New cycles found in air temperature data and proxy series

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The Continuous Wavelet transform

The wavelet analysis provides a two-dimensional unfolding of a one-dimensional signal by decomposing it into scale–time coefficients.

The continuous wavelet transform turns a signal s into a function W

$$W[s](t,a) = \int s(x)\bar{\psi}(\frac{x-t}{a})\,\frac{dx}{a},$$

where $\bar{\psi}$ denotes the complex conjugate of the function ψ, a the scale and t the time.

Conditions on ψ

The function ψ must be integrable, square integrable and satisfy some admissibility condition. Such a function is called a wavelet. The Morlet wavelet is particularly well conditioned for frequency-based study. It satisfies the following equality

$$\hat{\psi}(\omega) = \exp(-\frac{(\omega - \Omega)^2}{2}) - \exp(-\frac{\omega^2 + \Omega^2}{2}),$$

where $\Omega > 5$ is called the central frequency.

Some Properties of the Wavelet Transform

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- the wavelet transform allows to handle noisy data,
- the wavelet transform is blind to polynomial behaviors (up to a degree n, depending on ψ): $W[s+P] = W[s]$, where P is a polynomial of degree $\leq n$.

Consequently, non-zero mean and linear tendencies do not affect the wavelet transform.

The Scale Spectrum

For wavelets such as the Morlet wavelet, we have

$$W[\cos(\omega_0 t)](t, a) = \exp(i\omega_0 t)\hat{\bar{\psi}}(a\omega_0),$$

so that the frequency ω_0 is given by the maximum of $\hat{\bar{\psi}}(a\omega_0)$: $a_\omega = \Omega/\omega_0$. Consequently, the unknown frequency ω_0 can be obtained through the maximum of $|W[\cos(\omega_0 t)]|$.

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Definition

The scale spectrum of a signal s is defined by

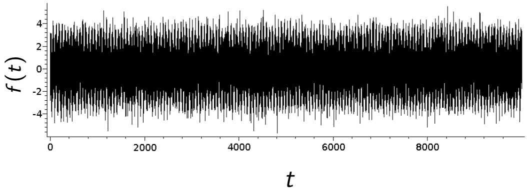
$$\Lambda(a) = E|W[s](t,a)|,$$

where E denotes the mean over the time t.

For What Purpose?

The scale spectrum should be useful for signals which are not stationary but whose characteristics do not evolve too quickly: the scale spectrum allows to recover frequencies even if they "kindly depend on the time", i.e. if $\omega_0 = \omega_0(t)$ with $\frac{d}{dt}\omega_0 \ll 1$, one should be able to recover $E\omega_0$.

A Visual Example



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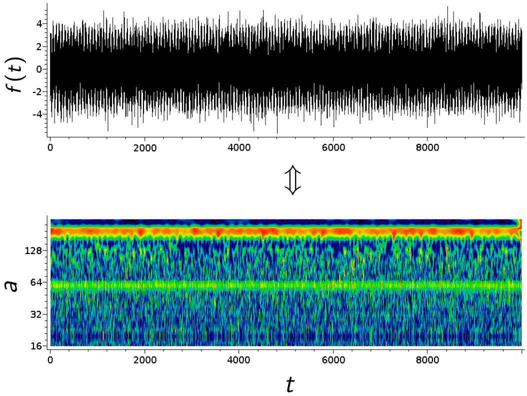

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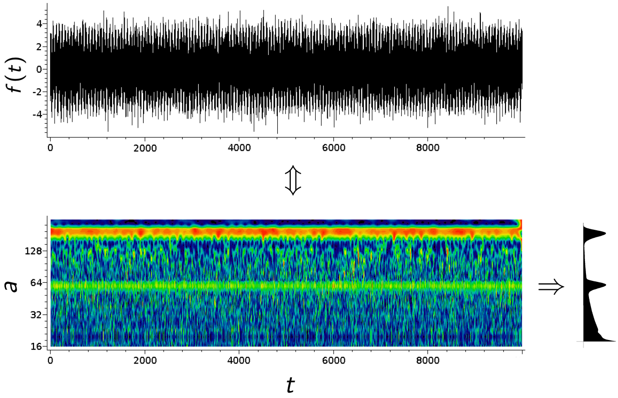
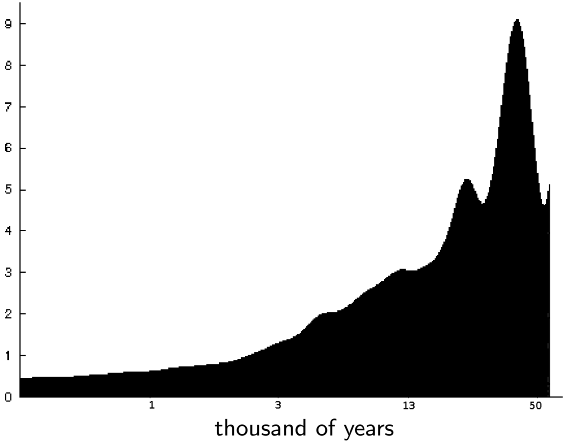


