New cycles found in air temperature data and proxy series using the Morlet wavelet method

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The Continuous Wavelet transform

The wavelet analysis provides a two-dimensional unfolding of a one-dimensional signal by decomposing it into scale–time coefficients.
The continuous wavelet transform turns a signal s into a function W

$$W[s](t,a) = \int s(x)\bar{\psi}(\frac{x-t}{a})\,\frac{dx}{a},$$

where $\bar{\psi}$ denotes the complex conjugate of the function ψ, a the scale and t the time.

Conditions on ψ

The function ψ must be integrable, square integrable and satisfy some admissibility condition. Such a function is called a wavelet. The Morlet wavelet is particularly well conditioned for frequency-based study. It satisfies the following equality

$$\hat{\psi}(\omega) = \exp(-\frac{(\omega - \Omega)^2}{2}) - \exp(-\frac{\omega^2 + \Omega^2}{2}),$$

where $\Omega > 5$ is called the central frequency.

Some Properties of the Wavelet Transform

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- the wavelet transform allows to handle noisy data,
- the wavelet transform is blind to polynomial behaviors (up to a degree n, depending on ψ): $W[s+P] = W[s]$, where P is a polynomial of degree $\leq n$.

Consequently, non-zero mean and linear tendencies do not affect the wavelet transform.

The Scale Spectrum

For wavelets such as the Morlet wavelet, we have

$$W[\cos(\omega_0 t)](t, a) = \exp(i\omega_0 t)\hat{\bar{\psi}}(a\omega_0),$$

so that the frequency ω_0 is given by the maximum of $\hat{\bar{\psi}}(a\omega_0)$: $a_\omega = \Omega/\omega_0$. Consequently, the unknown frequency ω_0 can be obtained through the maximum of $|W[\cos(\omega_0 t)]|$.

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Definition

The scale spectrum of a signal s is defined by

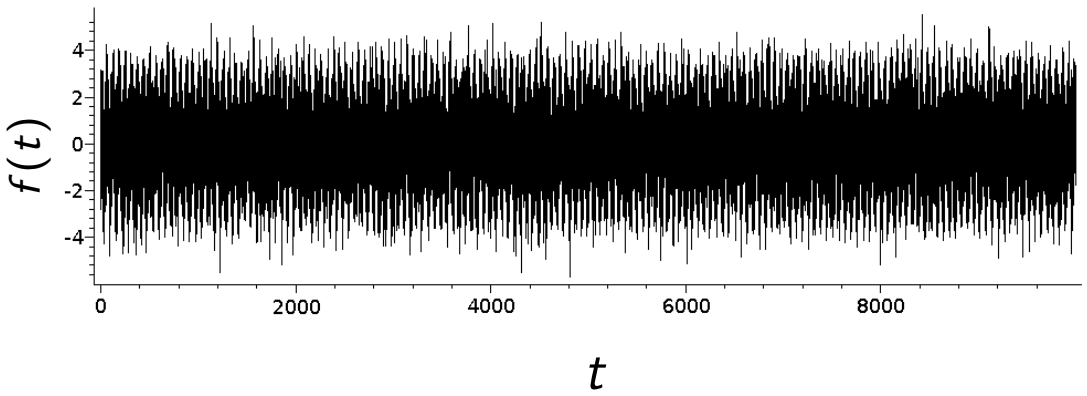
$$\Lambda(a) = E|W[s](t, a)|,$$

where E denotes the mean over the time t.

For What Purpose?

The scale spectrum should be useful for signals which are not stationary but whose characteristics do not evolve too quickly: the scale spectrum allows to recover frequencies even if they "kindly depend on the time", i.e. if $\omega_0 = \omega_0(t)$ with $\frac{d}{dt}\omega_0 \ll 1$, one should be able to recover $E\omega_0$.

A Visual Example



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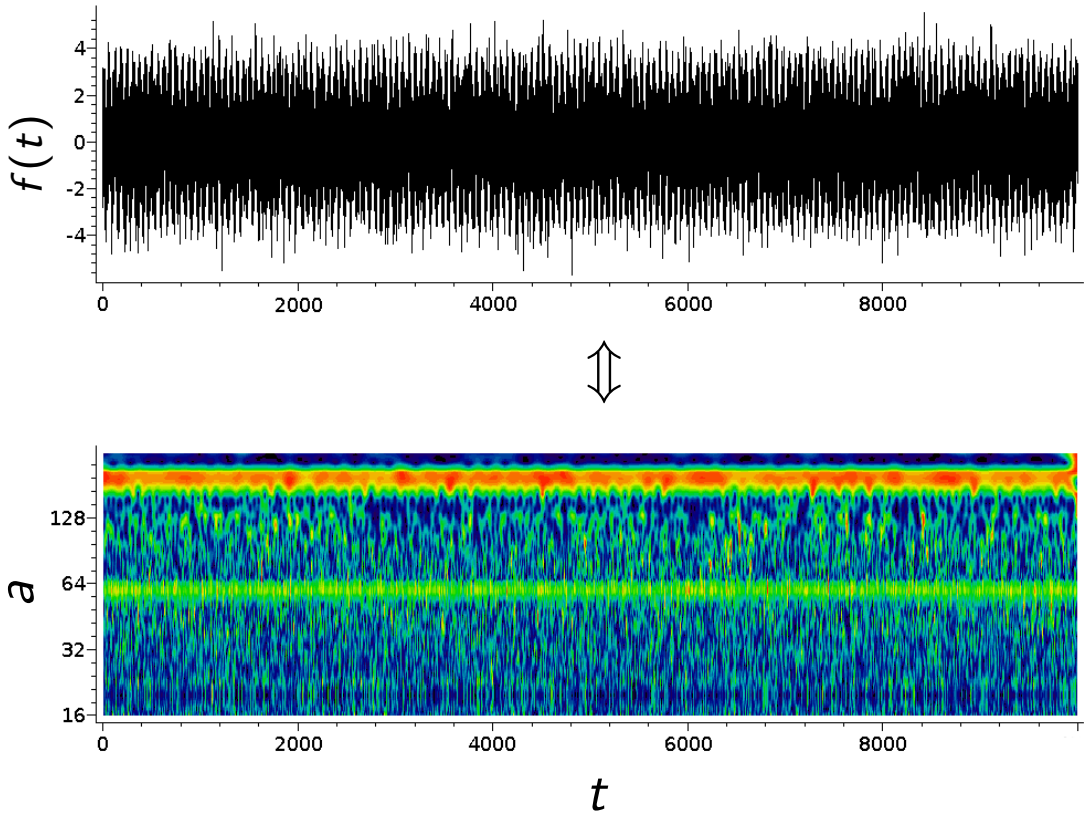

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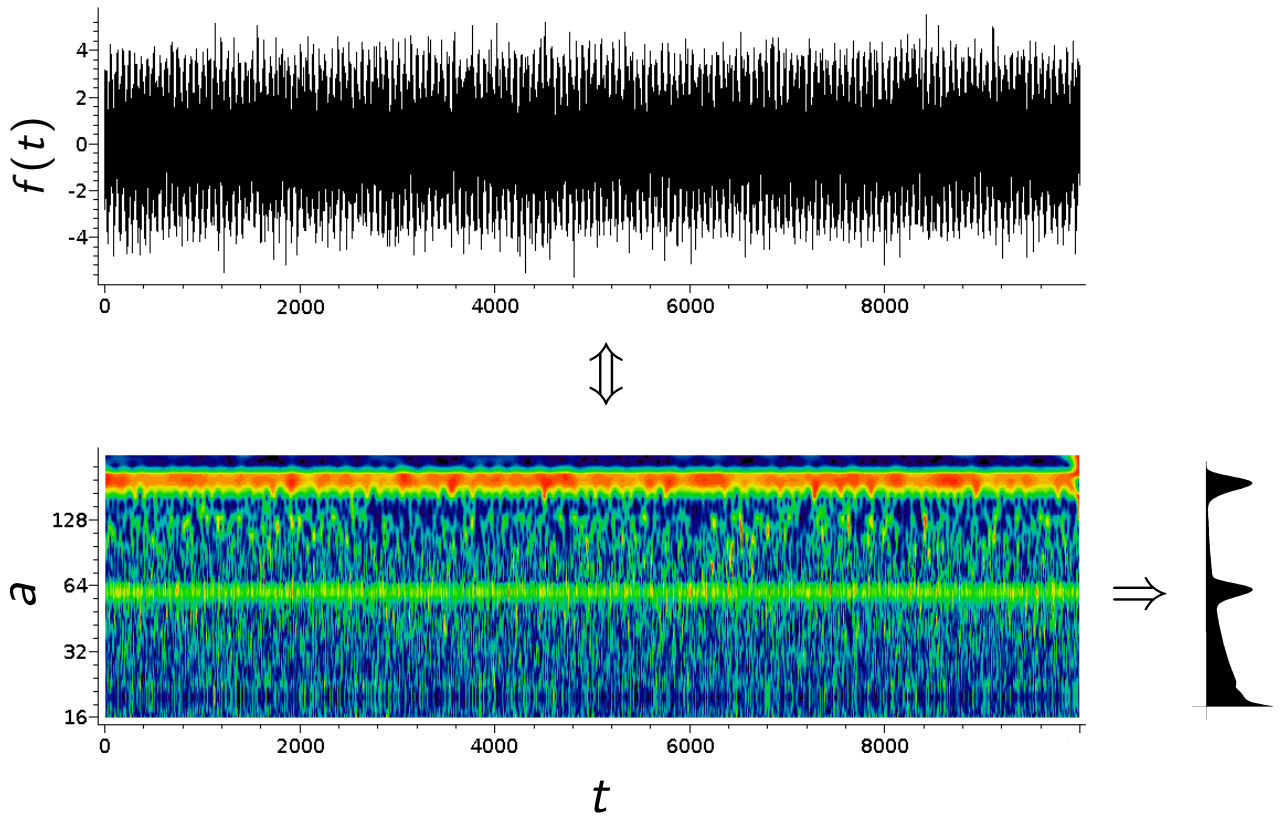
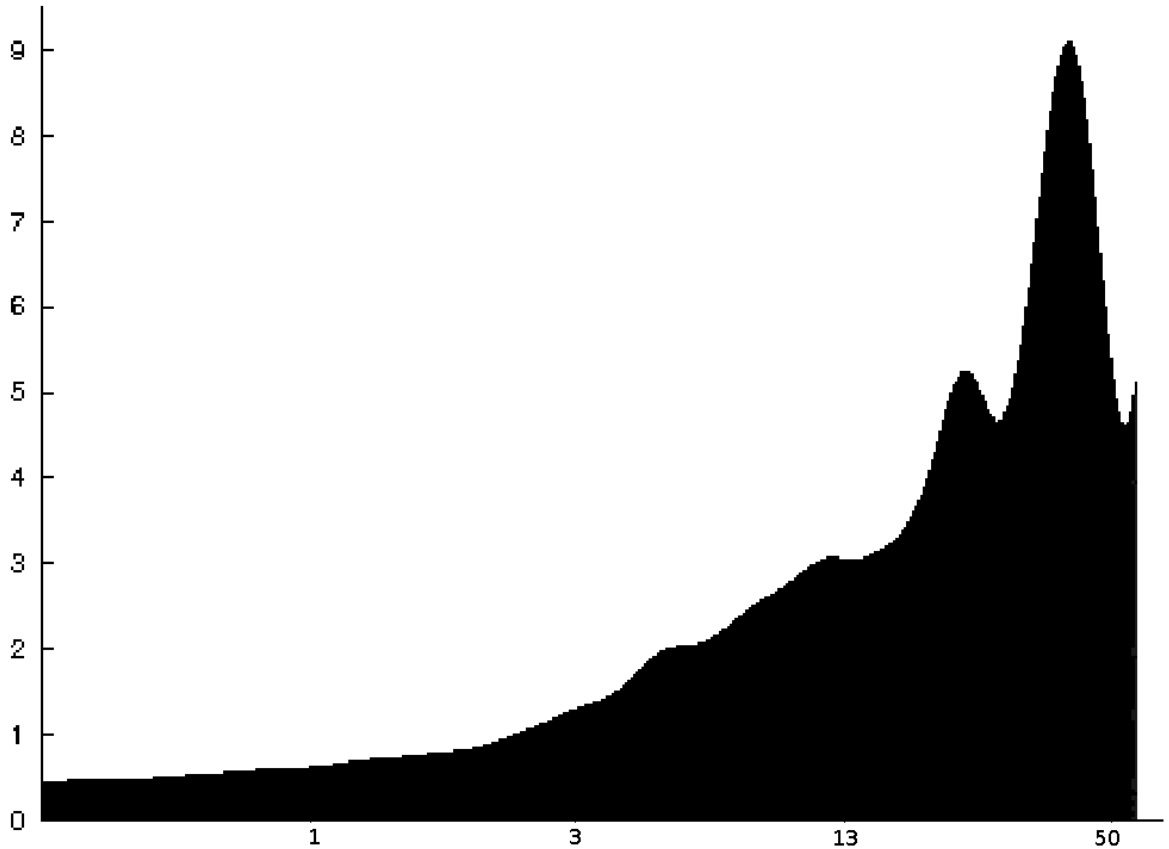


