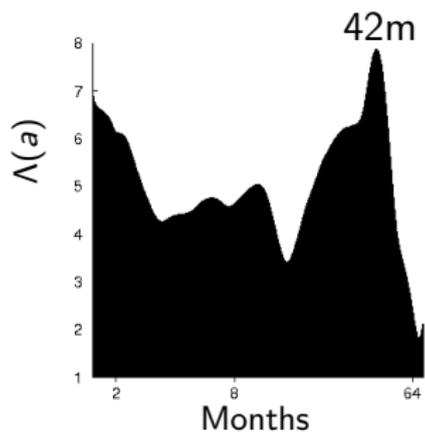


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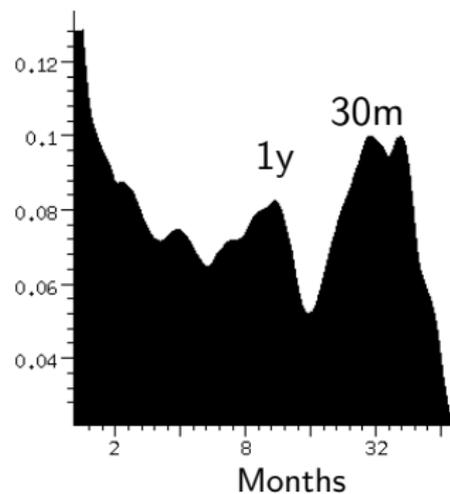
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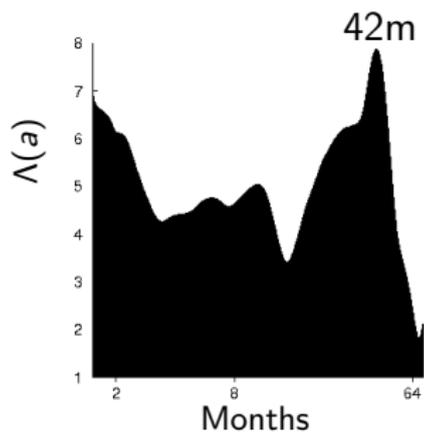


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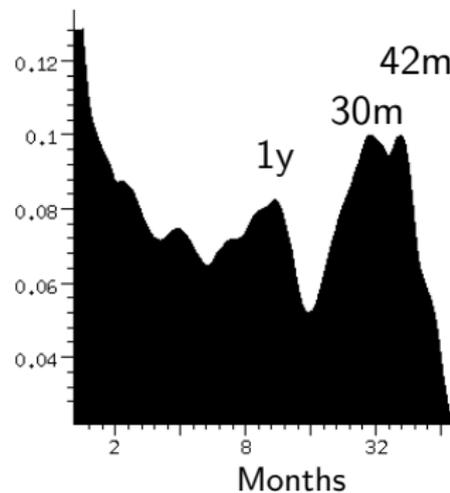
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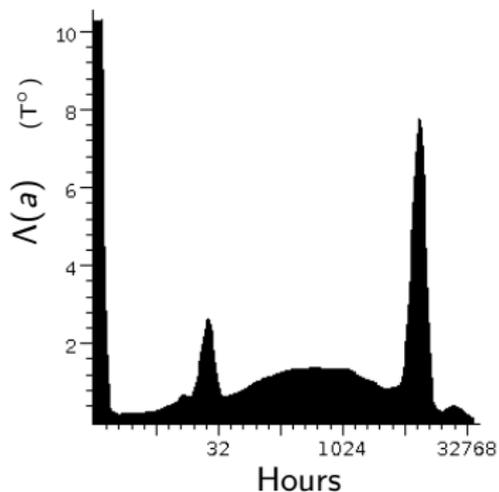


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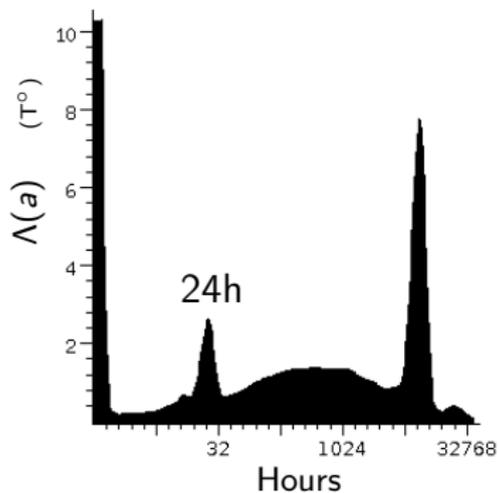
What About the Local Temperature Data?



