Stable $uudd\bar{s}$ pentaquarks in the constituent quark model

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The stability of strange pentaquarks $uudd\bar{s}$ is studied in a constituent quark model based on a flavor-spin hyperfine interaction between quarks. With this interaction model, which schematically represents the Goldstone boson exchange interaction between constituent quarks, the lowest lying strange pentaquark is a $p-$shell state with positive parity. The flavor-spin interaction lowers the energy of the lowest $p-$shell state below that of the lowest $s-$shell state, which has negative parity because of the negative parity of the strange antiquark. It is found that the strange pentaquark can be stable against strong decay provided that the strange antiquark interacts by a fairly strong spin-spin interaction with u and d quarks. This interaction has a form that corresponds to η meson exchange. Its strength may be inferred from the π^0 decay width of D_s^* mesons.

Renewed interest in the existence of pentaquarks [1,2] has been raised by the recent observation of an $S = 1$ baryon resonance, referred to as Z^+ or $\Theta^+(1540)$, in photoproduction of kaon pairs on neutrons: $\gamma n \to K^+K^-n$ [3]. This resonance has a peak at 1.54 ± 0.01 GeV/c^2 and a width, which is less than 25 MeV/c^2. It has been confirmed in photon deuteron collision experiments [4], in K^+-Xe collisions [5] and most recently in the γp reaction [6]. The latter experiment indicates that the $\Theta^+(1540)$ has isospin $I = 0$. The method for detecting pentaquarks has been discussed in [7]. The $\Theta^+(1540)$ may be interpreted as a strange meson-baryon resonance or as a pentaquark of the form $uudd\bar{s}$.

The expectation has been that stable pentaquarks should be likely to exist in the heavy flavor sectors [1,2,8,9], but experimental searches have remained inconclusive [10,11]. A constituent quark model study of pentaquark states of the form $qqqq\bar{s}$, indicates that such states are unstable against strong decay if the only interaction between the strange antiquark and the light flavor quarks is the confining interaction [12]. The prediction of a narrow strange pentaquark with positive parity at an energy close to that of the empirically found resonance was first made with a chiral soliton model, in which it was

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classified as the lowest state of an SU(3) antidecuplet [13]. Here it is shown that once an attractive spin dependent hyperfine interaction between the light flavor quarks and the strange antiquark is introduced, stable or narrow positive parity strange pentaquarks may also be accomodated within the constituent quark model. A constituent quark model can in fact accomodate the antidecuplet of Ref. [13] plus other SU(3) mutiplets. If one constructs a pentaquark from a q^3 (baryon) and a $q\overline{q}$ (meson) subsystems both being SU(3) flavor octets, one gets

$$(8)_F \times (8)_F = (27)_F + (10)_F + (\overline{10})_F + 2(8)_F + (1)_F \ , \tag{1}$$

thus the antidecuplet appears in this Clebsch-Gordan series.

The originally proposed pentaquarks, which were introduced in the context of the conventional one-gluon exchange model for the hyperfine interaction between constituent quarks, had negative parity, as they represented states with all the light flavor quarks as well as the strange antiquark in their lowest $s-$states. Once the chromomagnetic interaction is replaced by a spin and flavor dependent interaction, with the form, which corresponds to a Goldstone boson exchange (GBE) interaction between quarks, [14,15] the lowest lying pentaquarks will, however, have positive rather than negative parity [9].

The parity of the pentaquark is given by $\mathrm{P} = (-)^{\mathrm{L}+1}$. Here, we take L = 1 and analyze the case where the subsystem of two u and two d quarks is in a state of orbital symmetry $[31]_O$, which thus carries the angular momentum L = 1. Although the kinetic energy of such a state is higher than that of the orbitally symmetric state $[4]_O$, an estimate based on a schematic interaction model [9] shows that the $[31]_O$ symmetry should be the most favourable from the point of view of stability against strong decays. In Ref. [9] the antiquark was assumed to have heavy c or b flavor, and accordingly the interaction between a light quark and the heavy antiquark was neglected, which is justified in the heavy quark limit. As the constituent mass of the strange quark is not much larger than that of the light flavor quarks, that approximation cannot be invoked for strange pentaquarks. Below it is in fact shown that stable low lying strange pentaquarks only appear if an interaction between $\overline{s}$ and the light quarks is included explicitly in the constituent quark model.