Known long cycles are observed in the spectrum



EPICA dome

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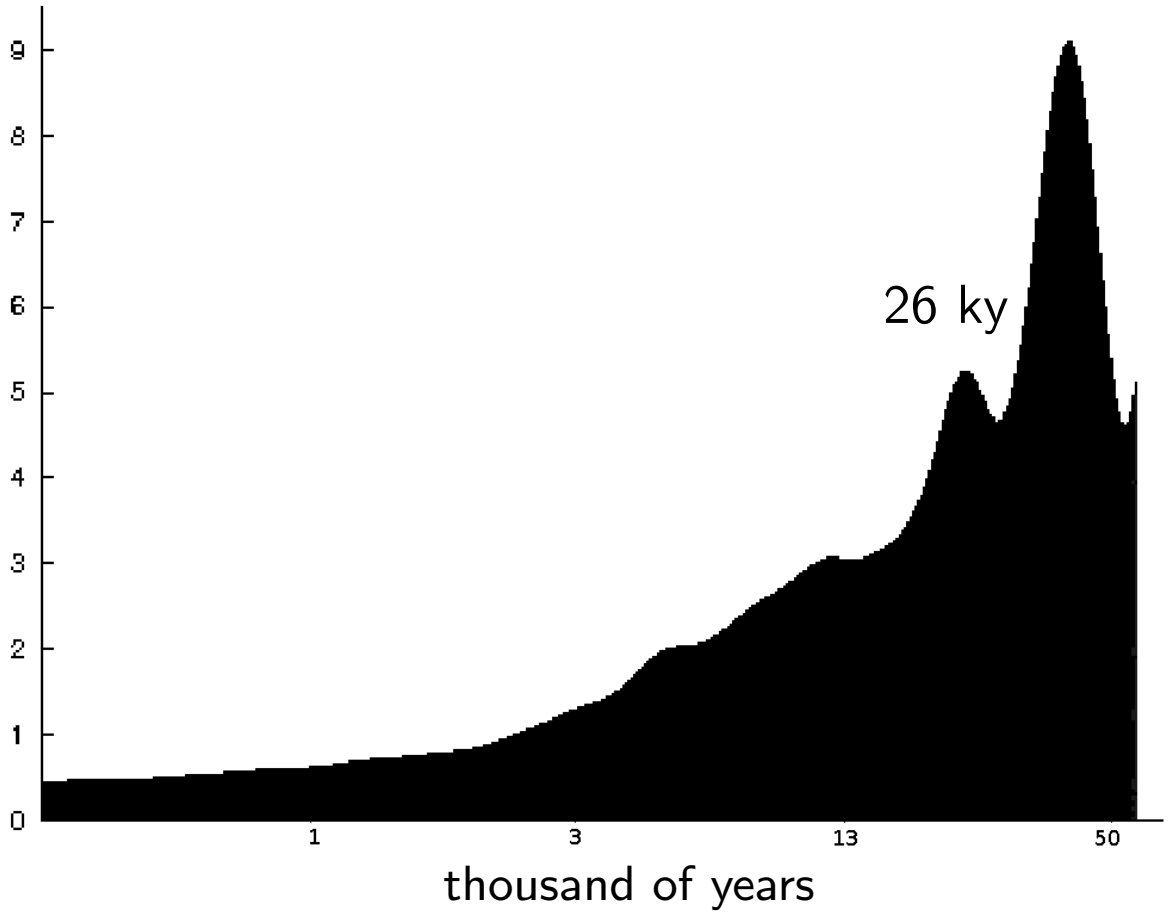

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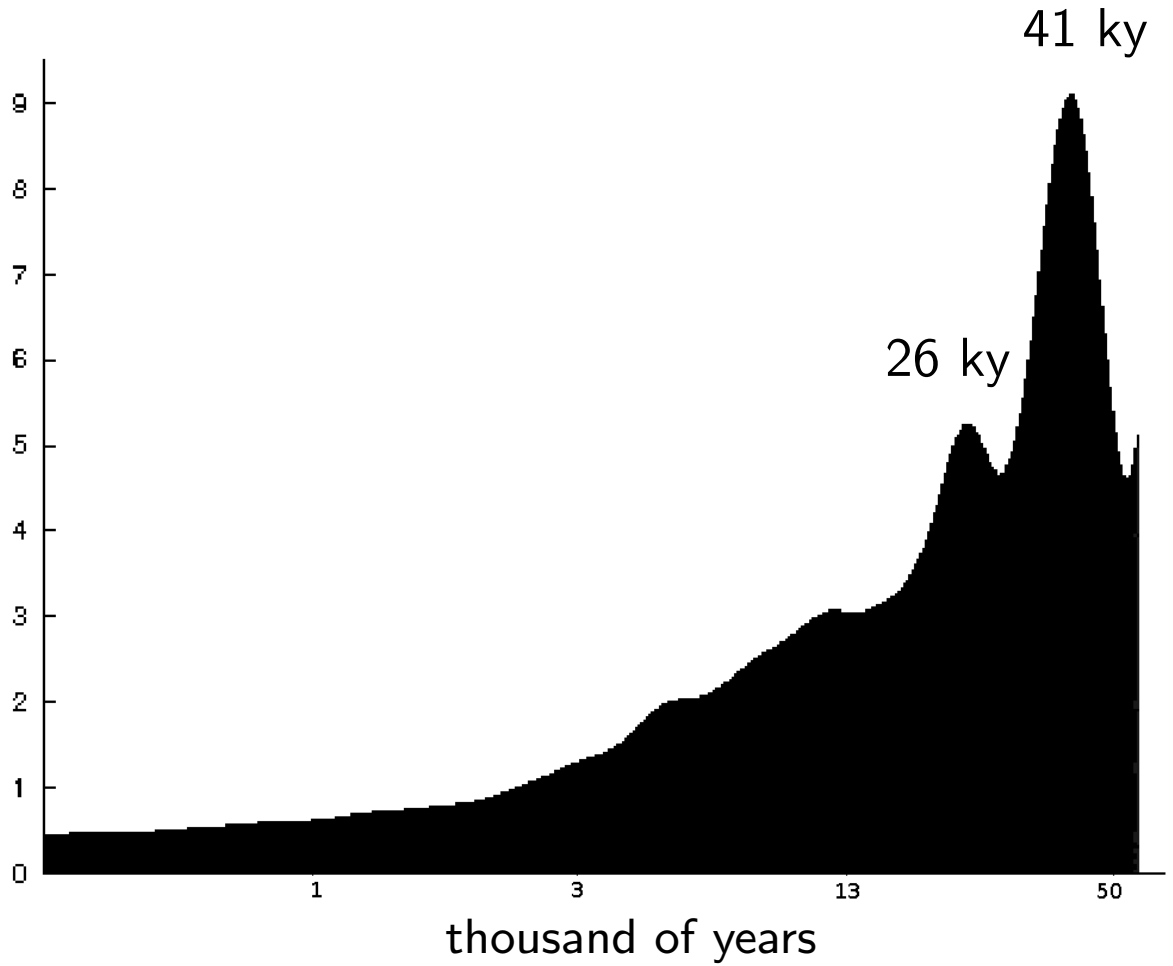
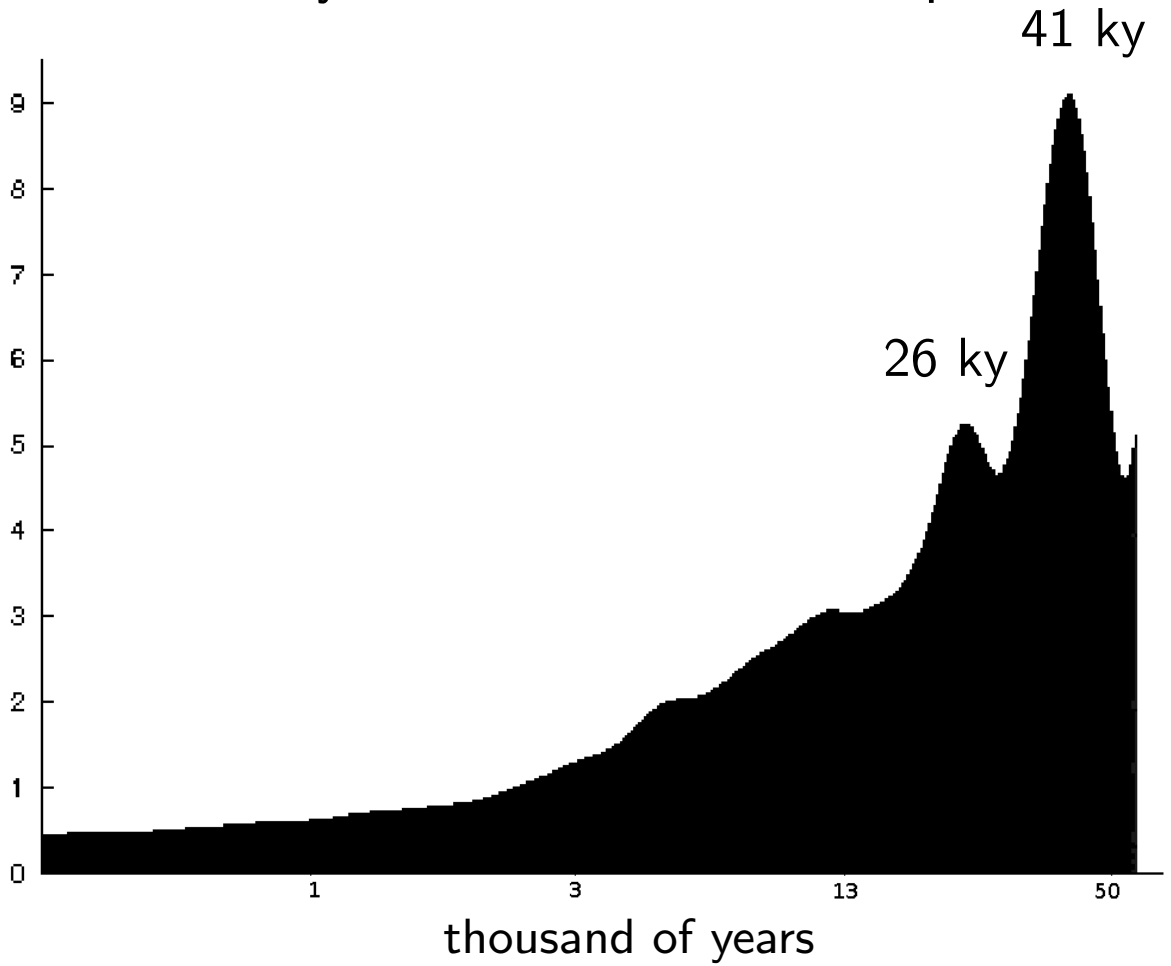


EPICA dome

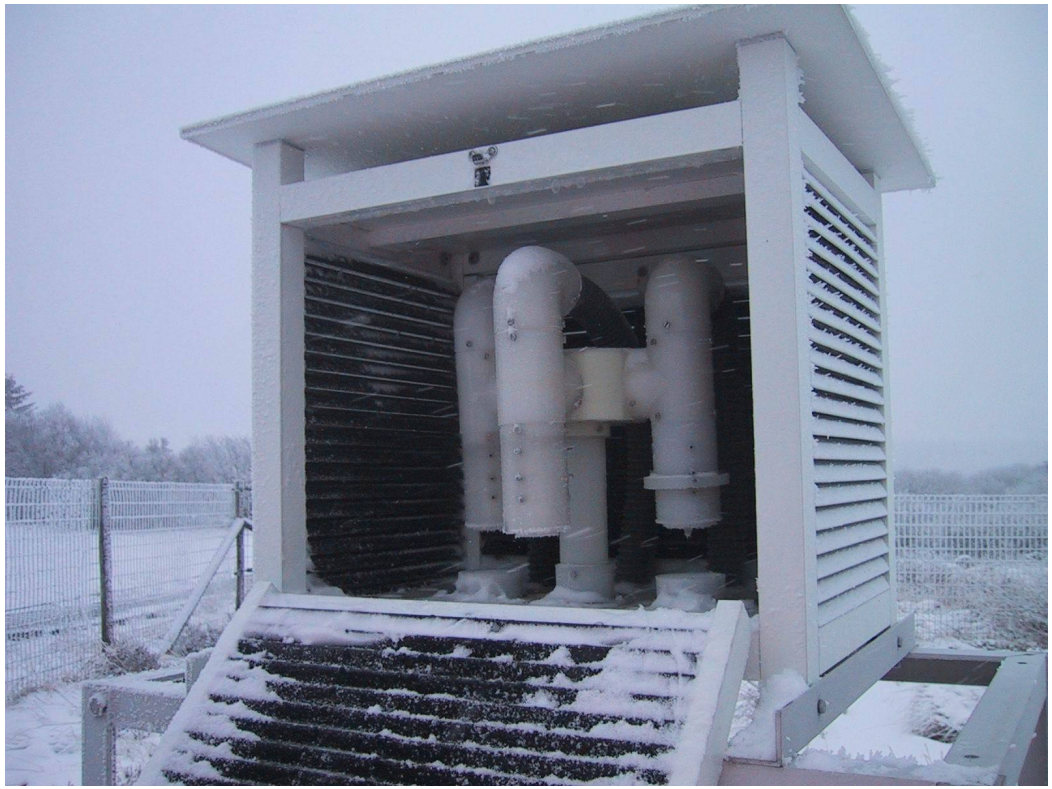
Known long cycles are observed in the spectrum

Milankovitch cycles are detected, as expected



EPICA dome

The time series we are interested in...



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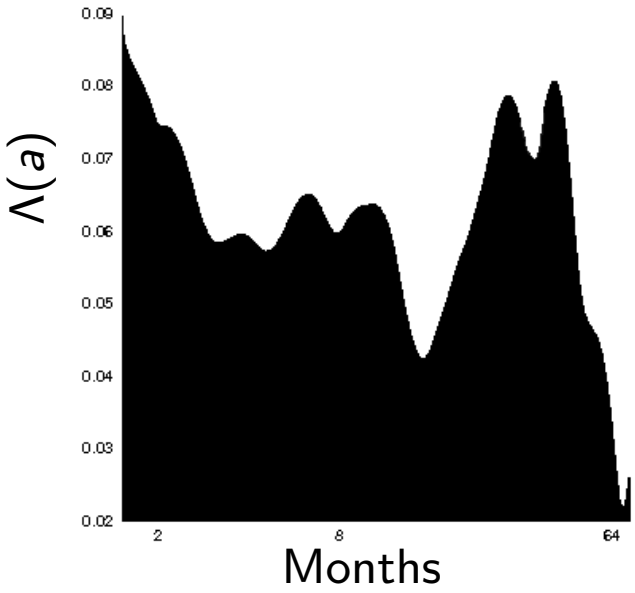

HadCRUT3, NH

1950–2007 monthly-sampled data

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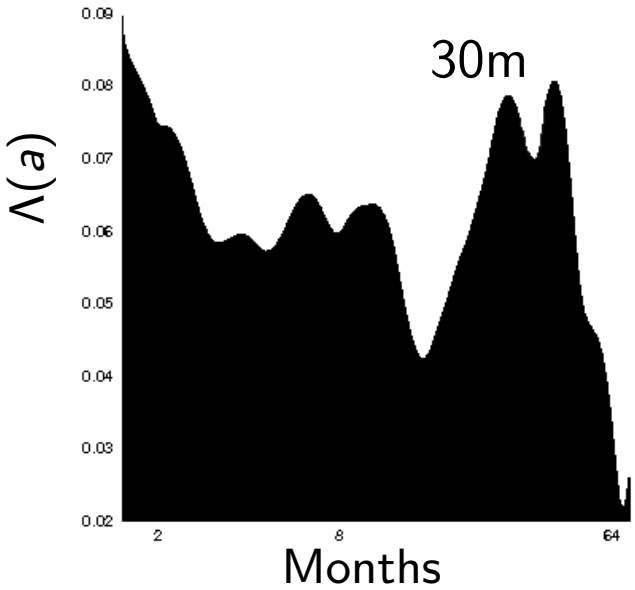