Bierset-aero, Belgium
1950–2007 hourly-sampled data

Global temperature data

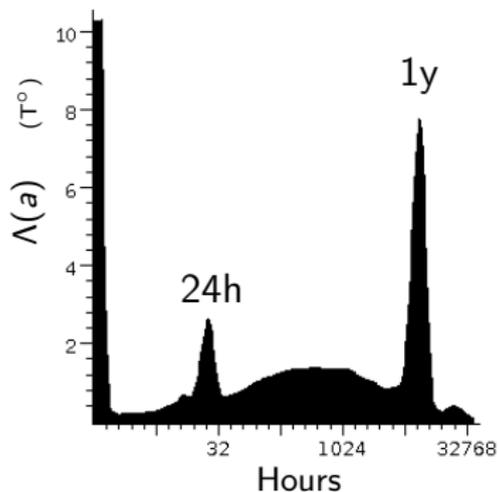
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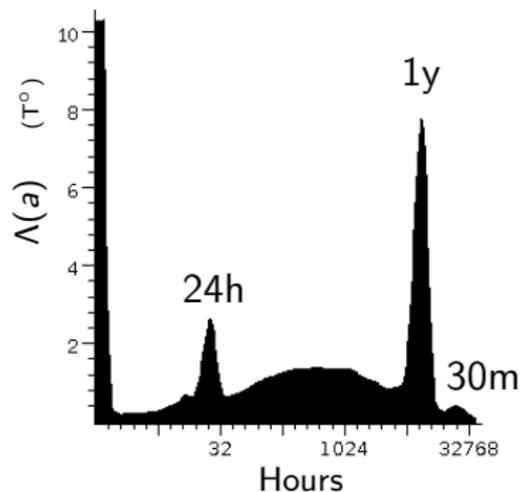
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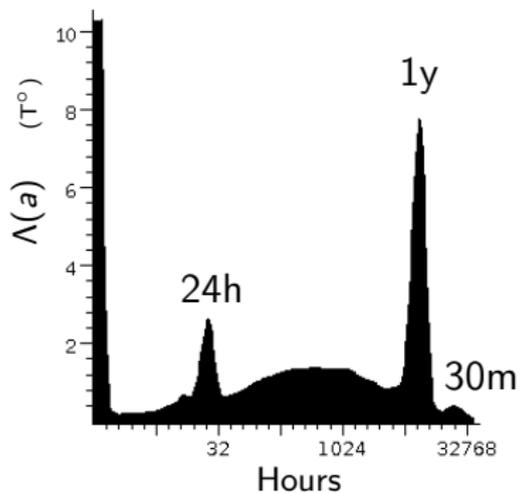
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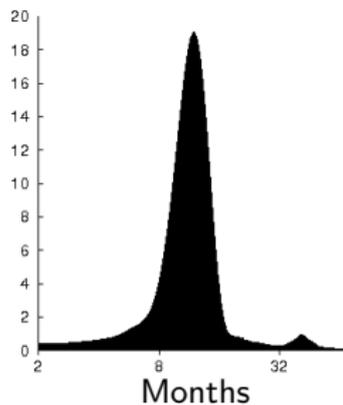
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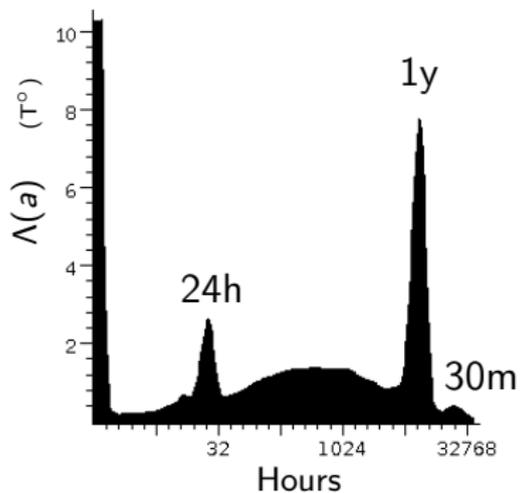
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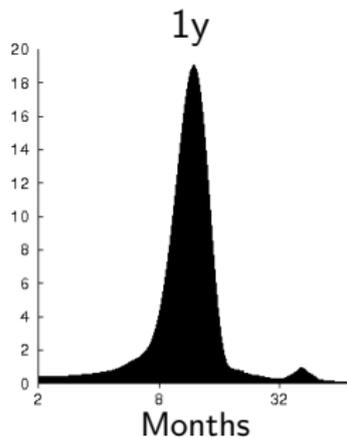
The Pas, Canada
1950–2007 monthly-sampled data

Global temperature data

What About the Local Temperature Data?



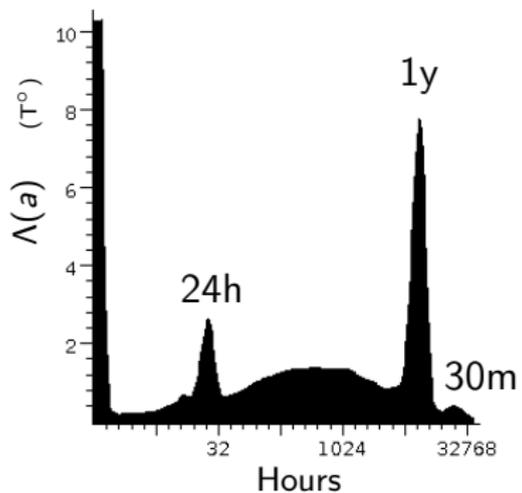
Bierset-aero, Belgium
1950–2007 hourly-sampled data



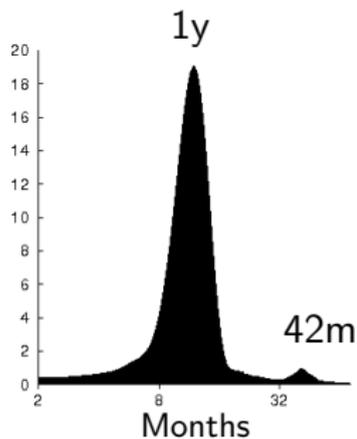
The Pas, Canada
1950–2007 monthly-sampled data

Global temperature data

What About the Local Temperature Data?



Bierset-aero, Belgium
1950–2007 hourly-sampled data



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A Random Process to Compare the Data With

To check if the observed cycles did not just occur by pure chance, we can compare the scale spectra of the data to spectra obtained from a random process “looking like” the data.