We shall employ the following schematic flavor-spin interaction between light quarks [14]:

$$V_\chi \ = \ - \ C_\chi \ \sum_{i\ <\ j}^{4} \lambda_i^F \cdot \lambda_j^F \ \vec{\sigma}_i \cdot \vec{\sigma}_j . \tag{2}$$

Here λ_i^F are Gell-Mann matrices for flavor SU(3), and $\vec{\sigma}_i$ are the Pauli spin matrices. The constant C_χ may be determined from the Δ-N splitting to be $C_\chi \cong 30$ MeV [14]. The interaction (2) is the simplest model for the hyperfine interaction between quarks, which can describe the empirical baryon spectrum in the constituent quark model [14]. It may be interpreted as arising from pion and (mainly) two-pion exchange, or more generally from exchange of the octet of light pseudoscalar mesons ("Goldstone bosons") and vector mesons between the constituent quarks [15,16]. The flavor-spin dependent interaction may also be interpreted as a quark interchange interaction.

The pion decay $D_s^* \to D_s \pi^0$ implies, by $\pi^0 - \eta$ mixing, that η mesons couple to strange quarks and antiquarks [17]. It is then natural to assume that there is an η meson exchange interaction between $\bar{s}$ antiquarks and light flavor quarks. While this interaction does not admit a similar quark interchange interpretation as the interaction (2), it should lead to a spin-dependent interaction between the strange antiquark and the 4 light flavor quarks, which is similar to (2). This may be schematically be represented by the interaction:

$$V_\eta \;=\; V_0 \sum_i^4 \vec{\sigma}_i \cdot \vec{\sigma}_{\bar{s}}. \tag{3}$$

Here V_0 is a constant, which should correspond to the ground state matrix element of the spin-spin part of the η exchange interaction, but which here will be taken to be a phenomenological constant. The total hyperfine interaction is then

$$V = \; V_\chi + V_\eta. \tag{4}$$

The quark model values for the pseudovector coupling constant between light flavor and η mesons and strange constituent quarks and η mesons are

$$f_{\eta qq} = \frac{m_\eta}{2\sqrt{3}f_\eta} g_A^q, \quad f_{\eta ss} = -\frac{m_\eta}{\sqrt{3}f_\eta} g_A^q. \tag{5}$$

These expressions suggest that $f_{\eta qq}$ falls within the range 1.25 – 1.4 and that $f_{\eta ss}$ falls in the range -2.5 and -2.8. Here $f_\eta = 112$ MeV is the η meson decay constant and g_A^q is the axial coupling constant of the quark. The value of the latter is expected to fall within the range 0.75 – 1.0 [18].

The strength of the coupling between η mesons and strange constituent quarks may be derived from the known empirical π^0 decay width of D_s mesons, which is mediated by η mesons. This suggests that $f_{\eta ss} \sim -1.66$ [17].

For pseudoscalar mesons the coupling to antiquarks has the same sign as that of quarks. Because of the negative sign of the coupling of strange quarks to η mesons and the positive sign of the coupling of strange quarks to light flavor constituent quarks, the potential coefficient V_0 is expected to be positive.

An estimate of the η meson exchange contribution to the strength of V_η may be obtained from the expectation value of the radial part of the η meson exchange interaction,

$$V_0(r) = \frac{f_{\eta qq} f_{\eta ss}}{12\pi}\{\frac{e^{-m_\eta r}}{r} - 4\pi\frac{\delta(\vec{r})}{m_\eta^2}\}, \tag{6}$$

in the ground state of a quark-antiquark pair described by a harmonic oscillator wave function

$$\varphi(\vec{r}) = (\frac{\alpha^2}{\pi})^{3/4} e^{-\alpha^2 r^2/2}, \tag{7}$$

where the parameter α may be adjusted to correspond to a realistic wave function model. This gives:

$$\langle V_0 \rangle = m_\eta \frac{f_{\eta qq} f_{\eta ss}}{3\pi\sqrt{\pi}} (\frac{\alpha}{m_\eta})^3 \{\frac{m_\eta^2}{2\alpha^2} - \sqrt{\pi}\frac{m_\eta^3}{4\alpha^3} e^{m_\eta^2/4\alpha^2} \mathrm{erfc}(\frac{m_\eta}{2\alpha}) - 1\}. \tag{8}$$