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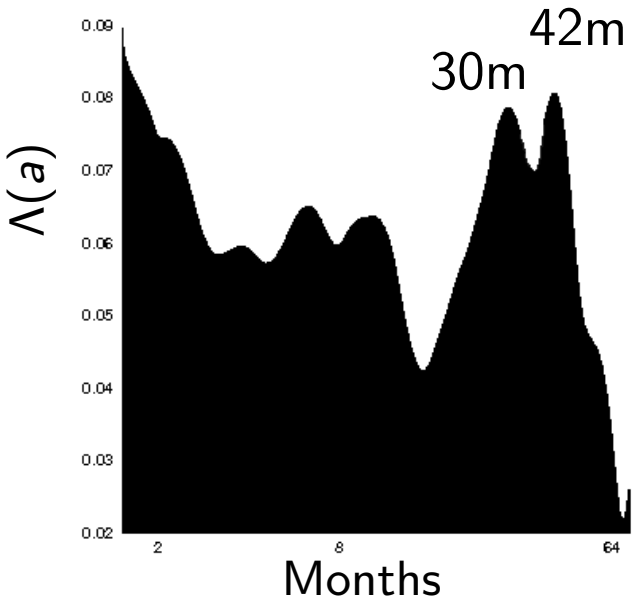

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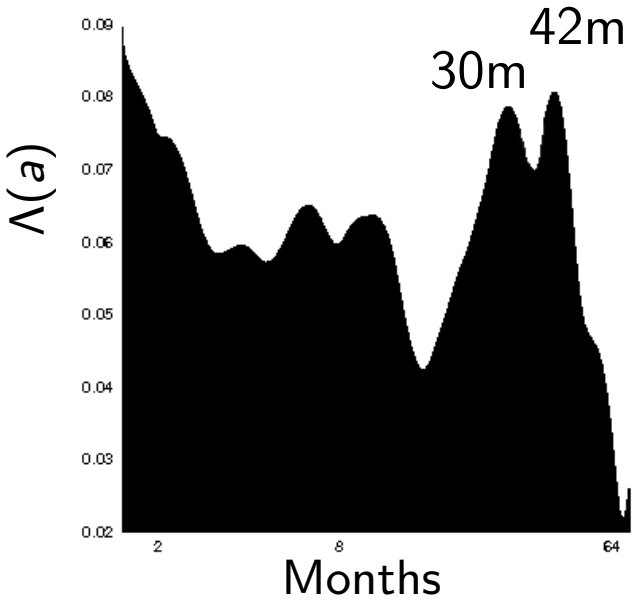

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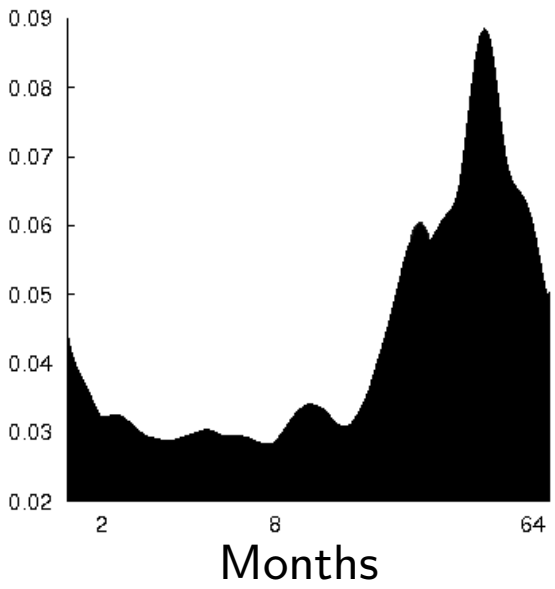

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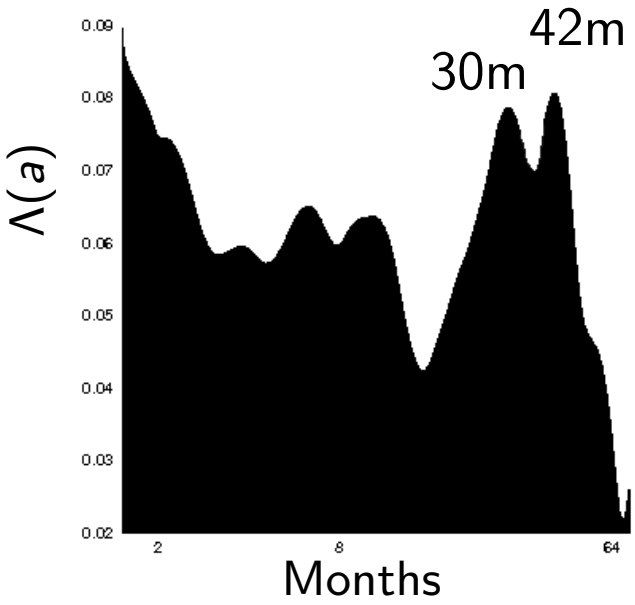

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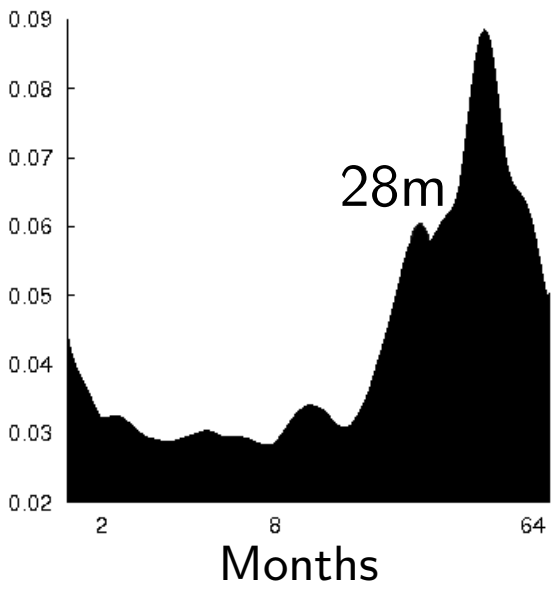

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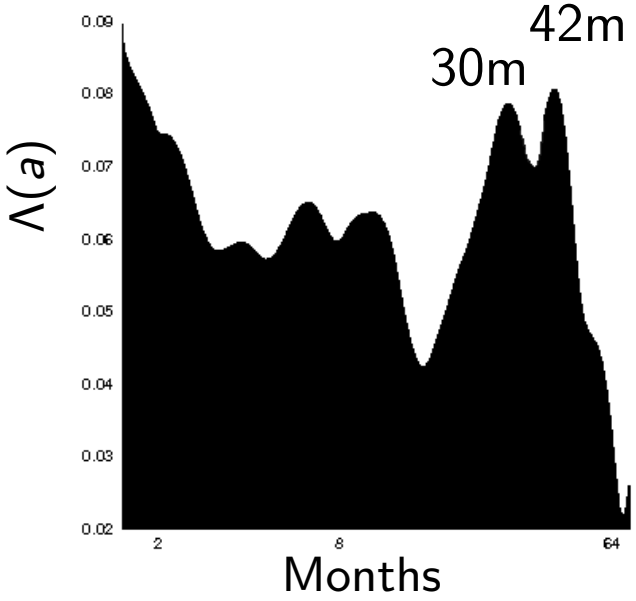

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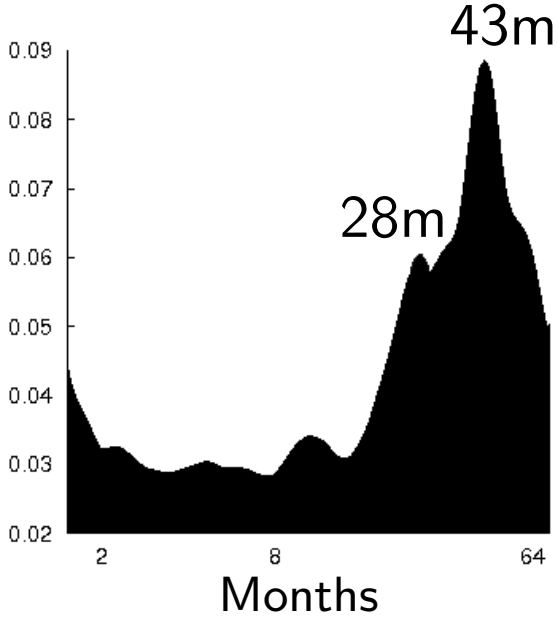

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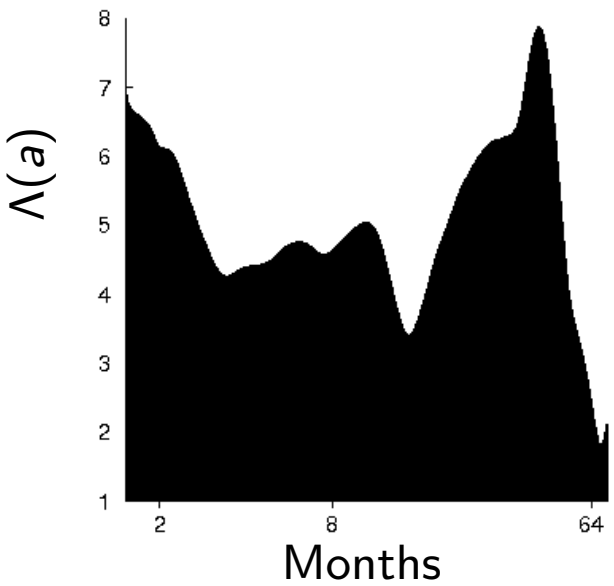

HaDSST2

1950–2007 monthly-sampled data

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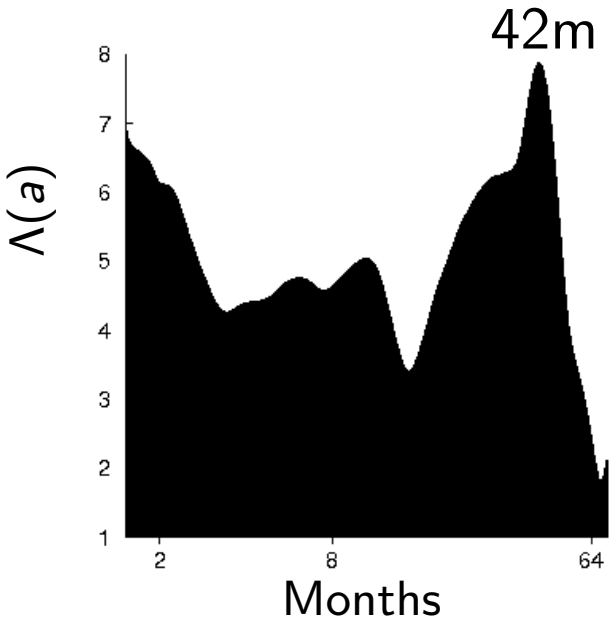

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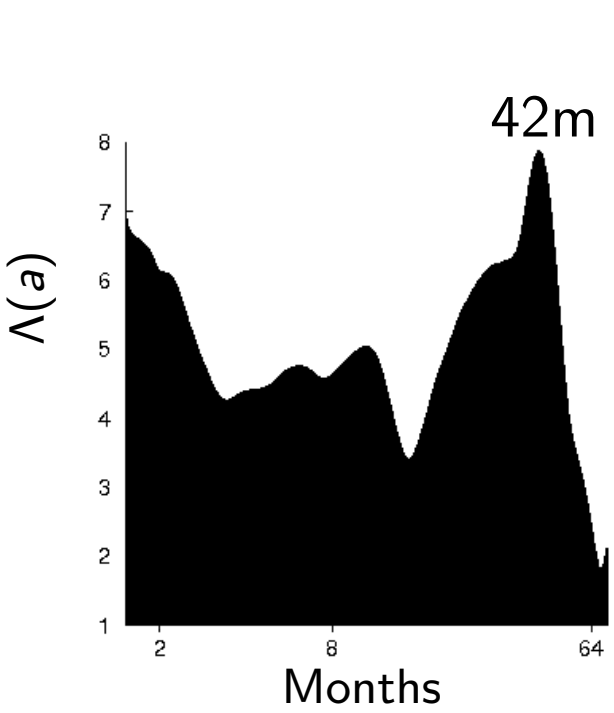

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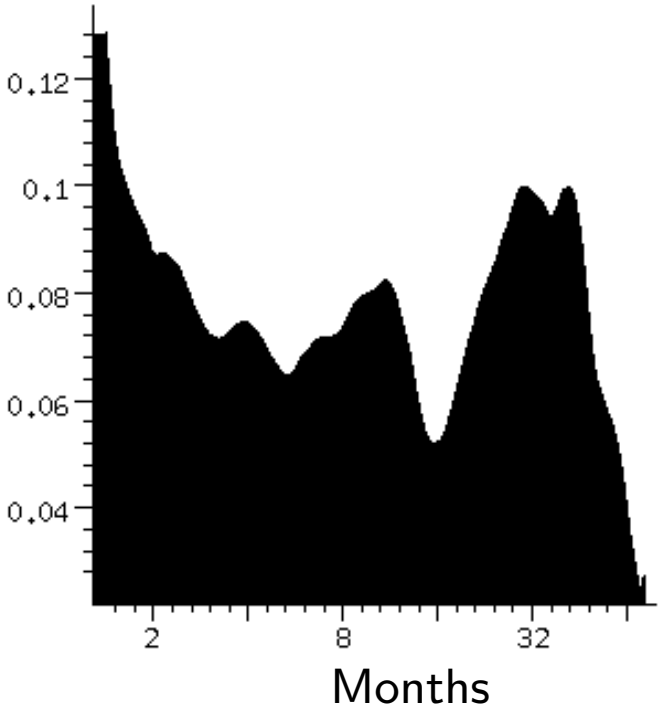

CRUTEM3gl

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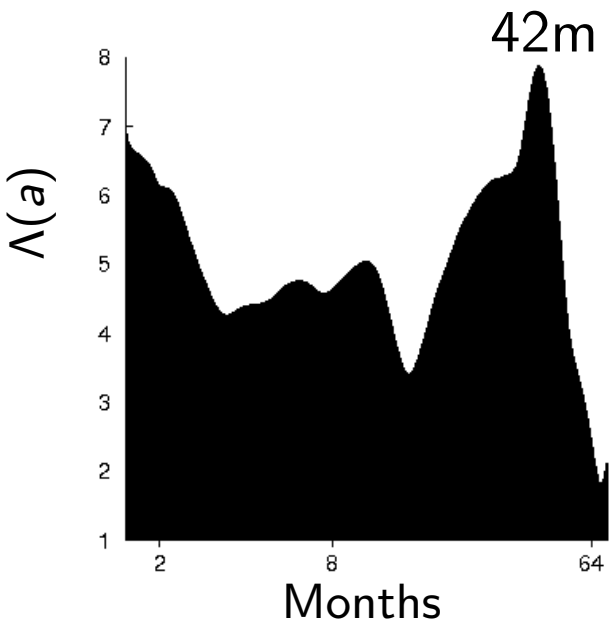