A Random Process to Compare the Data With

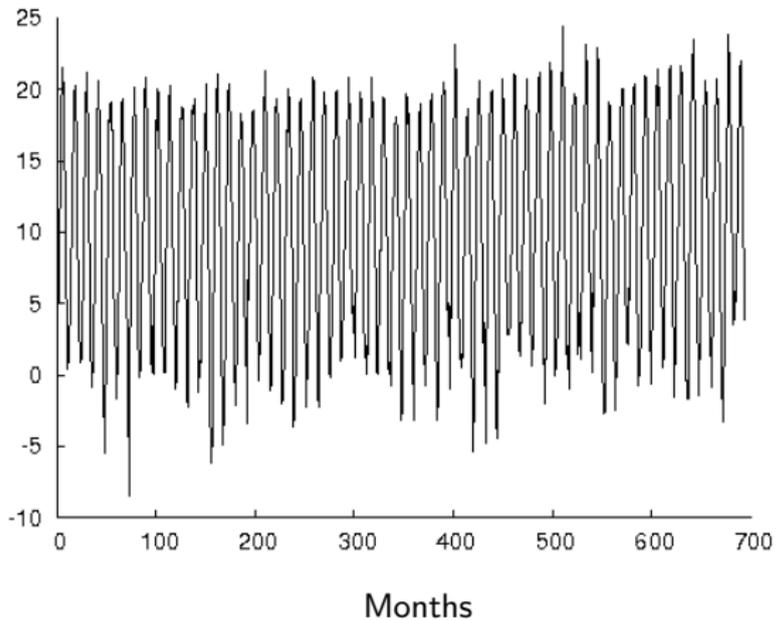
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The “traditional” model seems to be the AR(1):

$$X_{n+1} = \alpha X_n + \sigma \eta_{n+1},$$

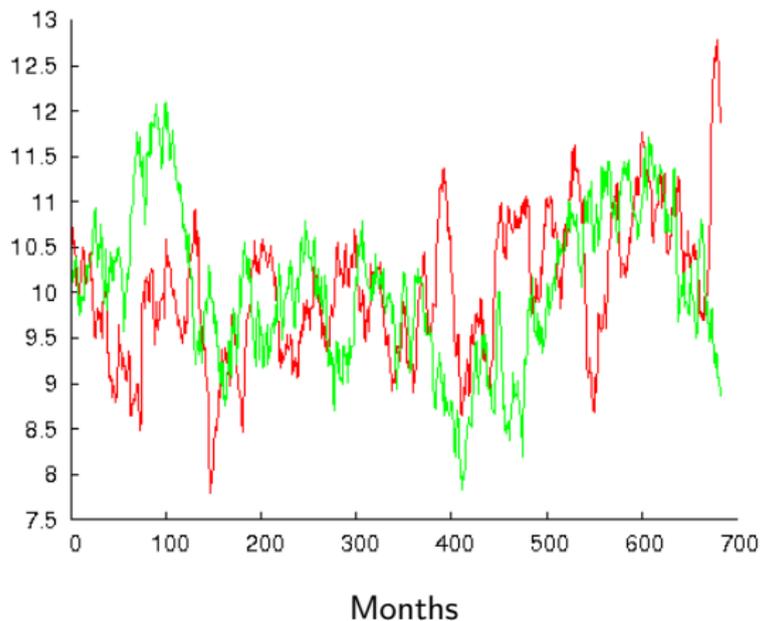
where , as usual, η_n is i.i.d., with zero mean and unit variance (e.g. Gaussian white noise $N(0,1)$).

A Visual Illustration



Wien, Austria

A Visual Illustration



Wien, Austria / A simulation

A Basic Idea

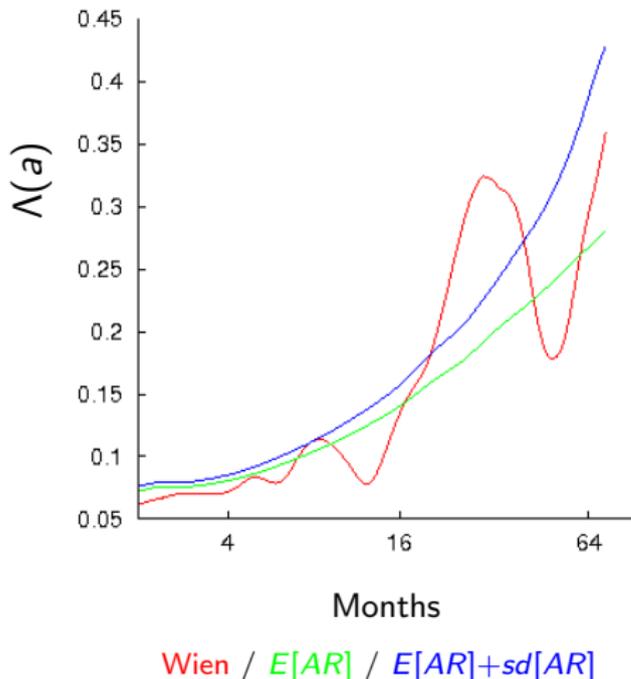
The significance of a cycle corresponding to a scale a can be roughly tested by computing the highest number for which

$$\Lambda(a) - (\bar{\Lambda}_N(a) + k\sigma_N(a)) > 0$$

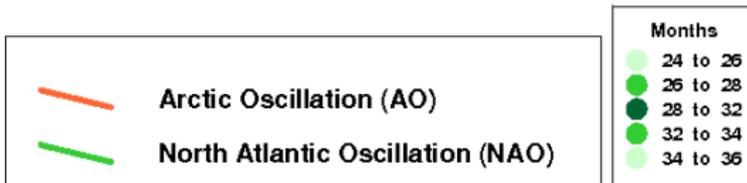
where Λ is the scale spectrum of the data, $\bar{\Lambda}_N$ is the mean scale spectrum of N realizations of the random process and σ_N is the associated standard deviation.