With the values of the $\eta-$quark couplings above, this expression yields values for $\langle V_0 \rangle$, which are of the same order of magnitude as that of C_χ, when the baryon wavefunctions are compact, with matter radii less than $1/m_\eta$. This condition is fullfilled for example by the model in [20], for which the ground state wavefunction may be approximated by a product of two oscillator functions of the form (7) of the two Jacobi coordinates, with $\alpha \simeq 650$ MeV [19]. With that value, and with $f_{\eta qq} = 1.3$ and $f_{\eta ss} =$ - 1.66, eqn.(8) yields $\langle V_0 \rangle \sim 90$ MeV. This number would be somewhat reduced by the contribution from singlet pseudoscalar exchange mechanisms like η'-meson exchange [20].

For the construction of the wave functions for the pentaquark it is convenient to first consider the light quark q^4 subsystem. For this the Pauli principle allows for the following two totally antisymmetric states with $[31]_O$ symmetry, written in the flavour-spin (FS) coupling scheme [9,21]:

$$|1\rangle \;=\; \left([31]_O[211]_C\left[1^4\right]_{OC}\;;\;[22]_F[22]_S[4]_{FS}\right), \tag{9}$$

$$|2\rangle \;=\; \left([31]_O[211]_C\left[1^4\right]_{OC}\;;\;[31]_F[31]_S[4]_{FS}\right). \tag{10}$$

Asymptotically, a ground state baryon and a meson, into which a pentaquark can split, would give $[3]_O \times [2]_O = [5]_O + [41]_O + [32]_O$. By removing the antiquark, one can make the reduction $[41]_O \to [31]_O \times [1]_O$ or $[32]_O \to [31]_O \times [1]_O$. Thus, the symmetry $[31]_O$ of the light quark subsystem is compatible with an L = 1 asymptotically separated baryon plus meson system.

Each one of these two states, (9) or (10), has to be coupled to the antiquark state. The total angular momentum $\vec{J} = \vec{L} + \vec{S} + \vec{s_{\overline{q}}}$, where $\vec{L}$ and $\vec{S}$ are the angular momentum and spin of the light flavor subsystem respectively and $\vec{s_{\overline{q}}}$ the spin of the antiquark s, takes the values $J = \frac{1}{2}$ or $\frac{3}{2}$. The resulting pentaquark states mix through the quark-antiquark spin-spin interaction (3). Here we study the lowest case, $J = \frac{1}{2}$.

For the stability problem the relevant quantity is

$$\Delta E = \; E(q^4\overline{q}) - \; E(q^3) - \; E(q\overline{q}), \tag{11}$$

where $E(q^4\overline{q})$, $E(q^3)$ and $E(q\overline{q})$ are the masses of the pentaquark, of the ground state baryon and of the meson into which the pentaquark decays, respectively. The multiquark system Hamiltonian used to calculate E is formed of a kinetic energy term, a confining interaction and the hyperfine interaction (4).

Consider first the contribution of (2) only. In the q^4 subsystem the expectation value of (2) is $-28\; C_\chi$ for $|1\rangle$ and $-64/3\; C_\chi$ for $|2\rangle$. Thus $|1\rangle$ is the lowest state. For the ground state q^3 system (the nucleon) the contribution is $-14\; C_\chi$. There is no such interaction in the $q\overline{q}$ pair. For the moment, we assume that the confinement energy roughly cancels out in ΔE. This is a simplifying assumption, which will be abandoned in the more realistic estimate given below. Then, the kinetic energy contribution to ΔE is $\Delta KE = 5/4\, \hbar\omega$ in a harmonic oscillator model. It follows that for the state $|1\rangle$ the GBE contribution is $\Delta V_\chi = -14\; C_\chi$. With $\hbar\omega \approx 250$ MeV, determined from the N(1440) - N splitting [14], this would give