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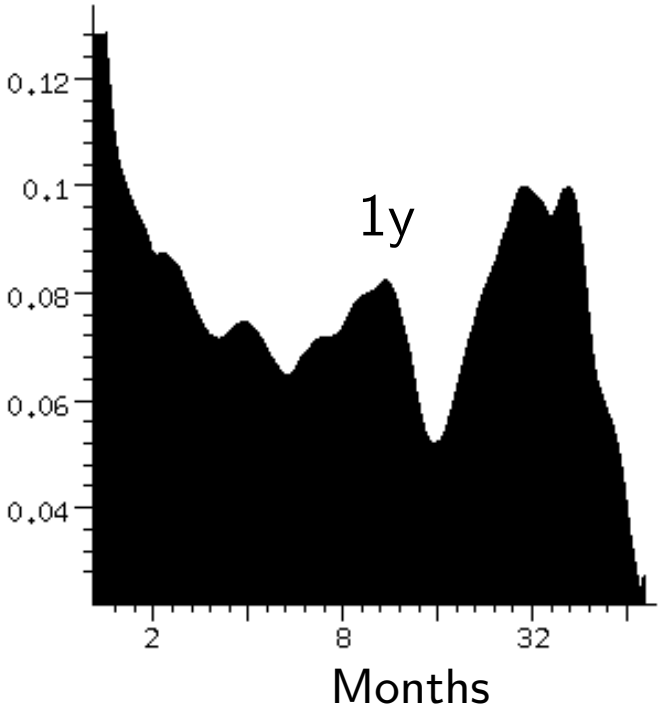

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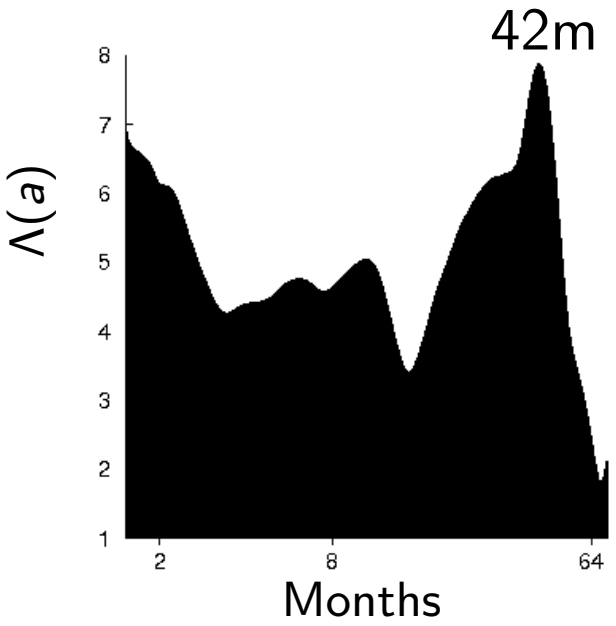

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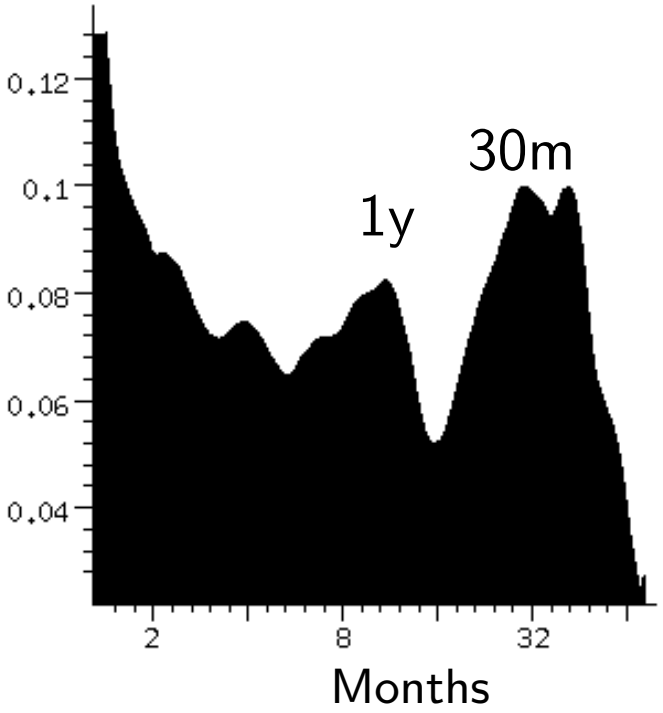

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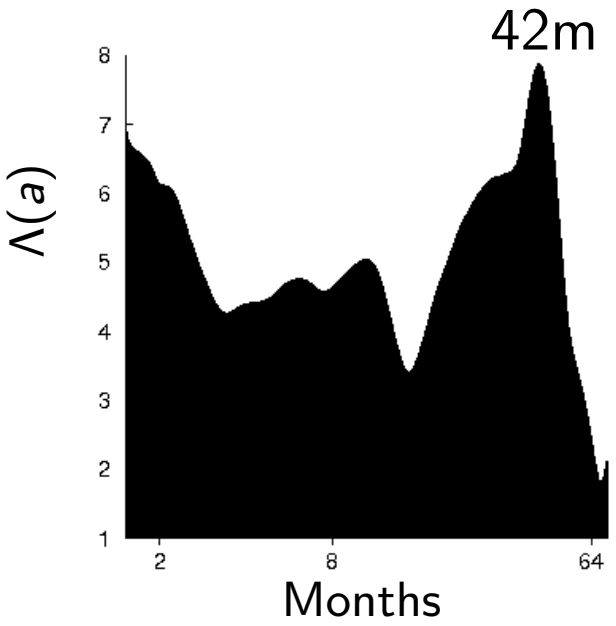

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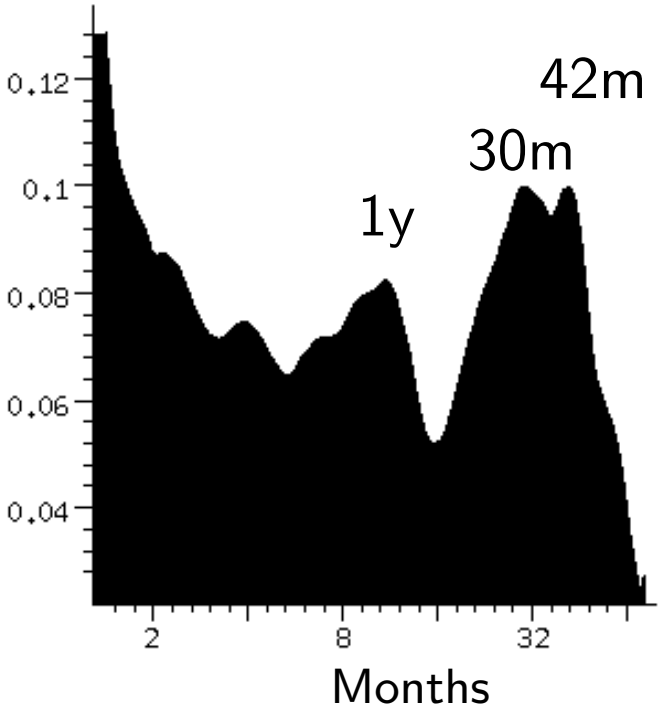

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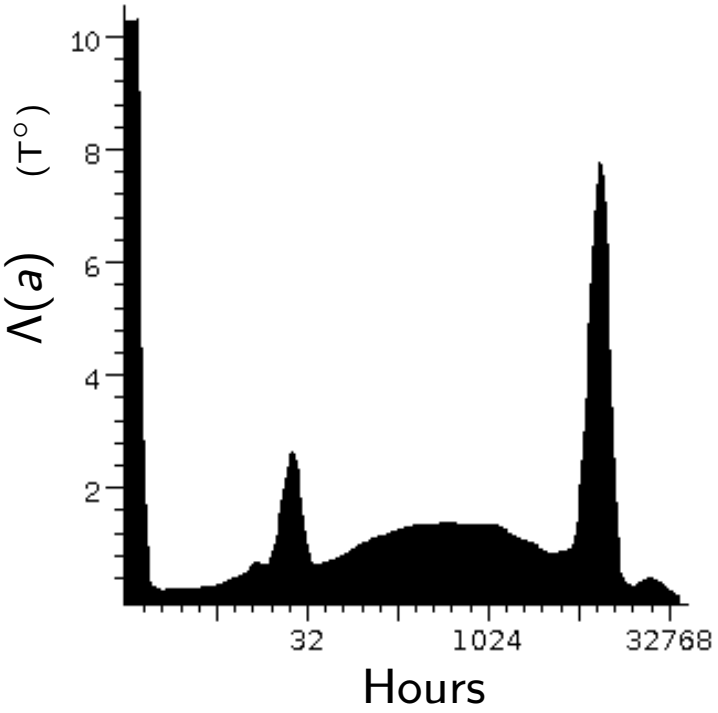

Bierset-aero, Belgium

1950–2007 hourly-sampled data

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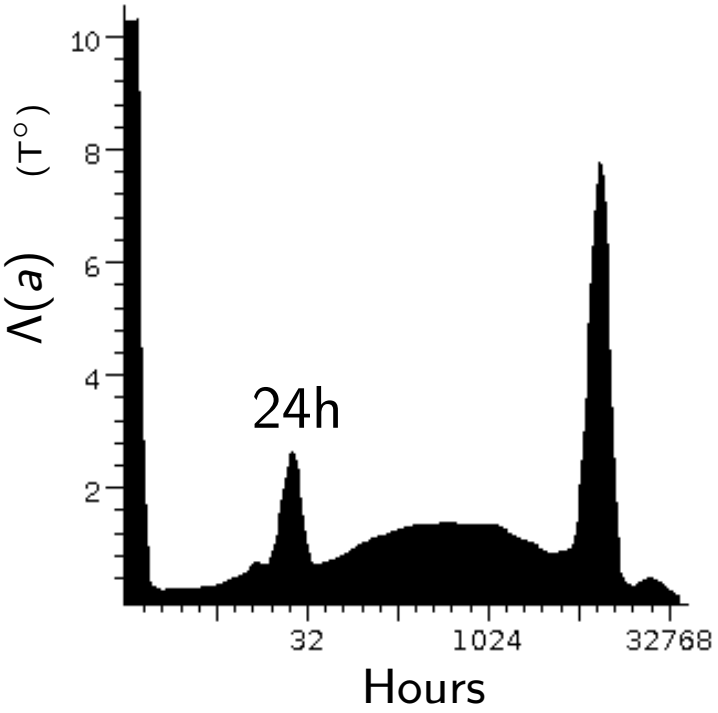

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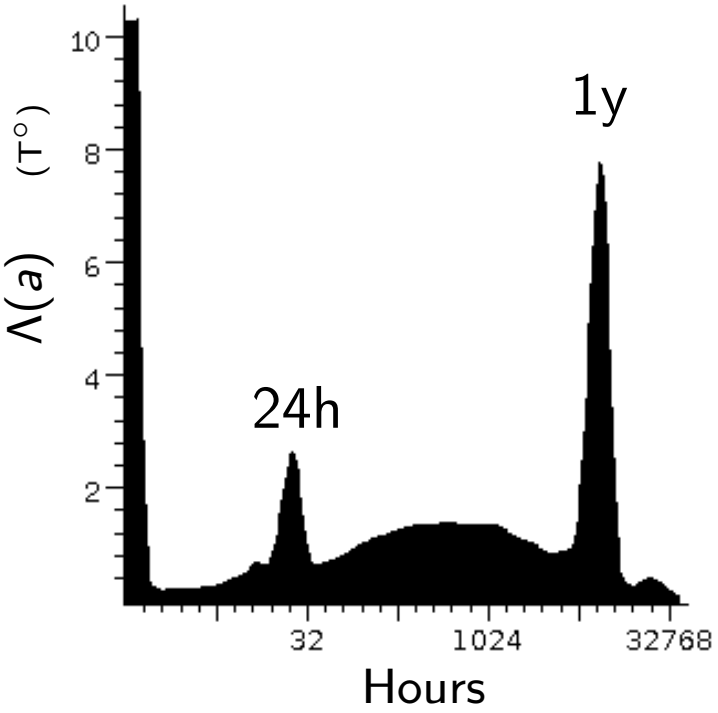

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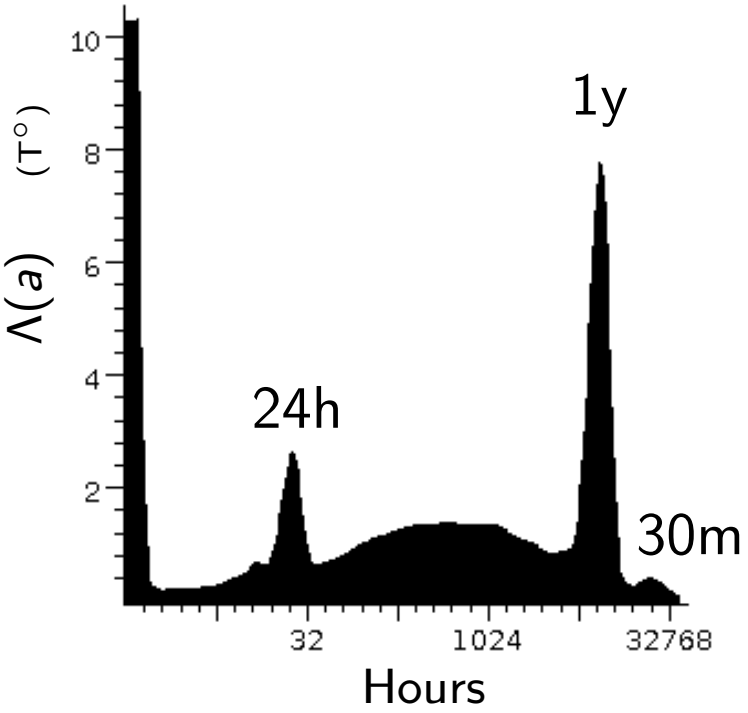

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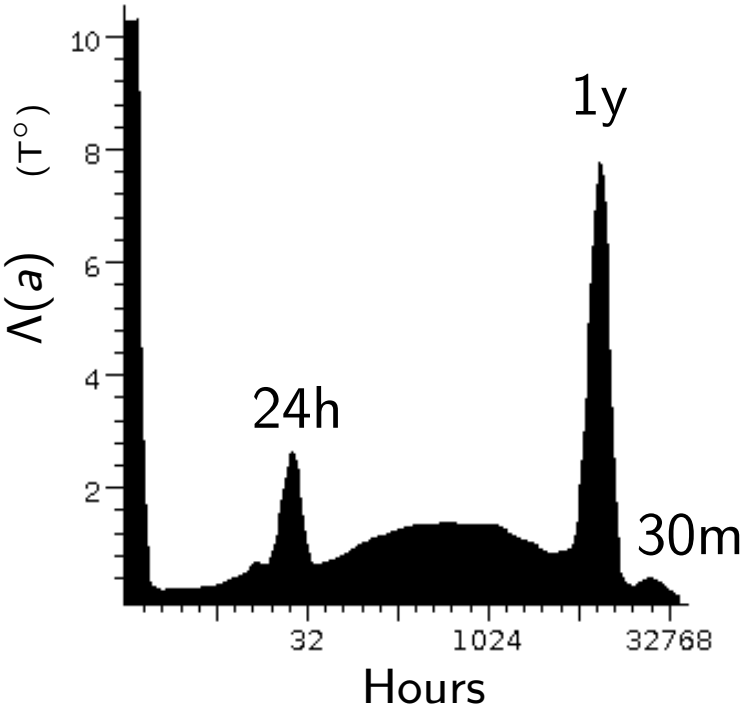