Known long cycles are observed in the spectrum



EPICA dome

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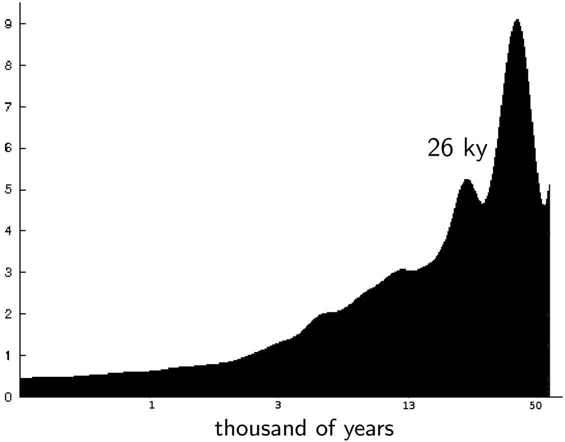

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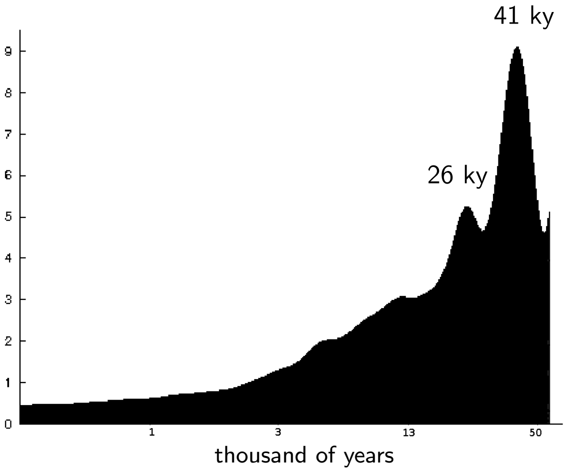

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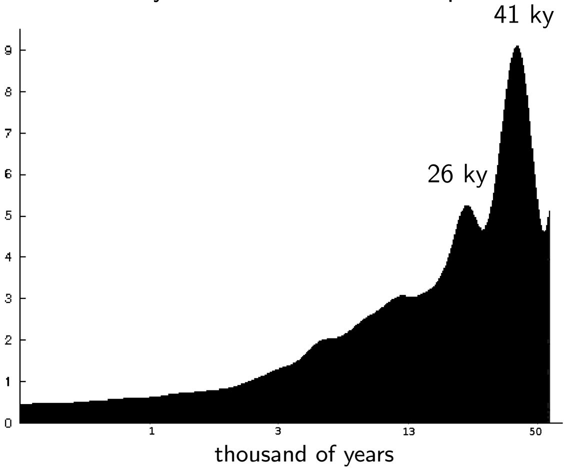
Milankovitch cycles are detected, as expected



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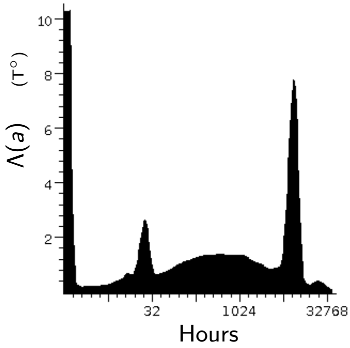
What About the Temperature Data?