$$\Delta E \;=\; \frac{5}{4}\,\hbar\omega \;-\; 14\; C_\chi \;=\; -\; 107.5\,\mathrm{MeV} \tag{12}$$

i.e. a substantial binding [9]. This is to be contrasted with the negative parity pentaquarks studied in Ref. [22] within the same model, but where the lowest state has the orbital symmetry $[4]_O$ so that one has $\Delta E = 3/4\ \hbar\omega - 2\ C_\chi = 127.5$ MeV, i.e. instability, in agreement with the detailed study made in [22].

The estimate (12) is a consequence of the flavor dependence of the chiral interaction (2). For a specific spin state $[f]_S$, a schematic color-spin interaction of type $V_{c\ m} = - C_{c\ m} \sum \lambda_i^c \cdot \lambda_j^c\ \vec{\sigma}_i \cdot \vec{\sigma}_j$, which may represent the one gluon exchange interaction, does not make a distinction between $[4]_O$ and $[31]_O$. Consequently, the $[31]_O$ state would appear to lie above the state $[4]_O$, because of the kinetic energy term. The flavor-spin interaction (2) overcomes the excess of kinetic contribution in $[31]_O$ and generates a lower expectation value for $[31]_O$ than for $[4]_O$.

Consider now the more realistic model [23], that was used in Ref. [9] to describe the baryon spectrum, where in the coordinate space, instead of a constant, one has a specific radial form for a given meson $\gamma = \pi, K, \eta$ or η'

$$V_\gamma(r) = \frac{g_\gamma^2}{4\pi}\frac{1}{12m_i m_j}\{\theta(r-r_0)\mu_\gamma^2\frac{e^{-\mu_\gamma r}}{r} - \frac{4}{\sqrt{\pi}}\alpha^3\exp(-\alpha^2(r-r_0)^2)\} \tag{13}$$

with the parameters:

$$\frac{g_{\pi q}^2}{4\pi} = \frac{g_{\eta q}^2}{4\pi} = \frac{g_{Kq}^2}{4\pi} = 0.67,\ \frac{g_{\eta' q}^2}{4\pi} = 1.206,\ r_0 = 0.43\ \text{fm},$$
$$\alpha = 2.91\ \text{fm}^{-1}, m_{u,d} = 340\ \text{MeV},\ m_s = 440\ \text{MeV}. \tag{14}$$

We have performed a variational calculation similar to that of Ref. [9] with $\overline{s}$ in the place of $\overline{c}$ or $\overline{b}$. As in the case of heavy antiquarks the quark-antiquark interaction was ignored here. The radial part of the pentaquark wave function is given by Eqs. (15)-(17) of Ref. [9]. For the $uudd\overline{s}$ pentaquark described by the state (9) the expectation value of the total Hamiltonian contains the following contributions: $\langle KE \rangle = 1848$ MeV, $\langle V_{conf} \rangle = 461$ MeV, $\langle V_\chi \rangle =$ - 2059 MeV. In units of $C_\chi = 30$ MeV this means that $\langle V_\chi \rangle =$ -68.3 C_χ, i. e. a much stronger attraction than in the schematic model where $\langle V_\chi \rangle =$ - 28 C_χ. The two variational parameters in the pentaquark wave function take the values $a = 0.11$ GeV^2, similar to the heavy quark limit case, and $b = 0.019$ GeV^2, which is two to three times smaller than in the heavy quark limit, which indicates that the light pentaquark is less compact. In estimation of the threshold energy $E(q^3) + E(q\overline{q})$ we use $E(q^3) = E(N)$ = 969 MeV i.e. the nucleon mass calculated variationally and $E(q\overline{q}) = 793.6$ MeV, i. e. the average mass (M + 3 M*)/4 of the pseudoscalar K-meson mass M = 495 MeV and the vector K-meson mass M* = 893.1 MeV. This gives ΔE= 287 MeV, i. e. the system is unbound.