A Model for the Data

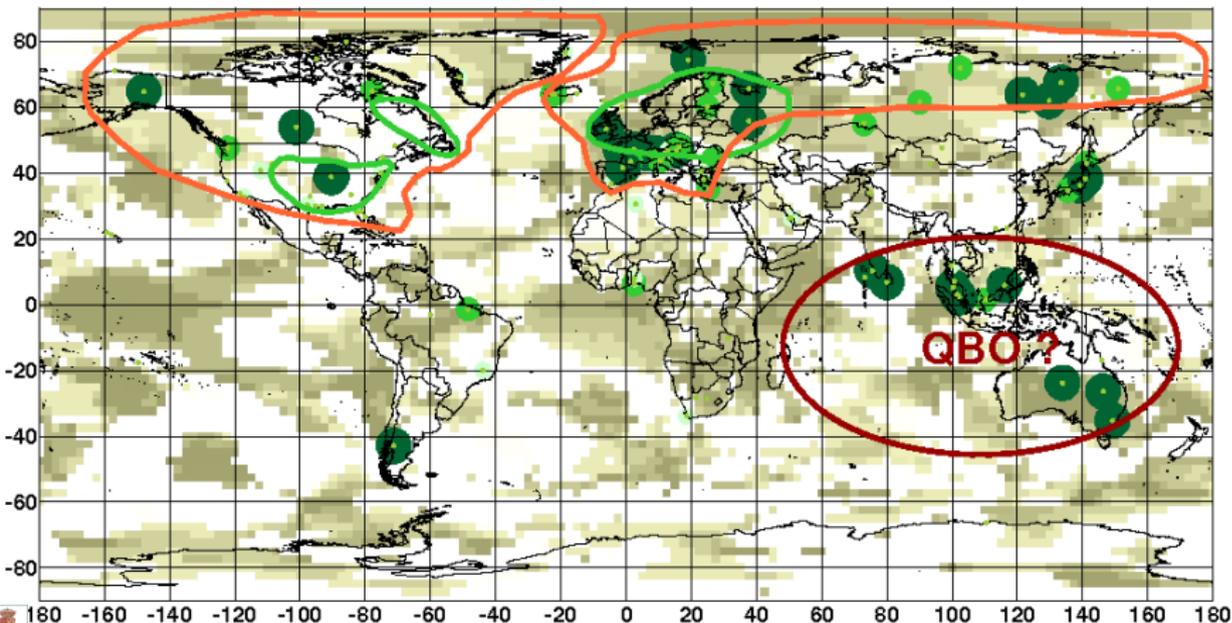
The scale spectra of the Wien Temperature Records vs. the Mean Spectra of the Model