Bierset-aero, Belgium
1950–2007 hourly-sampled data

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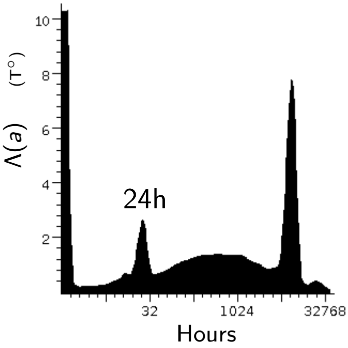

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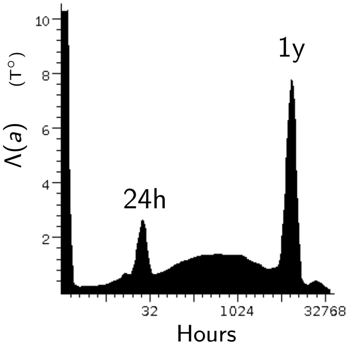

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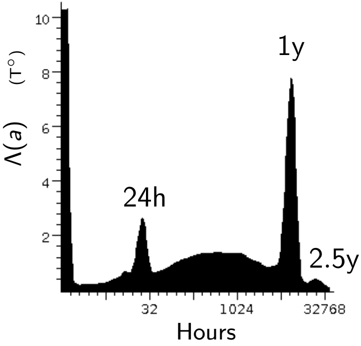

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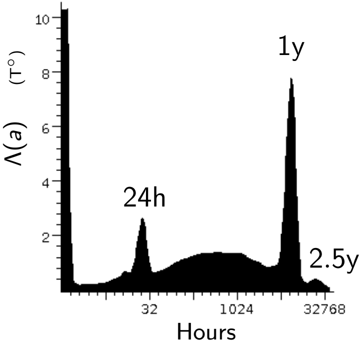

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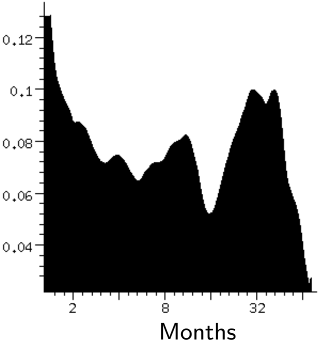

crutempgl3
1950–2007 monthly-sampled data

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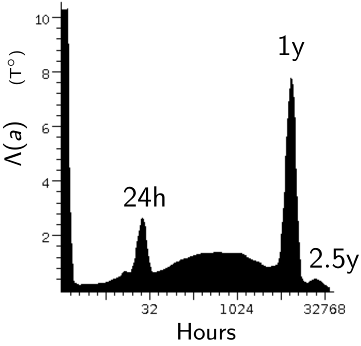

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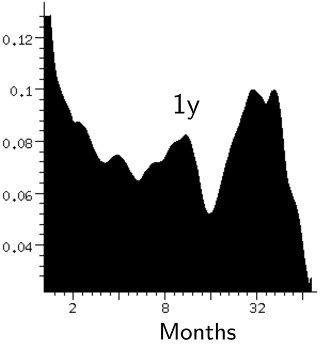

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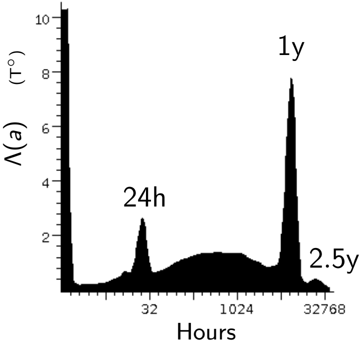

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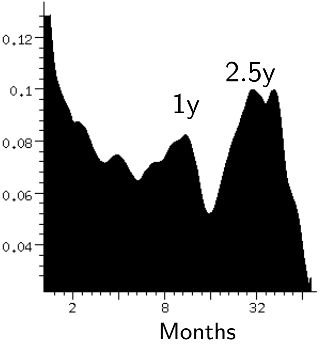

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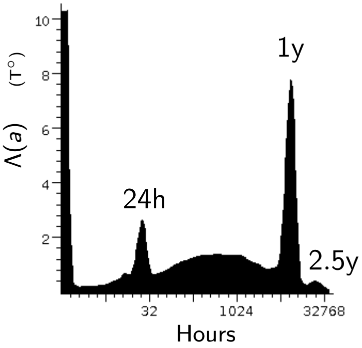

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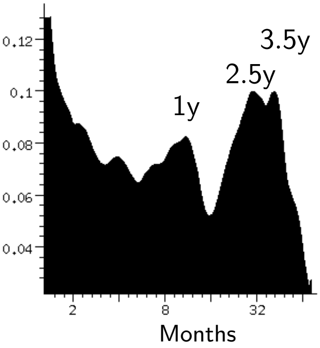