We now turn to the total hyperfine interaction (4) where V_χ is again described by the schematic model (2). The matrix elements of V_η of (3) are calculated with the five particle wave functions ψ_1 and ψ_2 given in the Appendix and obtained by coupling the antiquark $\overline{s}$ to the q^4 subsystem. The interaction (4) now leads to the following matrix to be diagonalized:

	$\langle\psi_1\vert\psi_1\rangle$	$\langle\psi_2\vert\psi_2\rangle$
$\langle\psi_1\vert\psi_1\rangle$	$-28C_\chi$	$\frac{8}{\sqrt{2}}V_0$
$\langle\psi_2\vert\psi_2\rangle$	$\frac{8}{\sqrt{2}}V_0$	$-\frac{64}{3}C_\chi - 4V_0$

(15)

Note that the contribution of V_η cancels out for the state ψ_1 derived from (9). Taking C_χ = 30 MeV, as mentioned above, the eigenvalues of this matrix become

$$\langle V\rangle = -(740 + 2V_0) \pm [10,000 - 400V_0 + 36V_0^2]^{1/2}. \tag{16}$$

When $V_0 = 0$, the lowest solution gives $\langle V\rangle = \langle V_\chi\rangle$ = - 840 MeV, consistent with Ref. [9].

In Fig. 1 the energy of the lowest solution (16) is plotted as a function of the strength V_0. One can see that for a value of $V_0 = C_\chi$ the energy $E(q^4\overline{q})$ can be lowered by about 130 MeV with respect to the case $V_0 = 0$. This implies a decrease by the same amount in ΔE of (11) and hence a substantial increase in the stability of the system $uudd\overline{s}$. To obtain a negative ΔE one needs $V_0 \approx 50$ MeV, i. e. $V_0 \approx 5/3\ C_\chi$, as one can see from Fig. 1. As inferred from above, this may be sufficient for ensuring stability in a realistic calculation.

The estimate obtained from Eq.(6) above suggests that such strength of the spin-spin interaction between the light flavor quarks and the strange antiquark is quite plausible. While η meson exchange is the most obvious source of such an interaction, other mechanisms as two-kaon exchange and η' exchange should also contribute.

The antidecuplet to which Θ^+ belongs contains two other pure pentaquark states, $uuss\overline{d}$ and $ddss\overline{u}$, located at the other two corners of the weight diagram. In the SU(3) symmetry limit represented by the matrix (15) all antidecuplet states are degenerate. However in a realistic model with broken SU(3) symmetry and radial dependence for the meson exchange as e.g. that of Eq. (13) the degeneracy is lifted. In particular the pure pentaquark states, $uuss\overline{d}$ and $ddss\overline{u}$ acquire larger masses than that of Θ^+ and are less likely to be bound. Their masses become larger due to two effects: 1) the presence of two strange quarks instead of one antiquark, like in Θ^+, and 2) a weaker attraction because the qq short range hyperfine interaction is inversely proportional to the product of the interacting quark masses. The other members of the antidecuplet are of interest in baryon spectroscopy as ordinary baryons may contain significant admixtures of such exotic configurations. This deserves a separate study especially in connection with decay modes, as described in Ref. [13].

The conclusion is that the stable strange pentaquarks with positive parity can be accomodated by the constituent quark model, provided that: 1^0 there is a flavor-spin dependent hyperfine interaction between the 4 light flavor quarks, which is sufficiently strong for reversing the order of the lowest states in the $s-$ and $p-$shells and that 2^0 there is an at least as strong spin-spin interaction between the light flavor and the strange antiquark. Recently the question of whether the hyperfine chromomagnetic interaction between the quarks can lead to a similar inversion of the parity ordering for pentaquarks has been considered in [24]. A possibility for inversion, although weaker than for the flavor-spin interaction was found for the $p-$state $uudd$ multiplet with color spin symmetry $[31]_{CS}$ combined with the $\bar{s}$ antiquark. While the presence of a strongly flavor dependent hyperfine interaction between constituent quarks originally was suggested by phenomenological arguments alone [14], and in particular by the requirement of reversal of normal ordering of the states in the constituent quark model with 3 valence quarks, it has received further indirect support by recent QCD lattice calculations, which show the same reversal of normal ordering for small quark mass values [25].