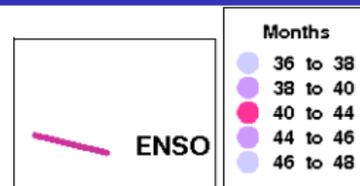
30 Months Cycle



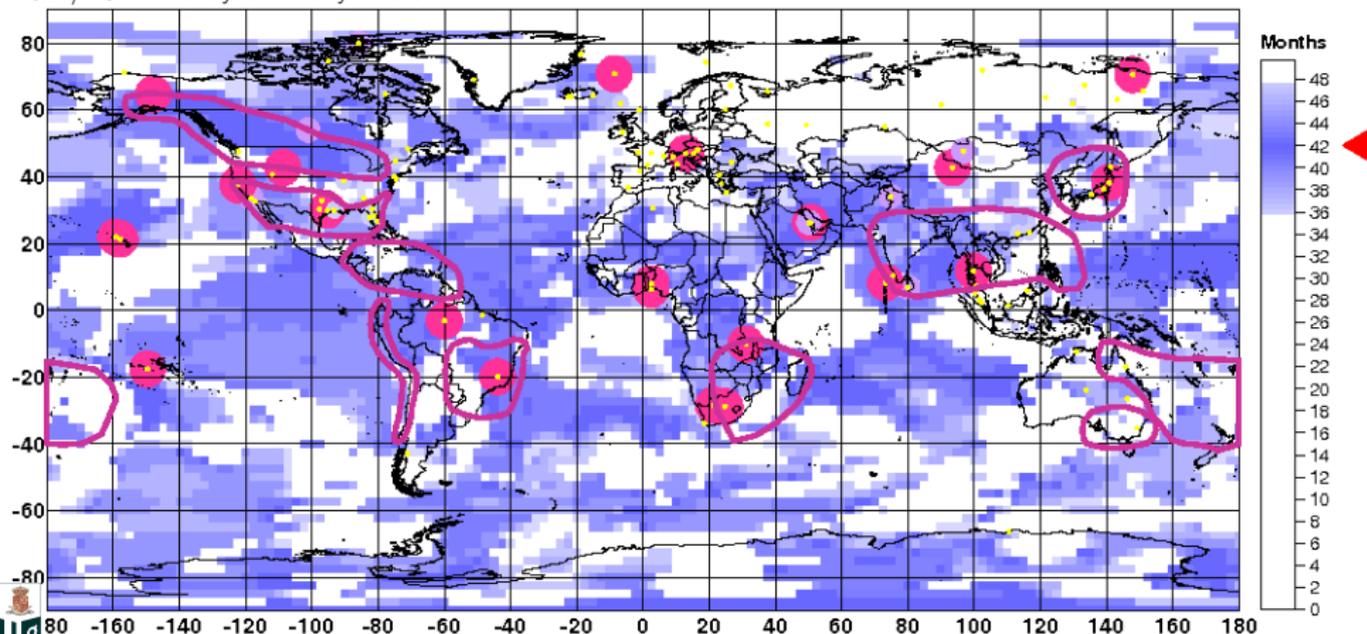
NCEP/NCAR Reanalysis Monthly Means



42 Months Cycle

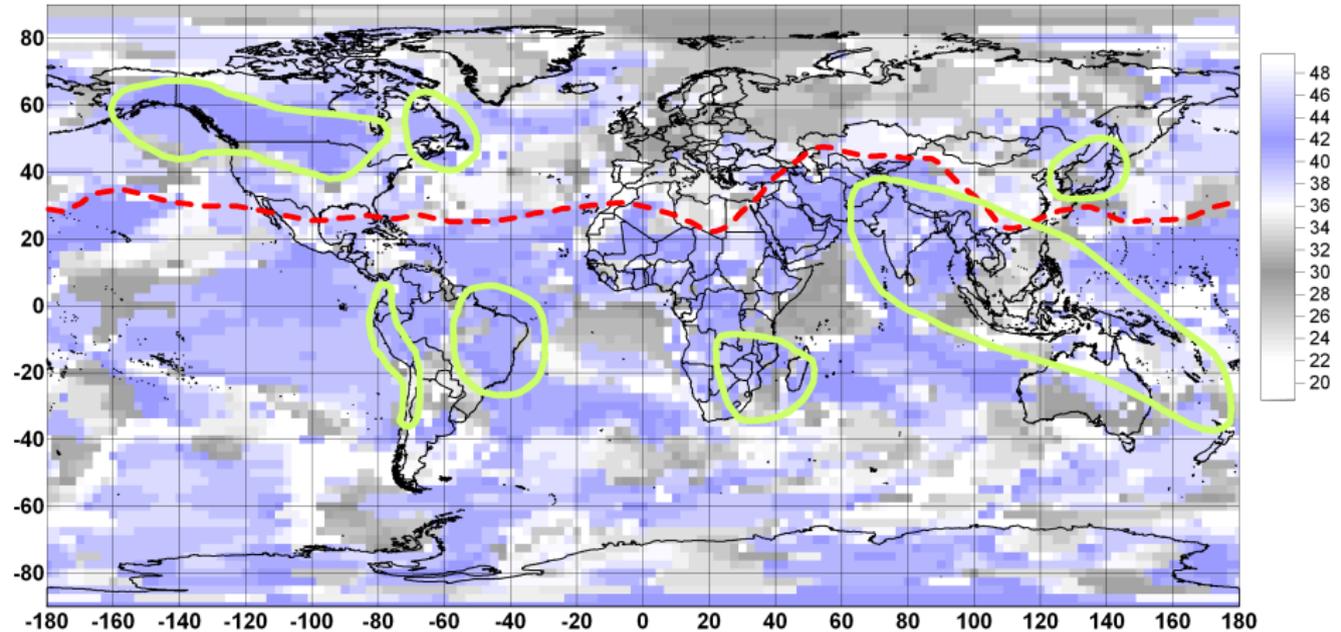


NCEP/NCAR Reanalysis Monthly Means

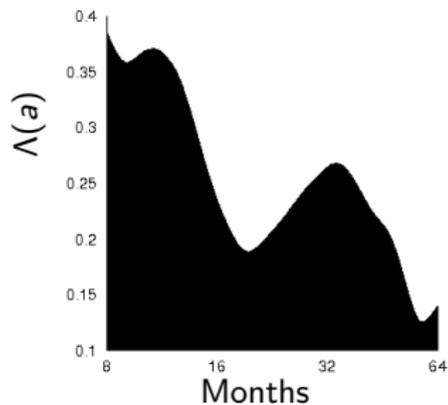


The Dominating cycle

NCEP/NCAR Reanalysis Monthly Means



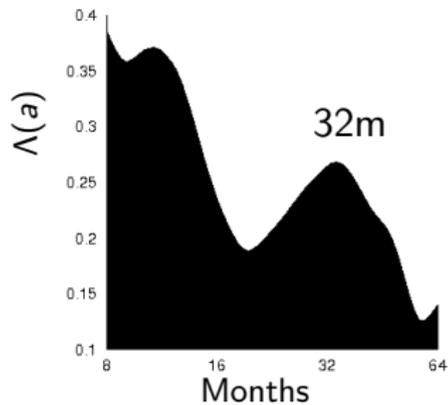
Scale Spectrum of Some Climatic Indices



NAO (CPC)

1950–2007 monthly-sampled data

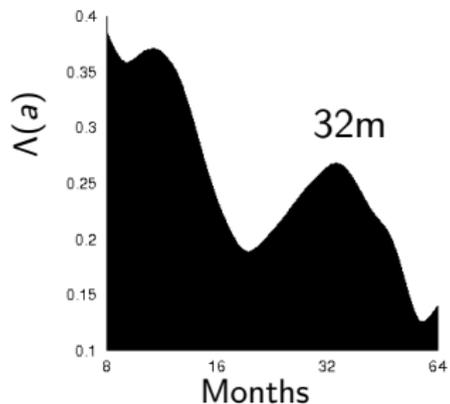
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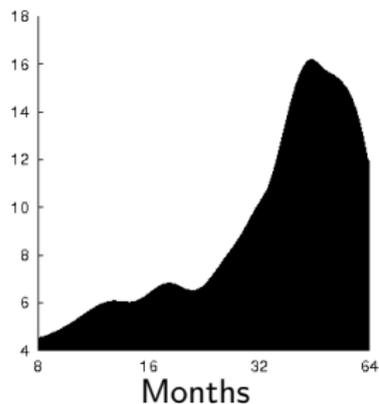
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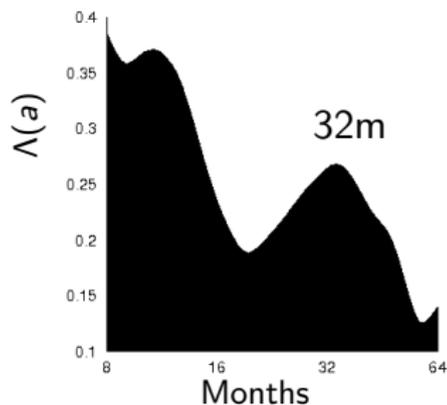
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ENSO (SST)