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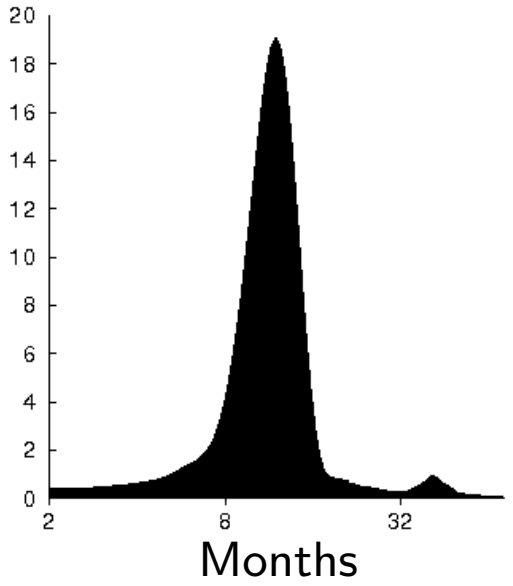

The Pas, Canada
1950–2007 monthly-sampled data

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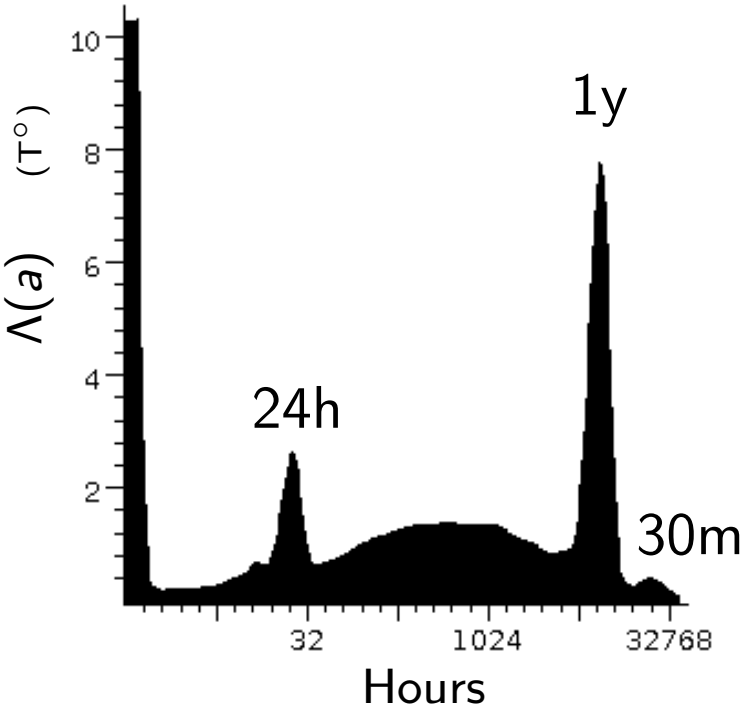

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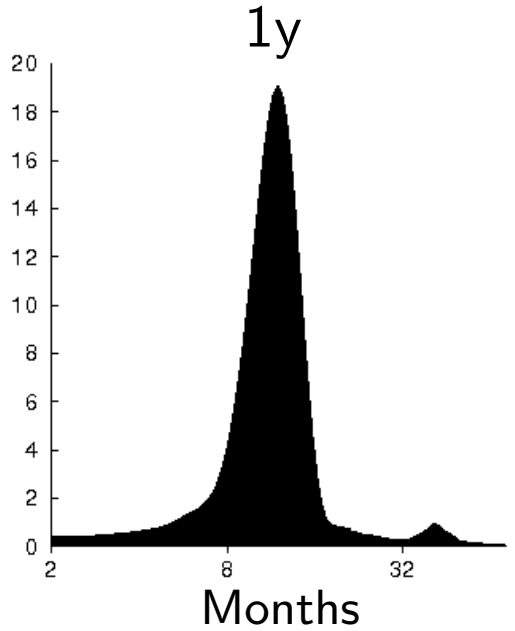

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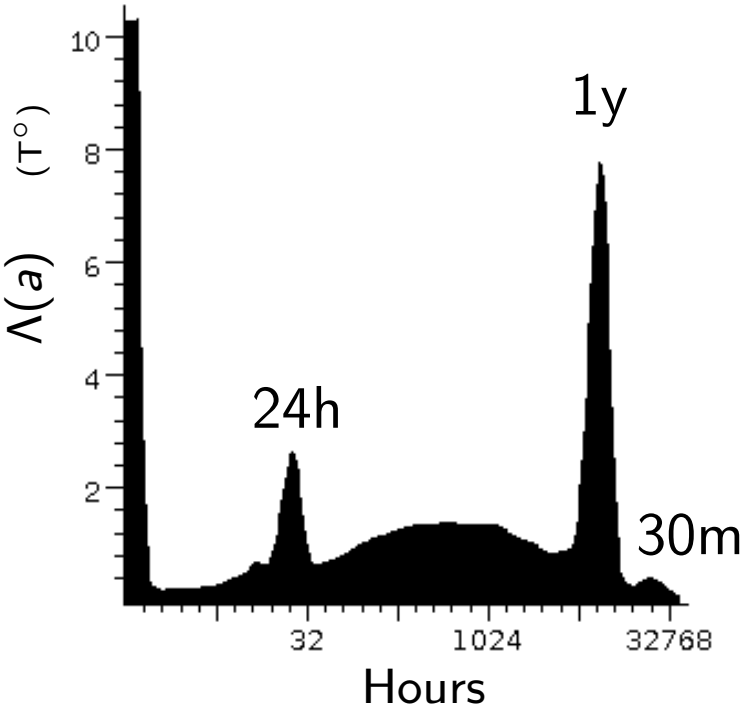

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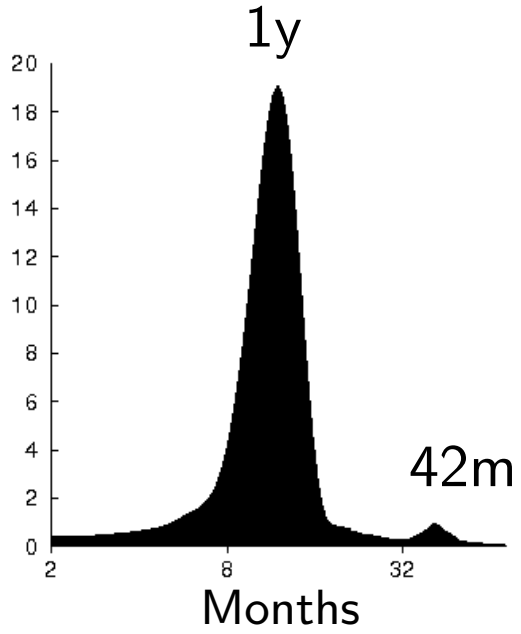

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A Random Process to Compare the Data With

One of the methods to check if "This cycle which is present in this time series did not just occur by pure chance" consists in comparing the scale spectra to spectra obtained from a random process "looking like" the data.

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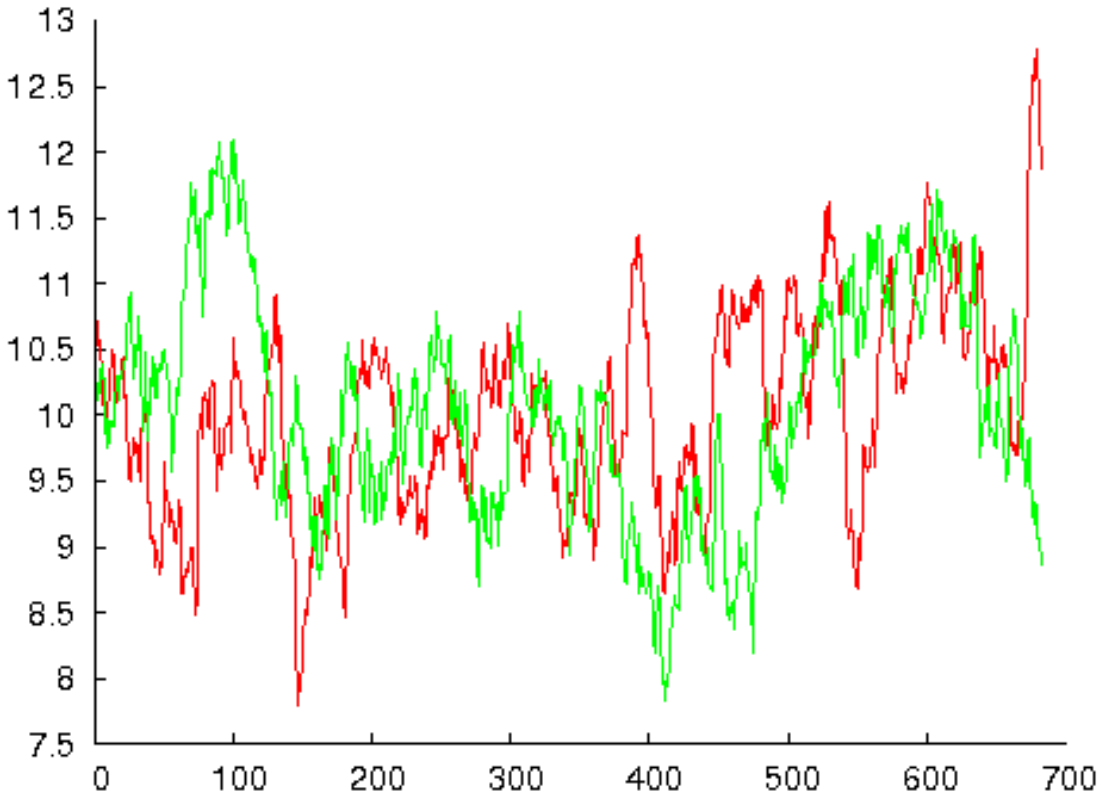
A good model seems to be the AR(1):

$$x_{n+1} = \alpha x_n + \sigma \eta_{n+1},$$

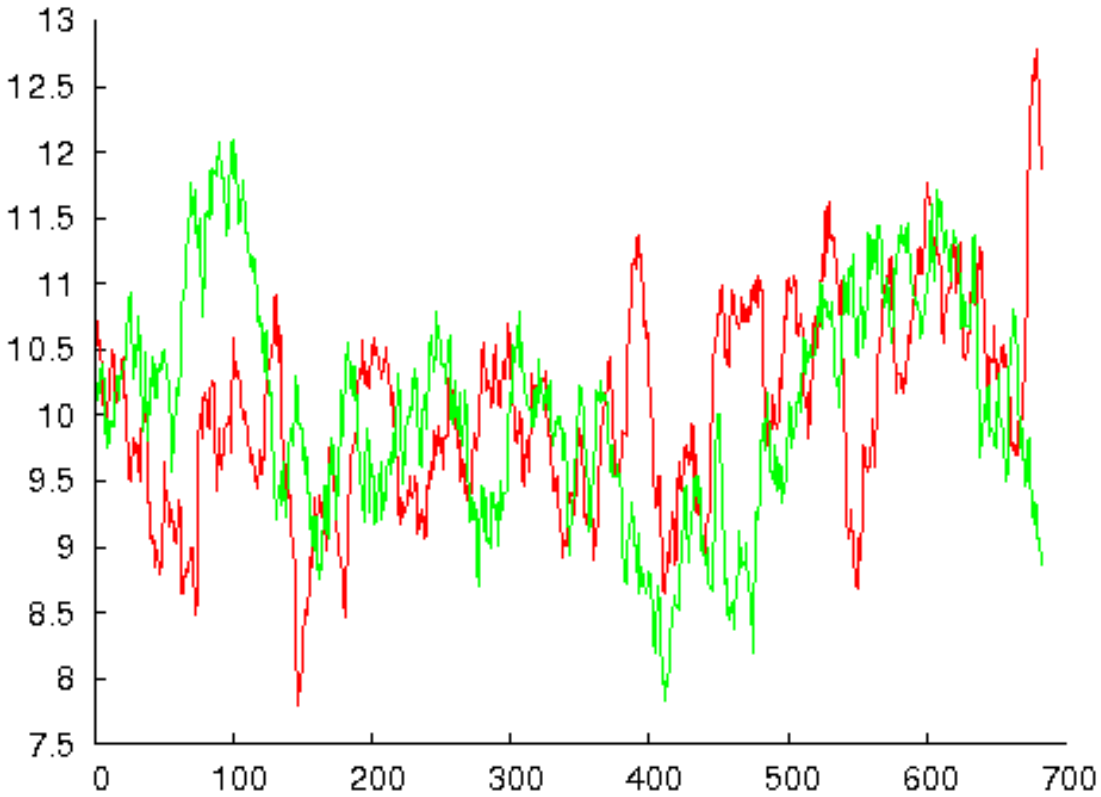
where , as usual, η_n is i.i.d., with zero mean and unit variance (e.g. Gaussian white noise N(0,1)).