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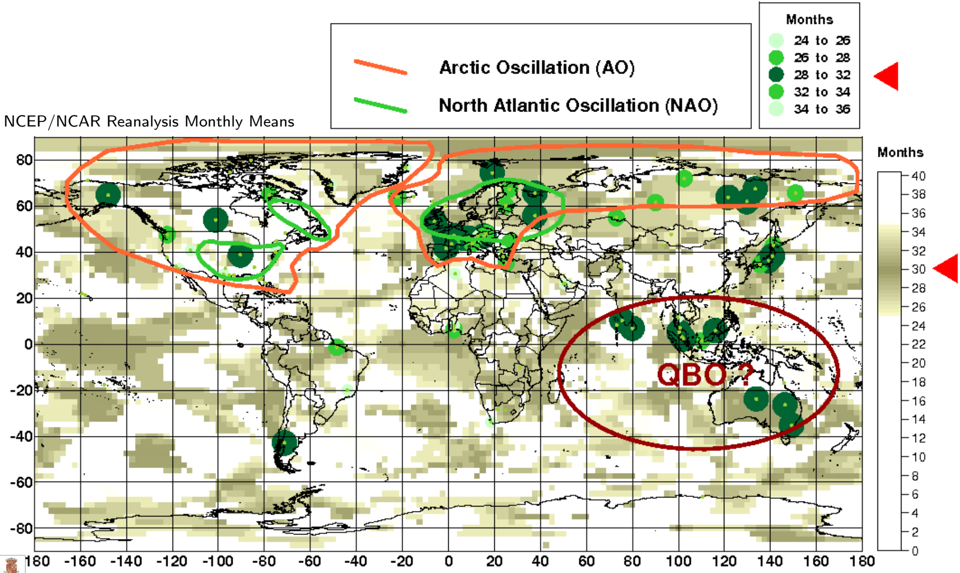
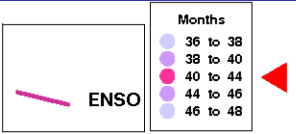
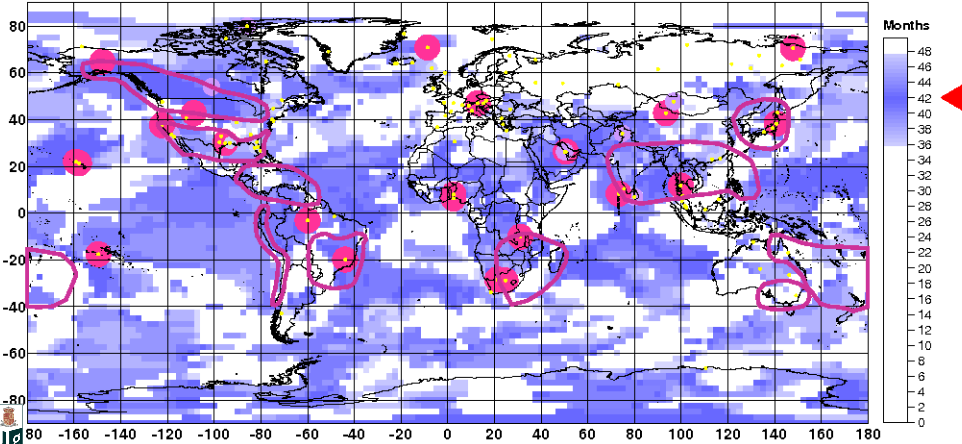
30 Months Cycle



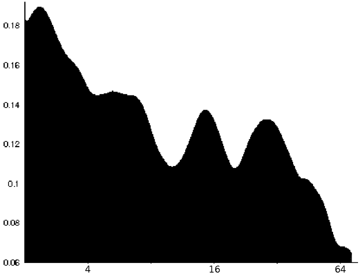
42 Months Cycle



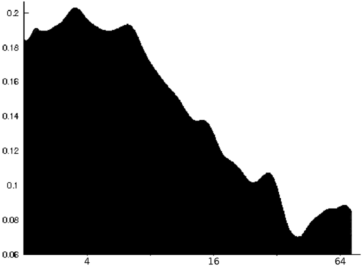
NCEP/NCAR Reanalysis Monthly Means



Millennial Temperature Reconstructions of Jones et al.