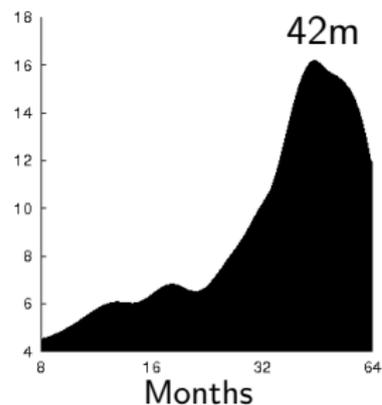
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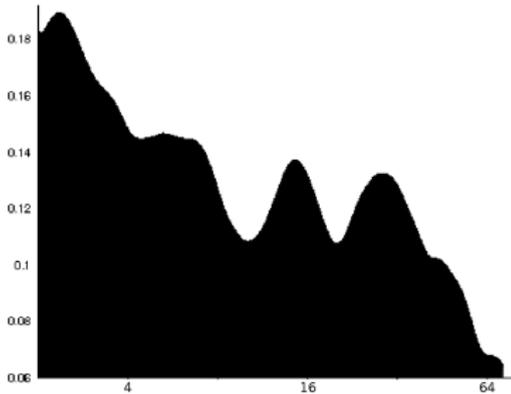
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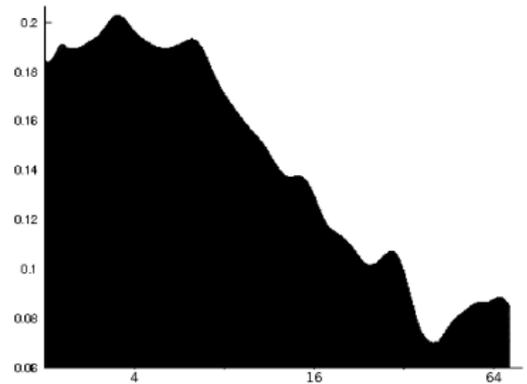
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Millennial Temperature Reconstructions of Jones et al.



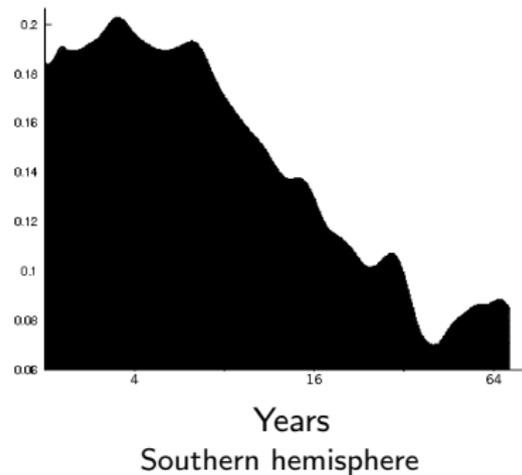
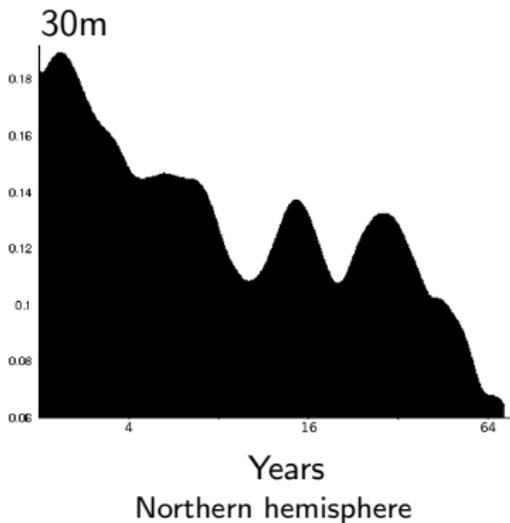
Years
Northern hemisphere



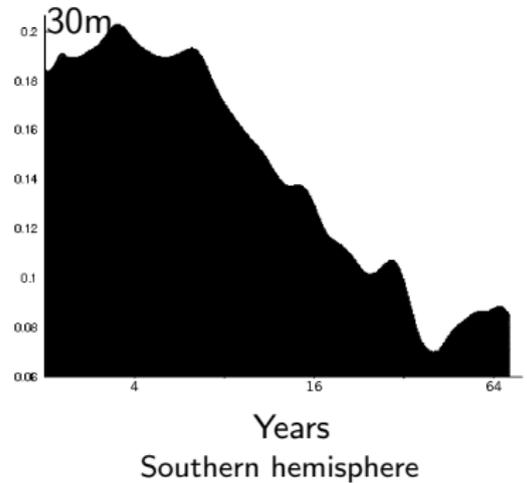
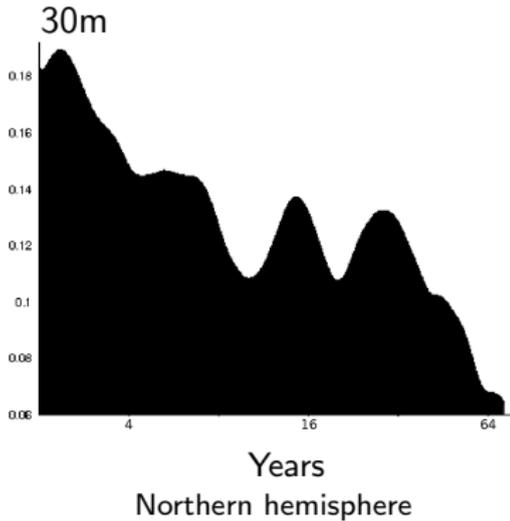
Years
Southern hemisphere