A Visual Illustration



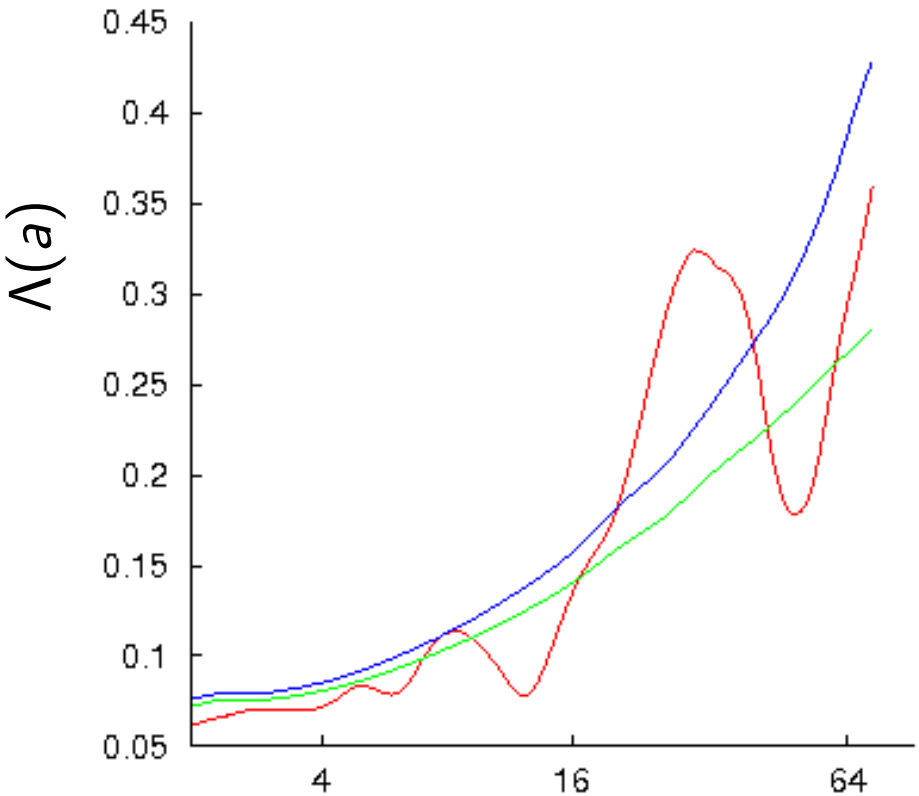
Wien, Austria

A Visual Illustration



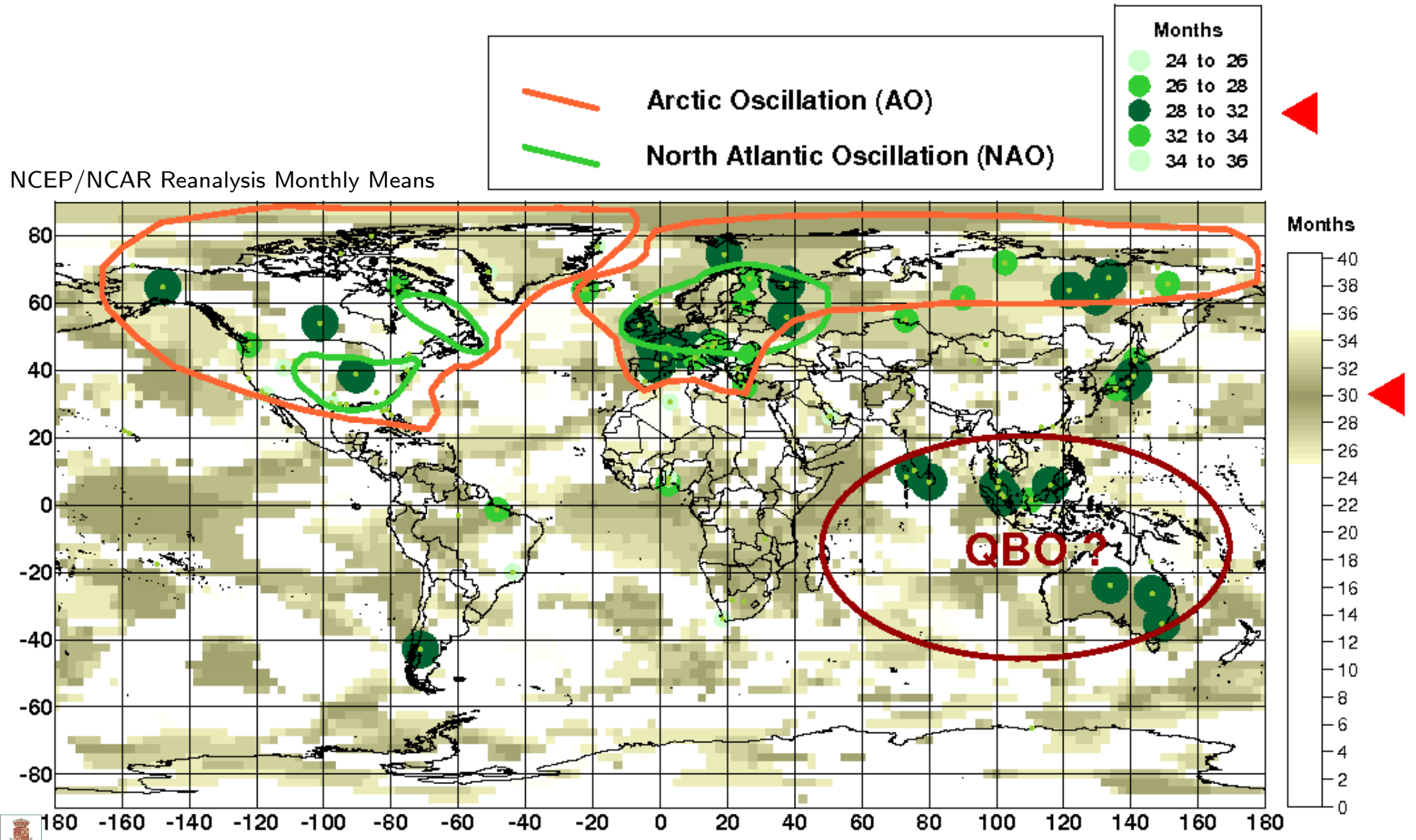
Wien, Austria / A simulation

The scale spectra of the Wien Temprature Records vs. the Mean Spectra of the Model

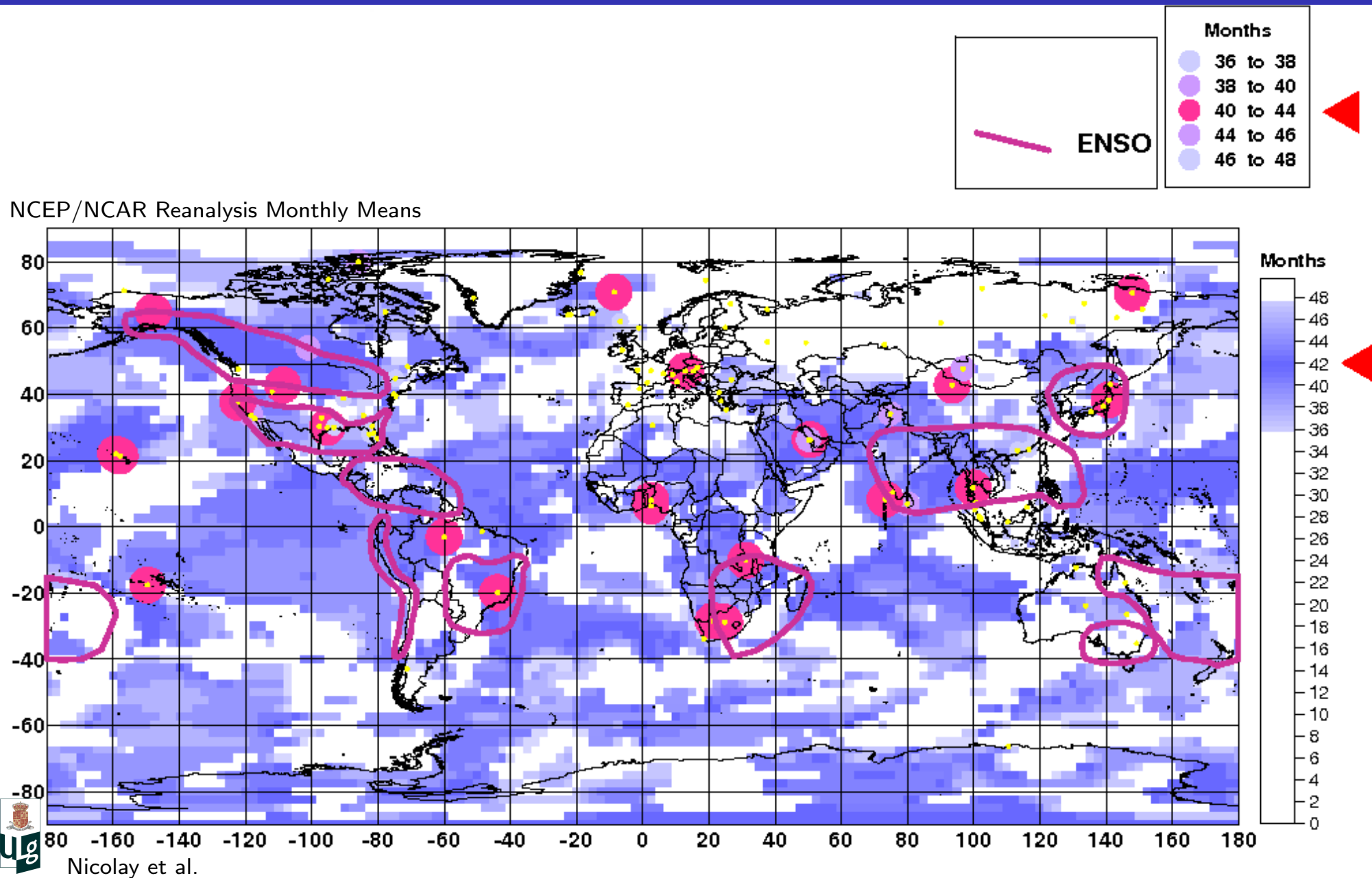


Wien / $E[AR]$ / $E[AR]+sd[AR]$

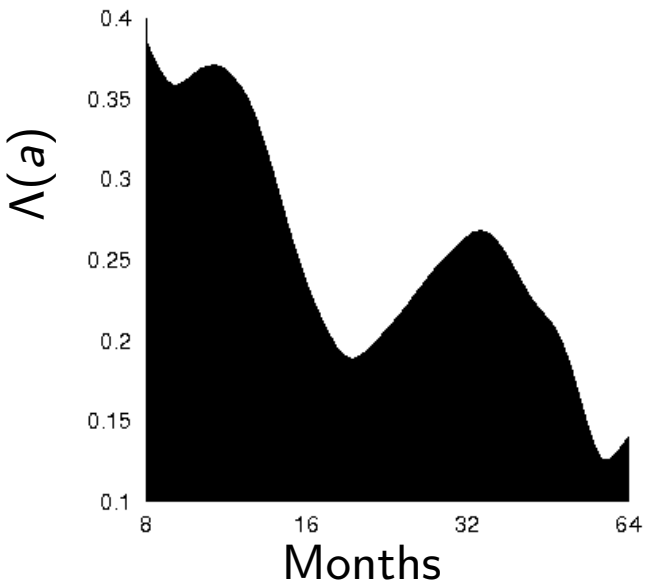
30 Months Cycle



42 Months Cycle



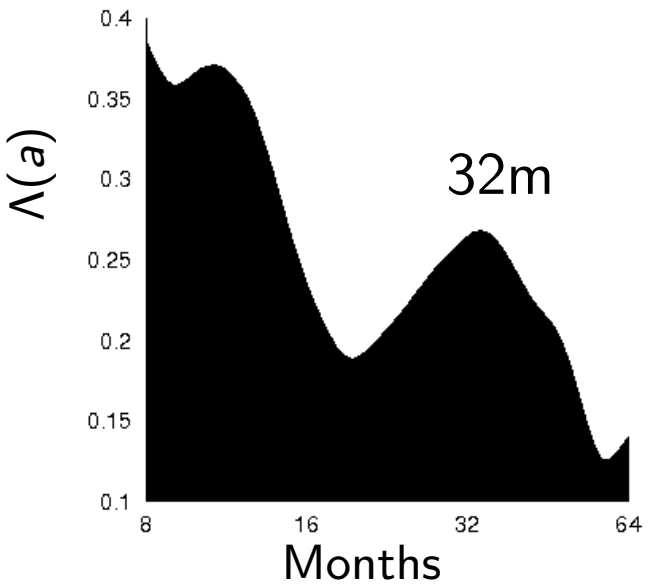
Scale Spectrum of Some Climatic Indices



NAO (CPC)

1950–2007 monthly-sampled data

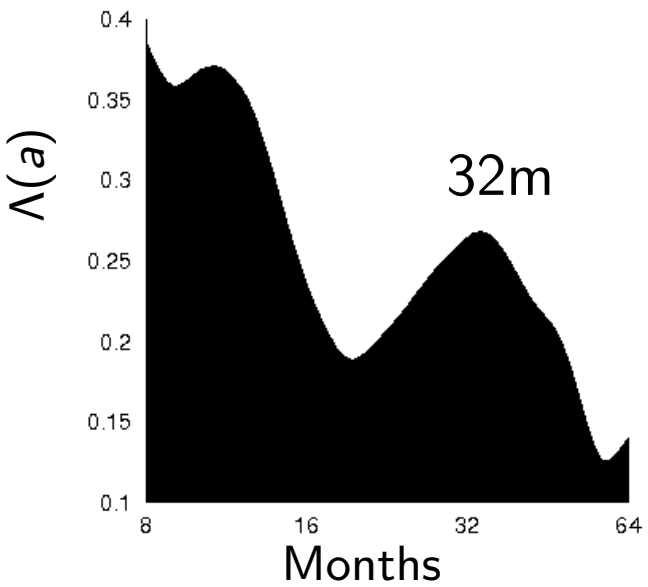
Scale Spectrum of Some Climatic Indices



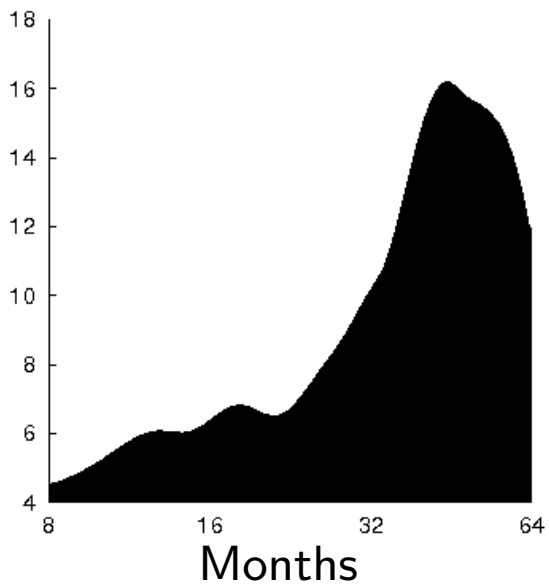
NAO (CPC)