Years
Northern hemisphere



Years
Southern hemisphere

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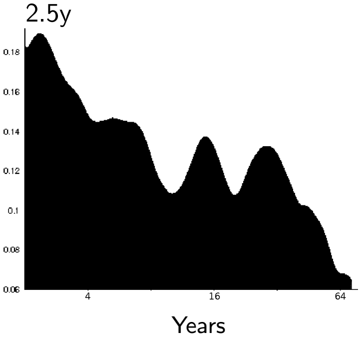

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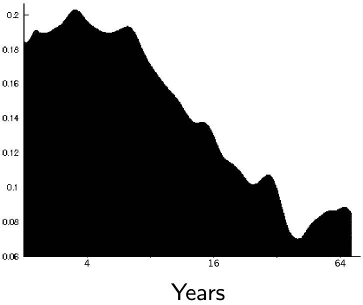

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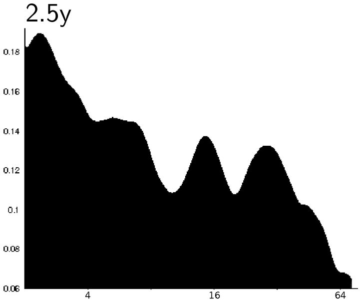

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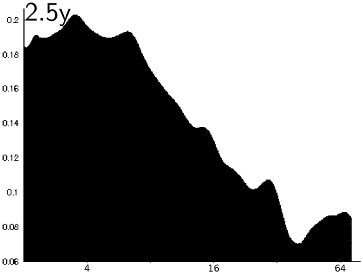

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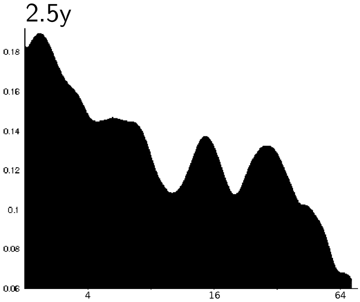

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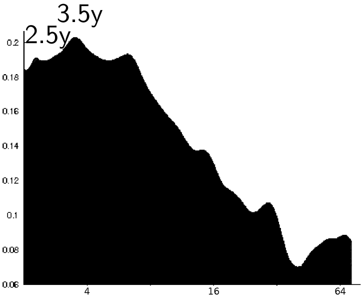

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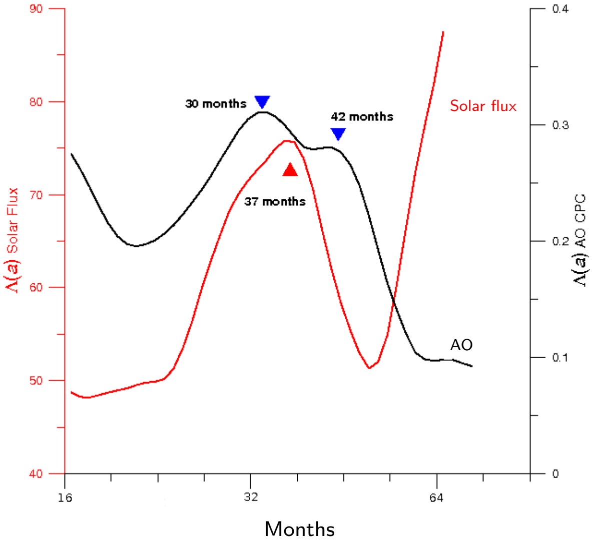
A Relation with the Sun?



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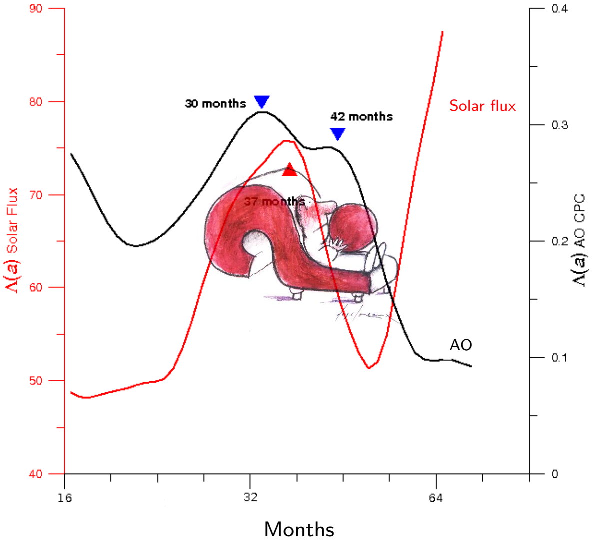

For Further Reading

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