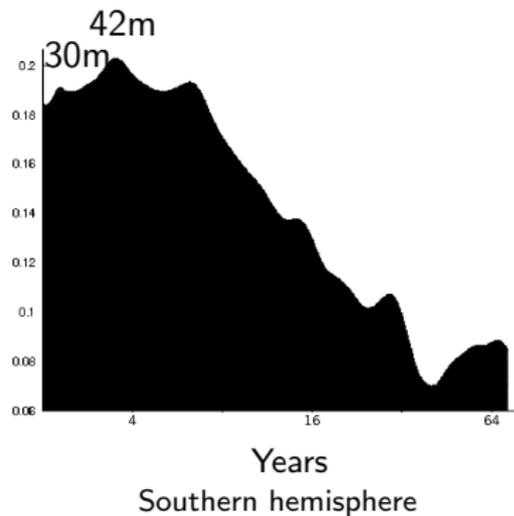
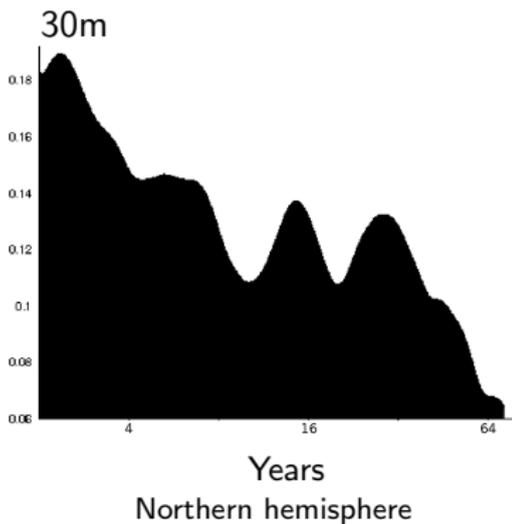
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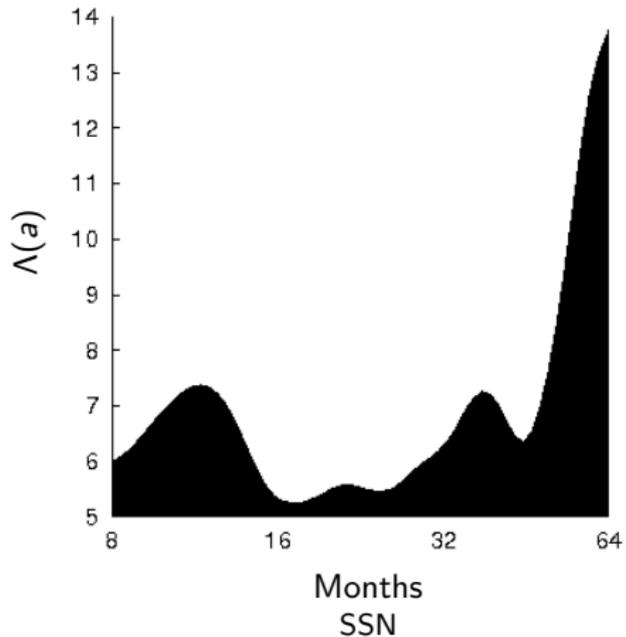
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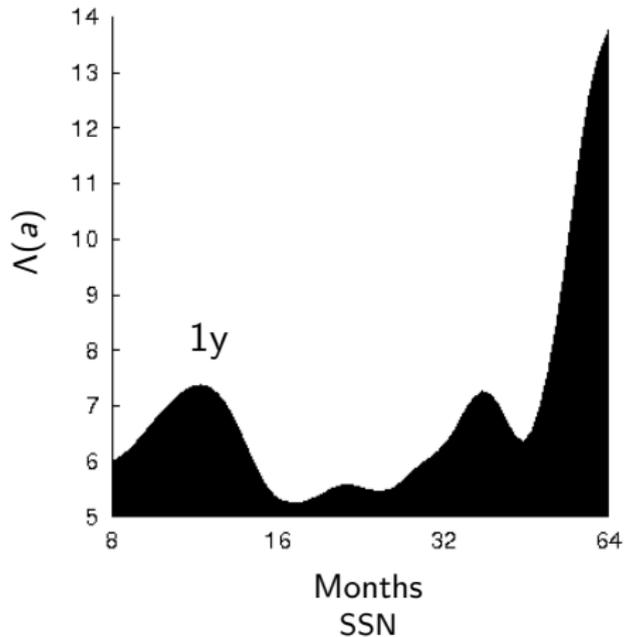