1950–2007 monthly-sampled data

Scale Spectrum of Some Climatic Indices

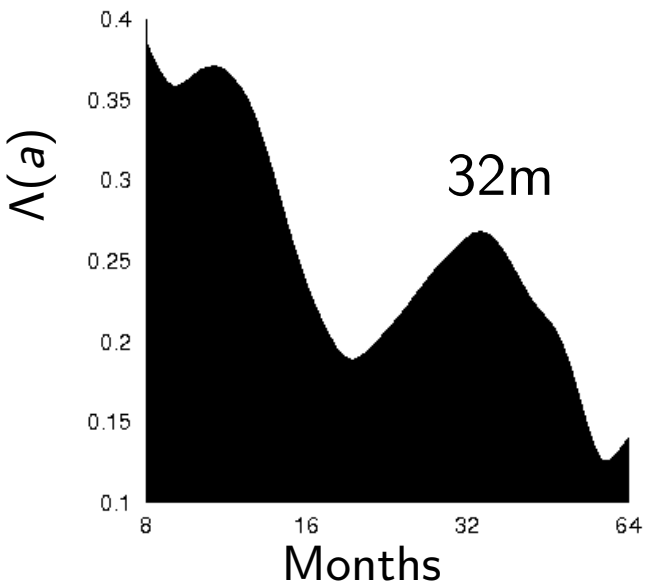


NAO (CPC)
1950–2007 monthly-sampled data

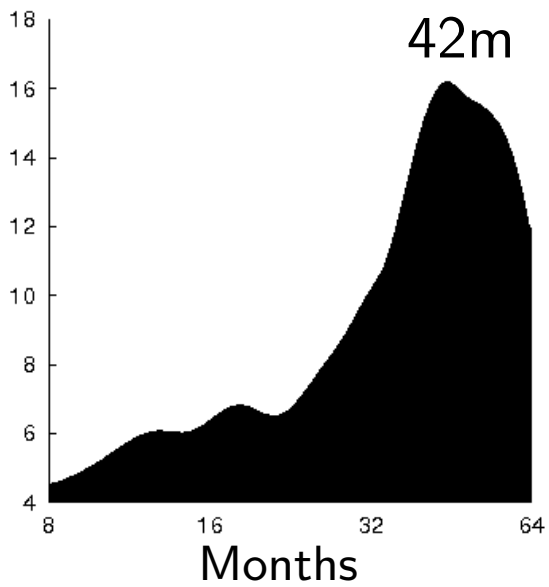


ENSO (SST)
1950–2007 monthly-sampled data

Scale Spectrum of Some Climatic Indices

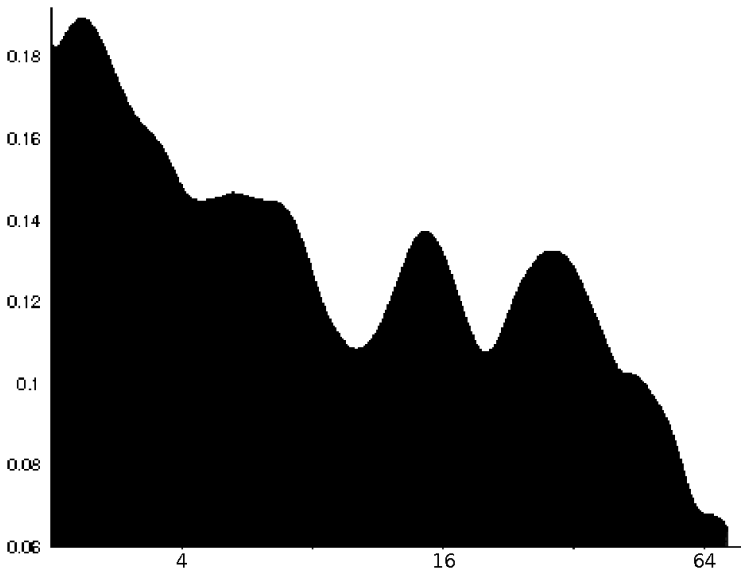


NAO (CPC)
1950–2007 monthly-sampled data



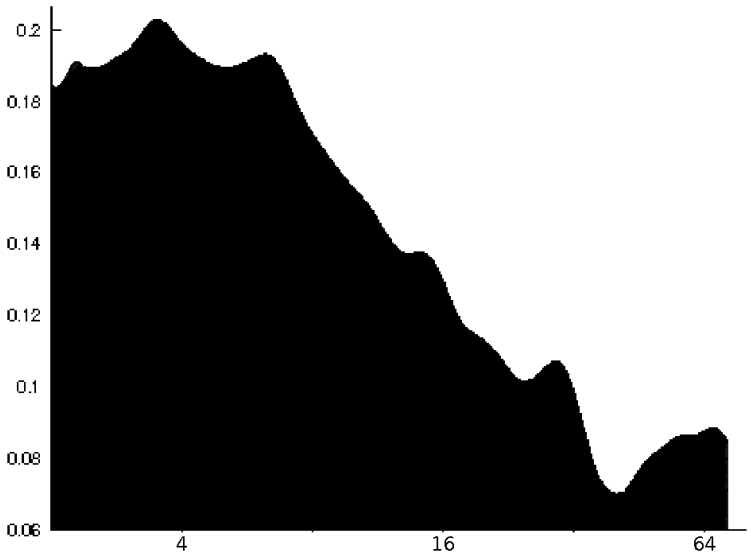
ENSO (SST)
1950–2007 monthly-sampled data

Millennial Temperature Reconstructions of Jones et al.



Years

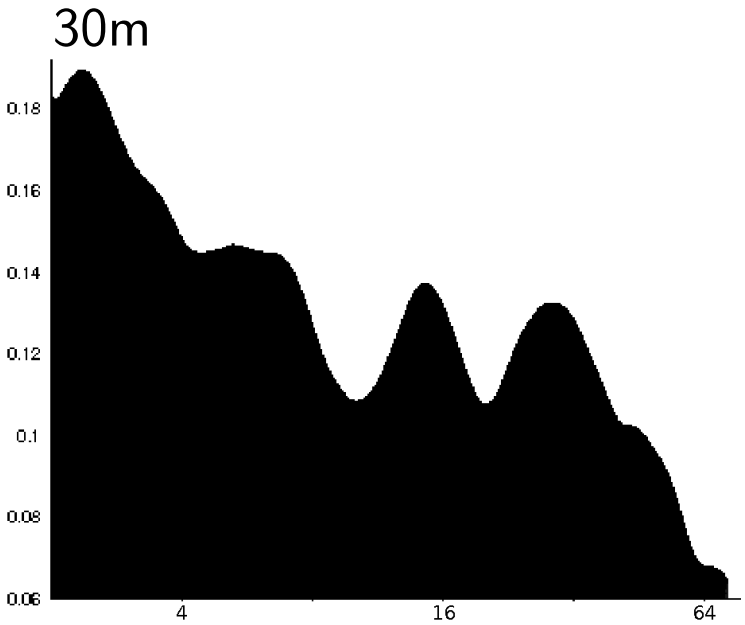
Northern hemisphere



Years

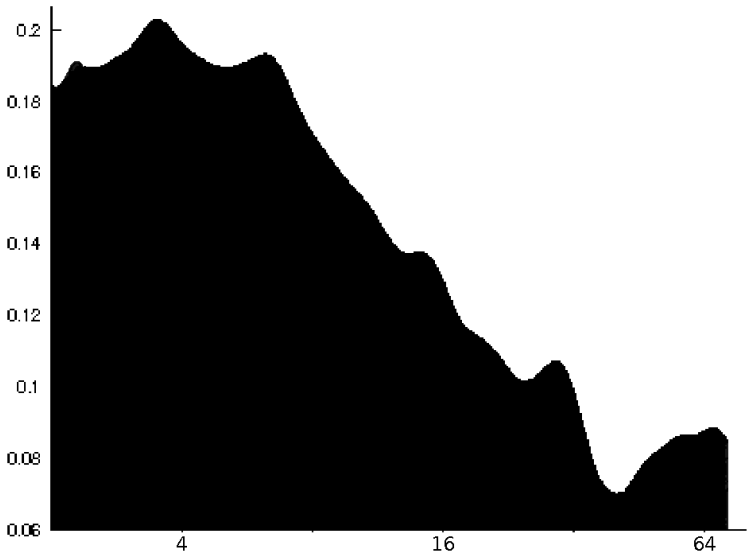
Southern hemisphere

Millennial Temperature Reconstructions of Jones et al.



Years

Northern hemisphere



Years

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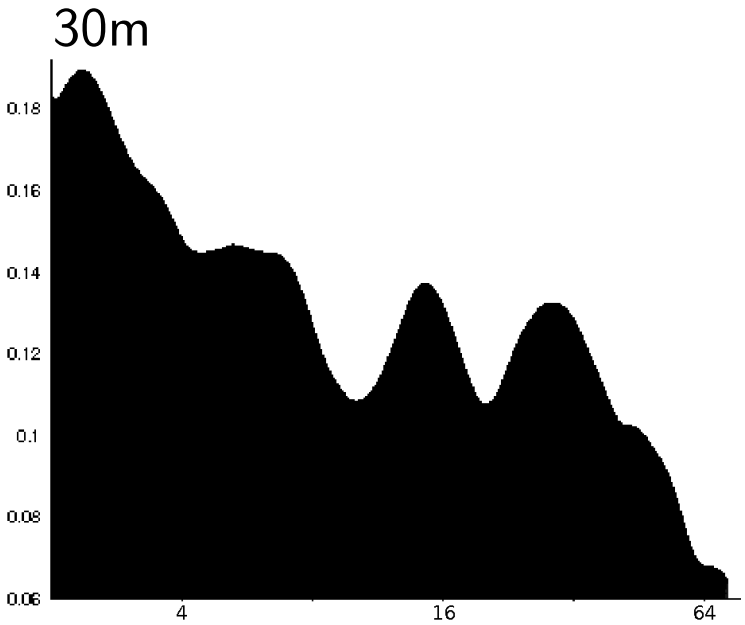

Years

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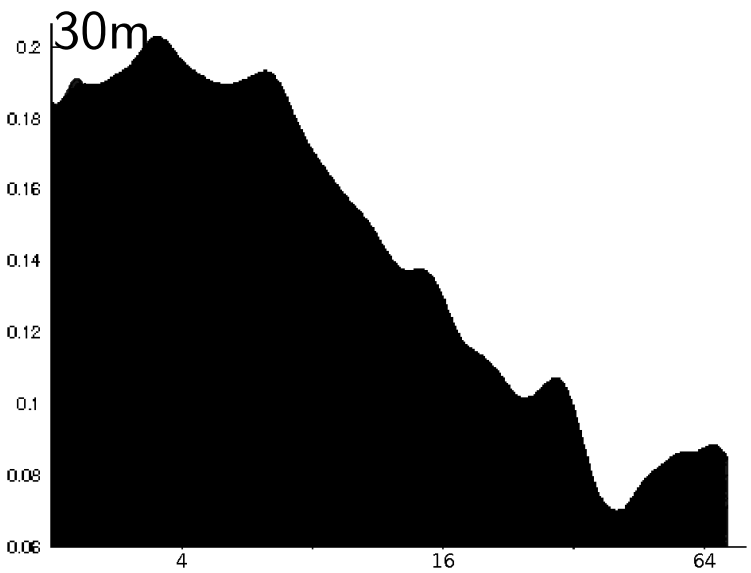

Years

Southern hemisphere

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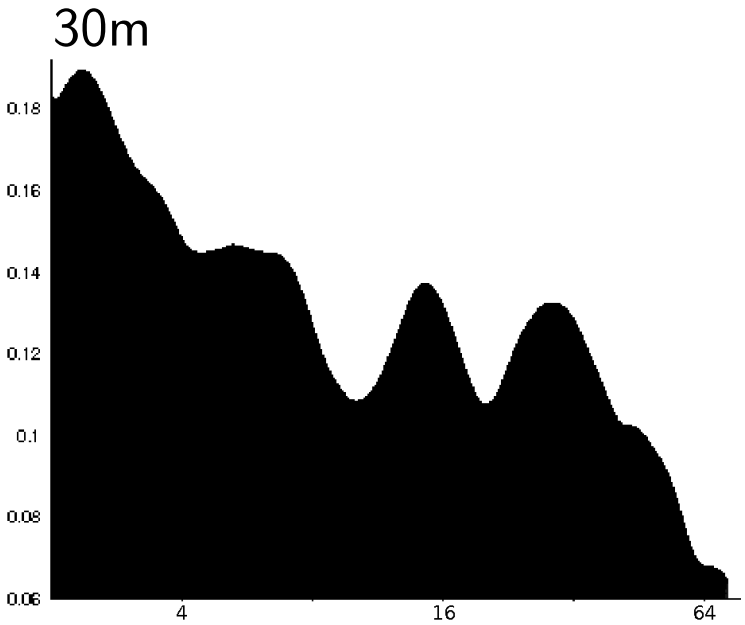
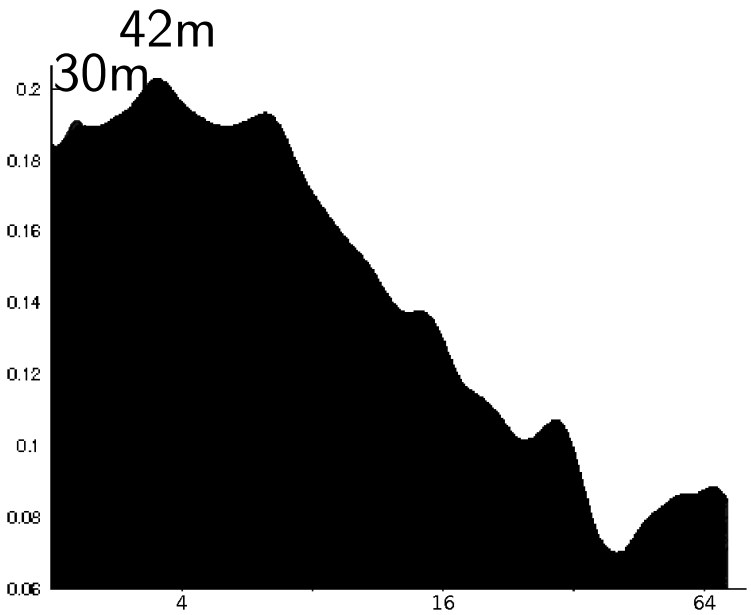