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Appendix

To calculate the matrix elements of the interaction (3) first one has to couple the antiquark to the subsystem q^4. Then one has to decouple a $q\bar{s}$ pair from the pentaquark system. One can work separately in the orbital, flavor, spin and color spaces. But as the interaction (3) concerns only the spin degree of freedom, the task is quite easy because in the spin space the antiquark is on the same footing with the quarks and the problem reduces to the usual recoupling, via Racah coefficients. The only care must be taken of is the symmetry properties of the states. Here we construct explicitly the flavor-spin part of the wave functions of the pentaquark.

Let us denote by $[f_{q^4}], [f_{q^3}], [f_{q^2}]$ and $[f]$ the partitions corresponding to the q^4, q^3, $q\bar{s}$ and $q^4\bar{s}$ respectively. The corresponding spins are denoted by J_q, j_1, S and J. For the two states (9) and (10) one has $[f_{q^4}] = [22]$, $J_q = 0$ and $[f_{q^4}] = [31]$, $J_q = 1$ respectively. The coupling to the antiquark spin must therefore lead to the only common case $[f] = [32]$, $J = 1/2$. The $q\bar{s}$ pair can have of course $[f_{q^2}] = [2]$, $S = 1$ or $[f_{q^2}] = [11]$, $S = 0$. Then the spin part of the wave function of the pentaquark reads

$$[\chi_{J_q}^{[f_q 4]} \chi_{1/2}^{[1]}]\,{}_{JM}^{[f]} = \sum_S [(2S+1)(2J_q+1)]^{1/2} W(j_1 \frac{1}{2} J \frac{1}{2}; J_q S)\; [\chi_{j_1}^{[f_q 3]} \chi_S^{[f_q 2]}]\,{}_{JM}^{[f]}. \quad (17)$$

In the recoupling one has however to keep track of the flavor-spin symmetry of the subsystem of 4 identical quark. This part of the wave function is symmetric, both in (9) and (10). The flavor part of the wave function of q^4 should be specified but the recoupling with the antiquark does not have to be explicit, inasmuch as the interaction (3) is flavor independent. However the coupling to the antiquark must give the same quantum numbers $(\lambda\mu) = (02)$ in the flavor space, in both cases, otherwise the scalar product cancels. The SU(3) irreducible representation $(\lambda\mu) = (02)$ corresponds to the antidecuplet of ref. [13]. It allows $Y = 2$ as the hypercharge of the $uudd\bar{s}$ system.

The two independent pentaquark flavor states associated with (9) are

$$\phi_1 = (\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array} \times \phi_{\overline{s}})^{(02)}, \tag{18}$$

$$\phi_2 = (\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array} \times \phi_{\overline{s}})^{(02)}, \tag{19}$$

where $\phi_{\overline{s}}$ is the flavor antiquark state. Replacing the corresponding Racah coefficients in the relation (17) the flavor-spin wave function of the pentaquark becomes

$$\begin{aligned} |\psi_1\rangle = |[22][1];[32]\rangle_{1/2M}\rangle \;\; = \;\; & \frac{1}{\sqrt{2}}\{\phi_1 \; [-\frac{1}{2}[\chi^{[21]}_{1/2}\chi^{[11]}_0]^{\;[32]}_{\;1/2M} + \frac{\sqrt{3}}{2}[\chi^{[21]}_{1/2}\chi^{[2]}_1]^{\;[32]}_{\;1/2M} \;] \\ & +\phi_2 \; [-\frac{1}{2}[\chi^{[21]}_{1/2}\chi^{[2]}_1]^{\;[32]}_{\;1/2M} + \frac{\sqrt{3}}{2}[\chi^{[21]}_{1/2}\chi^{[2]}_1]^{\;[32]}_{\;1/2M} \;]\}, \end{aligned} \tag{20}$$

where in each row $\chi^{[21]}_{1/2}$ is associated with a different Young tableau.