Scale spectrum of the Sunspot Number



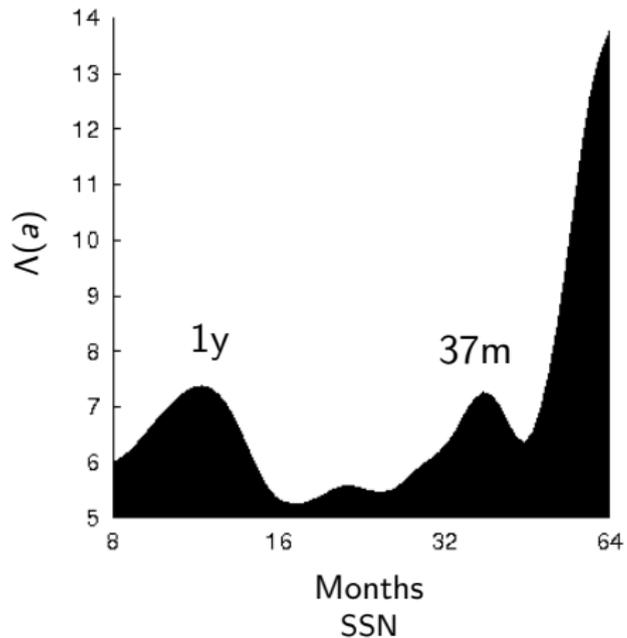
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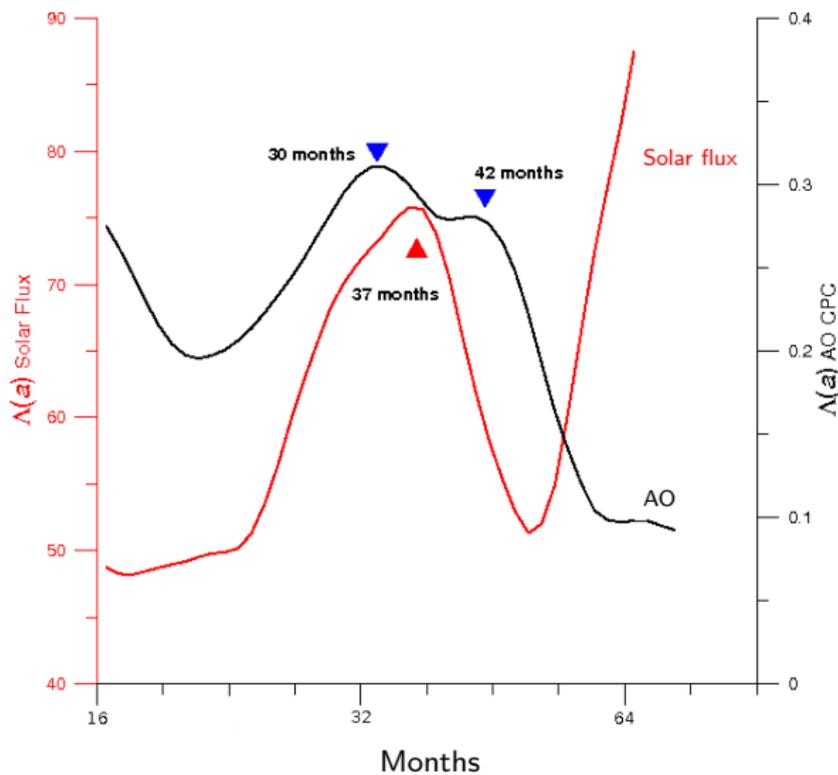
1950–2007 monthly-sampled data

Scale spectrum of the Sunspot Number



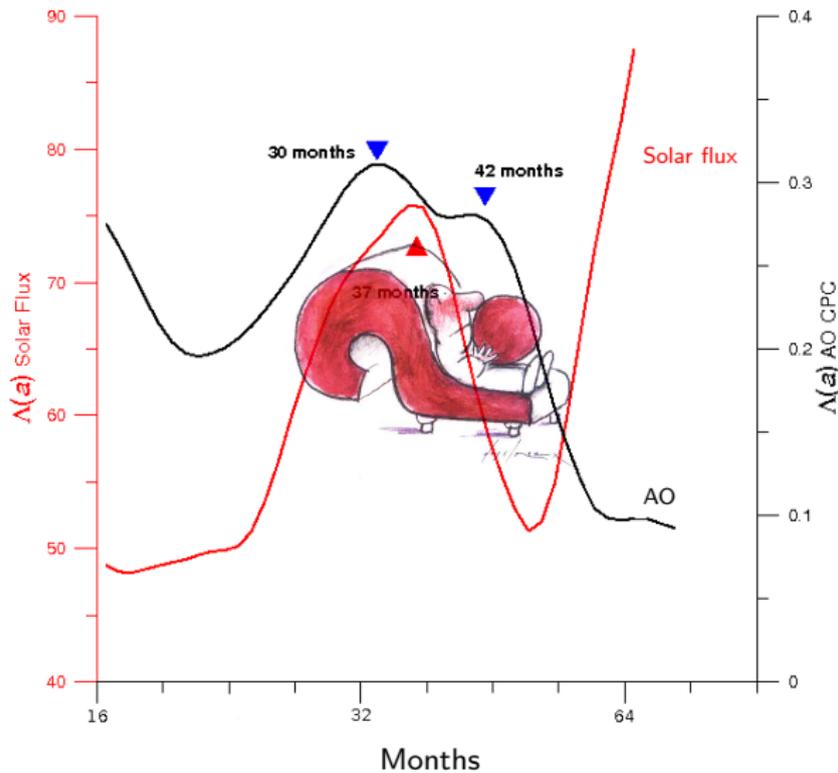
1950–2007 monthly-sampled data

A Relation with the Sun?



What About the Sun?

A Relation with the Sun?



Some values of k

Data time series	cycle	k
ENSO	43	3.4
AO	32	6.6
NAO	33	4.7
NH10	28	3.3
SH7	43	1.3

For Further Reading



S. Nicolay, G. Mabilie, X. Fettweis and M. Erpicum,
30 and 43 months period cycles found in air temperature time series using the
Morlet wavelet method,
Climate Dynamics, doi: [10.1007/s00382-008-0484-5](https://doi.org/10.1007/s00382-008-0484-5) (2008)



G. Mabilie and S. Nicolay,
New cycles found in air temperature data and proxy series,
submitted



http://reflexions.ulg.ac.be/cms/c_5030/mathematiques