Years

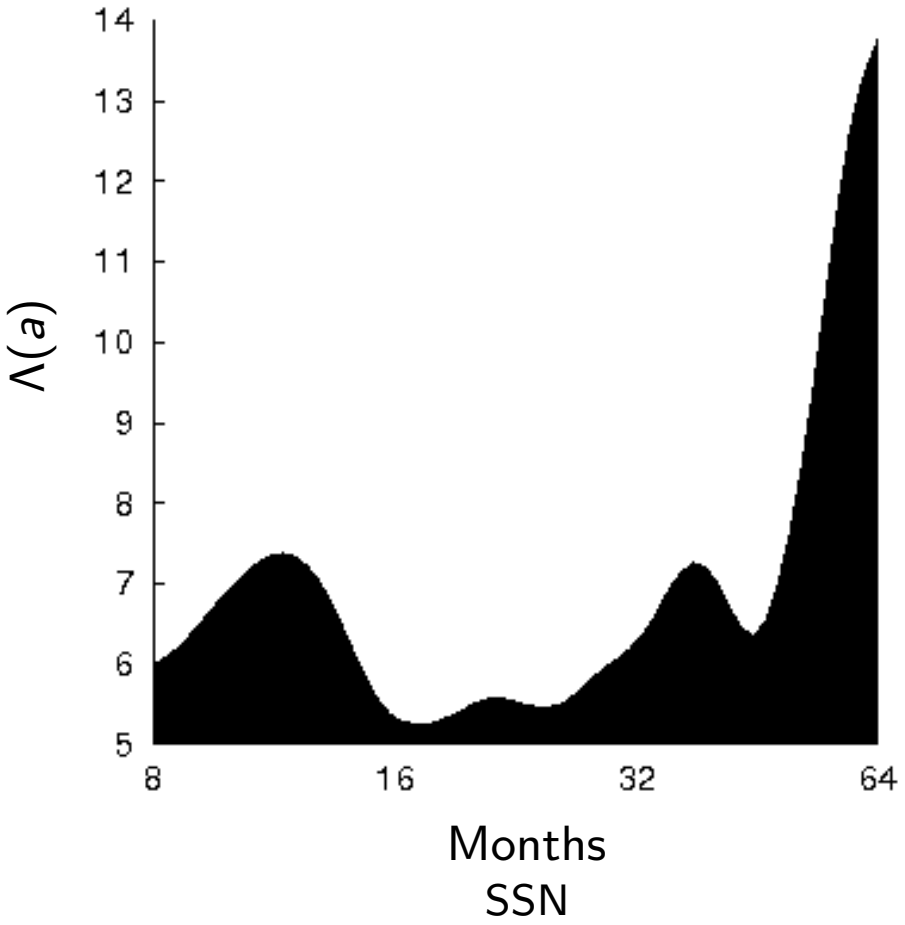
Northern hemisphere



Years

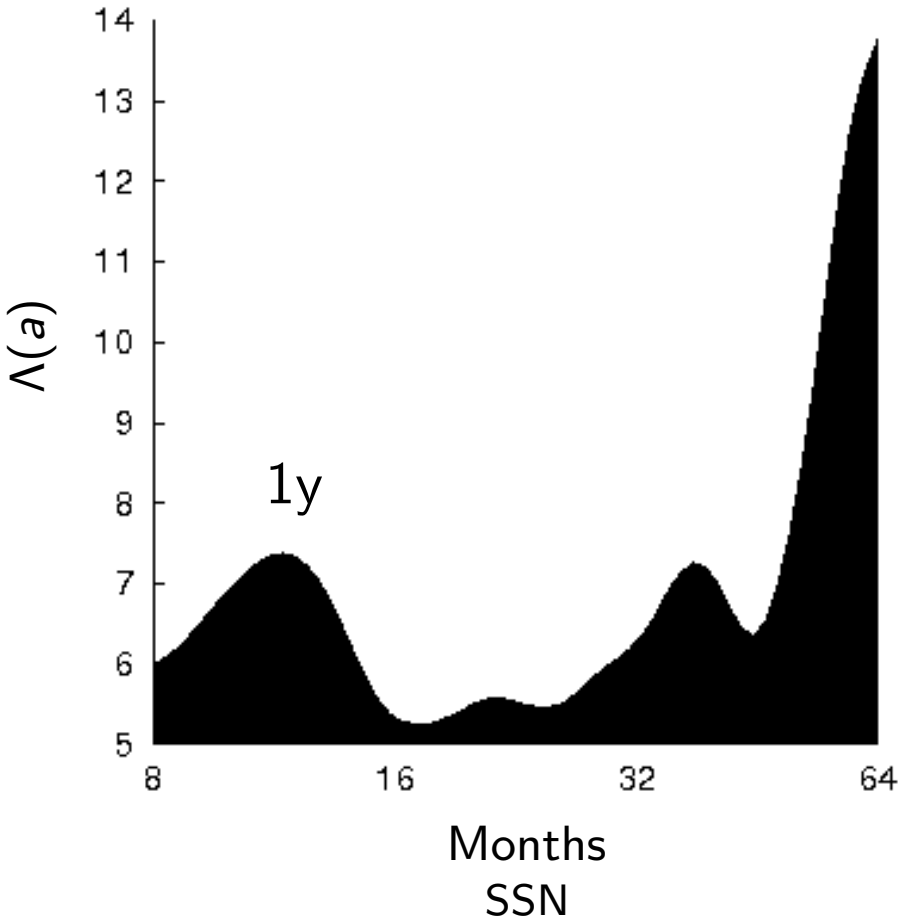
Southern hemisphere

Scale spectrum of the Sunspot Number



1950–2007 monthly-sampled data

Scale spectrum of the Sunspot Number



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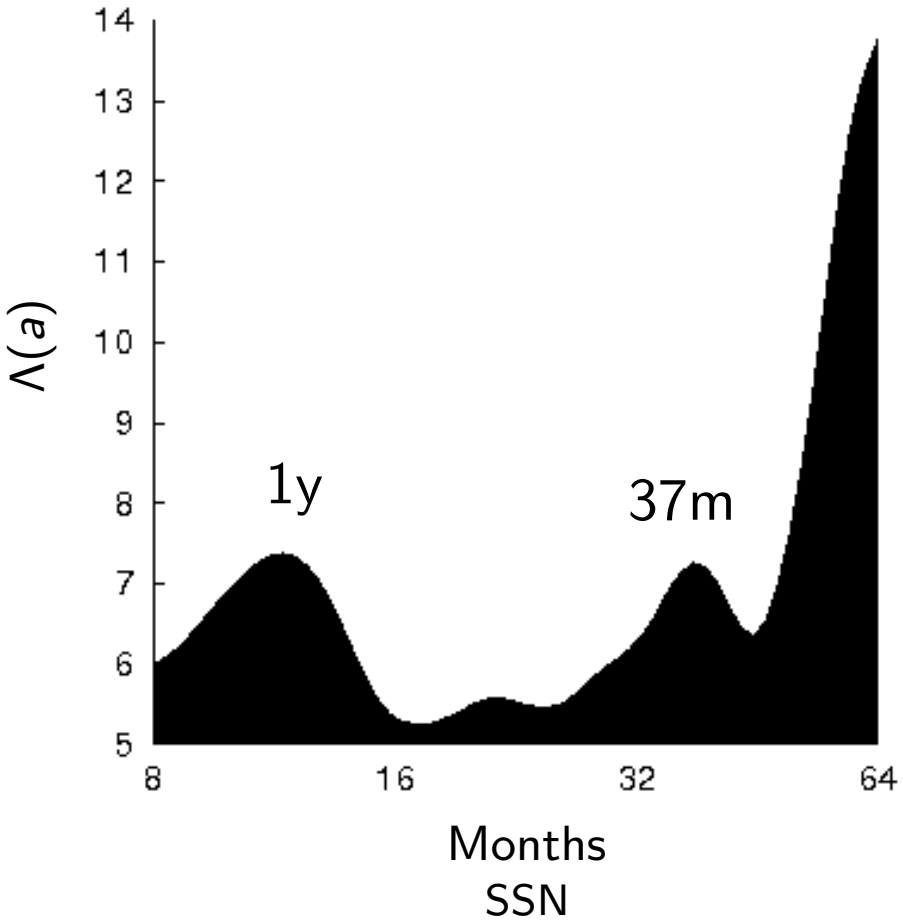
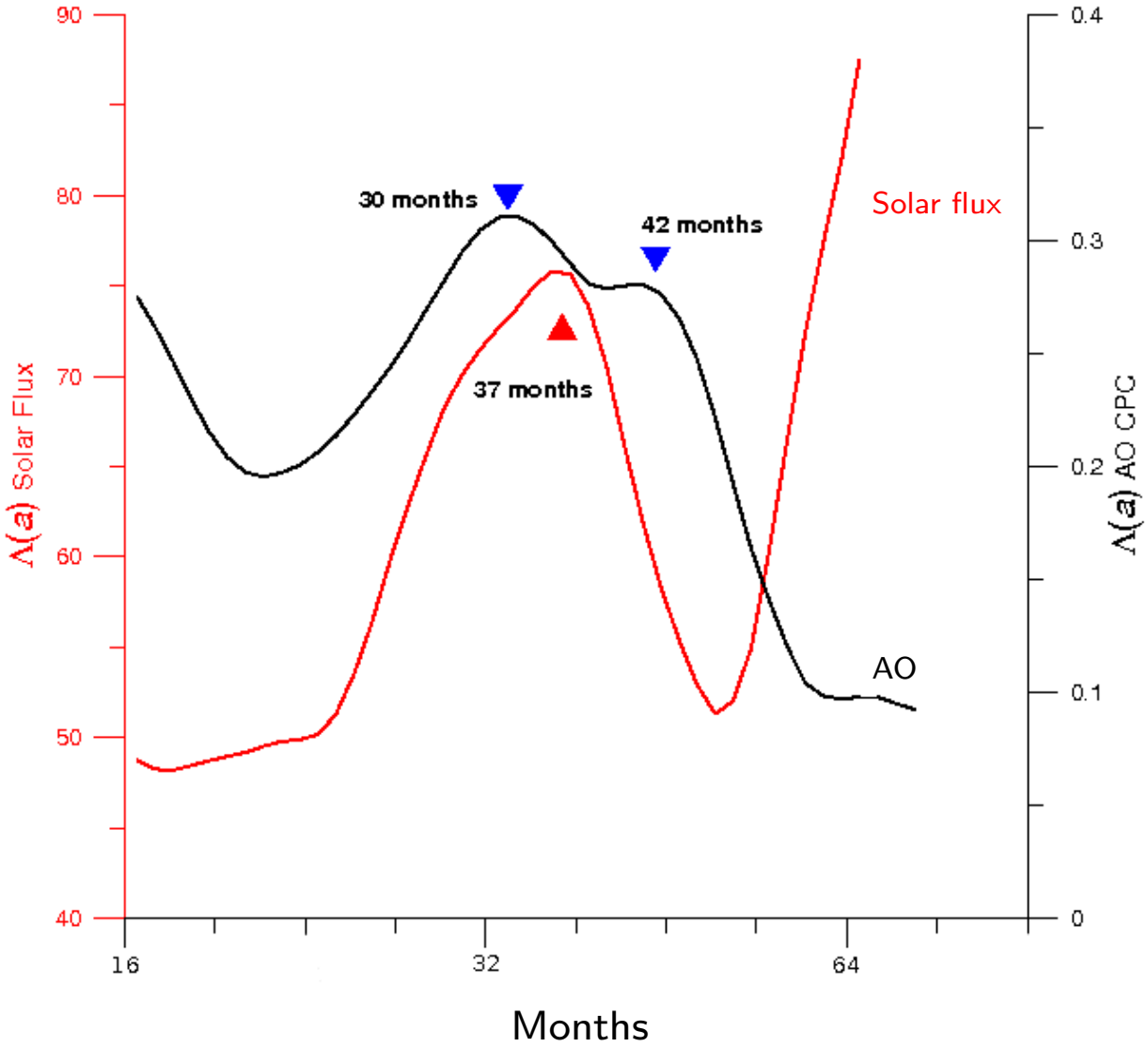
Scale spectrum of the Sunspot Number



1950–2007 monthly-sampled data

A Relation with the Sun?



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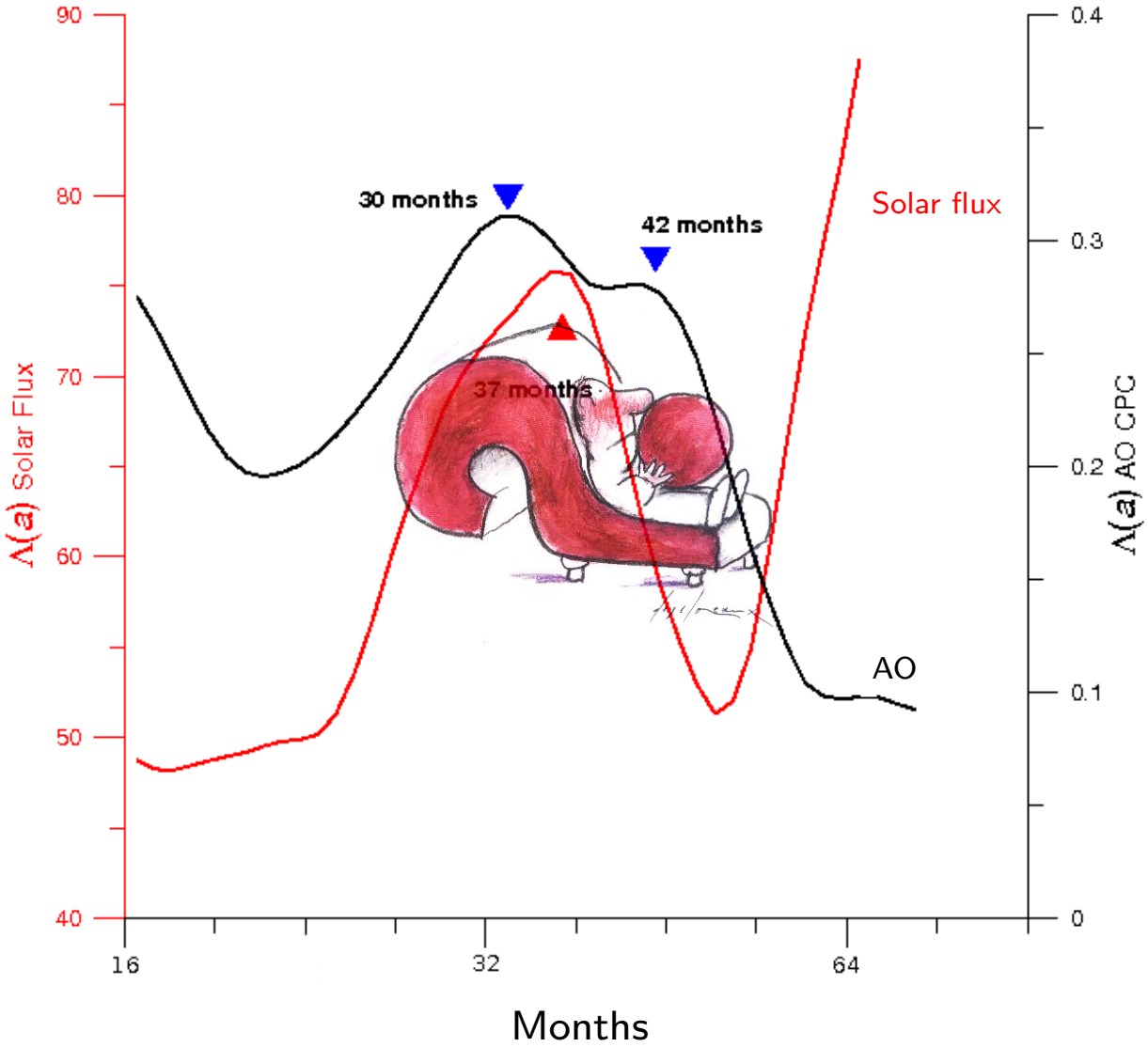

For Further Reading

S. Nicolay, G. Mabille, X. Fettweis and M. Erpicum,
30 and 43 months period cycles found in air temperature time series using the Morlet wavelet method,
Climate Dynamics, doi: 10.1007/s00382-008-0484-5 (2008)

G. Mabille and S. Nicolay,
New cycles found in air temperature data and proxy series,
submitted

http://reflexions.ulg.ac.be/cms/c_5030/mathematiques