The flavor-spin pentaquark state constructed from (10) contains the following three independent flavor states

$$\phi_3 = (\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & & \\ \cline{1-1} \end{array} \times \phi_{\overline{s}})^{(02)}, \tag{21}$$

$$\phi_4 = (\begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 & & \\ \cline{1-1} \end{array} \times \phi_{\overline{s}})^{(02)}, \tag{22}$$

$$\phi_5 = (\begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 & & \\ \cline{1-1} \end{array} \times \phi_{\overline{s}})^{(02)}. \tag{23}$$

Then using these states and the recoupling (17) with corresponding Racah coefficients we obtain the pentaquark flavor-spin state

$$\begin{aligned} |\psi_2\rangle = |[31][1];[32]\rangle_{1/2M} \;\; = \;\; & \frac{1}{\sqrt{3}}\{\phi_3 \; [\chi^{[3]}_{3/2}\chi^{[2]}_1]^{\;[32]}_{\;1/2M} \\ & +\phi_4[\;\frac{1}{2}[\chi^{[21]}_{1/2}\chi^{[2]}_1]^{\;[32]}_{\;1/2M} + \frac{\sqrt{3}}{2}[\chi^{[21]}_{1/2}\chi^{[11]}_0]^{\;[32]}_{\;1/2M}] \\ & +\phi_5[\frac{1}{2}[\chi^{[21]}_{1/2}\chi^{[2]}_1]^{\;[32]}_{\;1/2M} + \frac{\sqrt{3}}{2}[\chi^{[21]}_{1/2}\chi^{[11]}_0]^{\;[32]}_{\;1/2M}] \;\}. \end{aligned} \tag{24}$$

Again, in each row the function $\chi^{[21]}_{1/2}$ has a distinct Young tableau. The explicit form of q^3 and q^4 flavor or spin states associated with every Young tableau above can be found for example in Ref. [21].

The wave functions $|\psi_1\rangle$ and $|\psi_2\rangle$ are used to calculate the matrix elements of the interaction (3).

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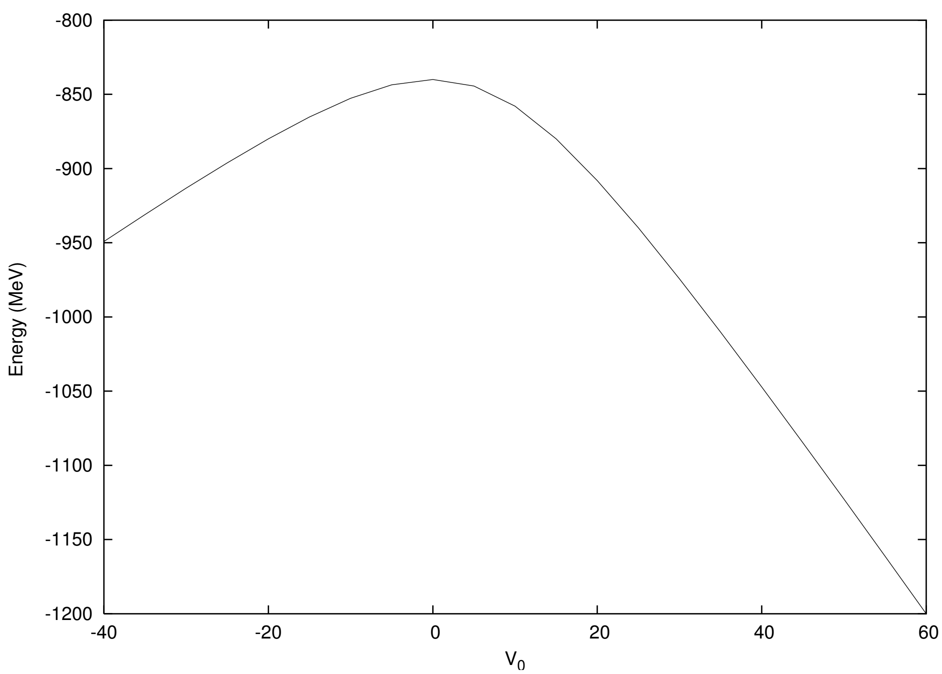

Figure 1. The lowest solution of (16) as a function of the parameter V_0.