Experimental analysis of 2:1 modal interactions with noncommensurate linear frequencies in an aerospace structure

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Summary. Nonlinear interactions between modes with noncommensurate linear frequencies are studied. It is experimentally evidenced that a strongly nonlinear, full-scale aerospace structure may exhibit such 2:1 interactions in typical testing conditions. The experimental observations are compared with numerical predictions.

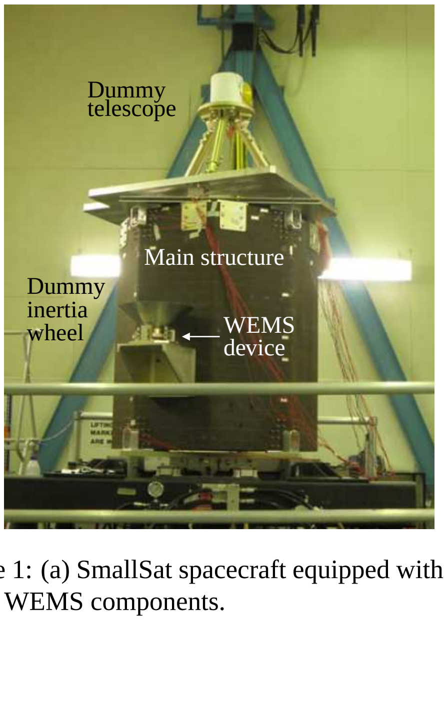
Introduction

The possibility of interactions between modes with well-separated frequencies is a salient feature of the theory of nonlinear oscillations. These interactions have been extensively studied in the technical literature [1, 2, 3], but they have been so far reported most frequently in the case of systems with low dimensionality. The objective of the present contribution is precisely to evidence that nonlinear modal interactions may also be observed in experimental conditions commonly endured by engineering structures in industry. The application of interest is the SmallSat spacecraft, a full-scale satellite structure developed by EADS-Astrium. Different sine-sweep measurements collected during a classical qualification test campaign are analysed. It is found that – potentially dangerous – 2:1 interactions between modes with noncommensurate linear frequencies are possible due to the frequency-energy dependence of nonlinear dynamics. The experimental observations are compared with numerical predictions obtained by applying a continuation algorithm to a finite element model of the satellite structure.

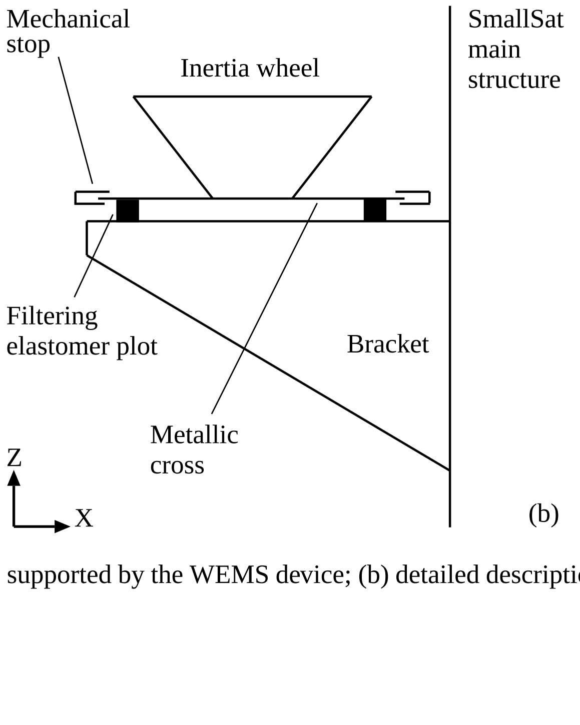
The SmallSat spacecraft structure

The SmallSat structure was conceived by EADS-Astrium as a low-cost platform for small satellites in low earth orbits [4]. It is a monocoque tube structure which is 1.2 m in height and 1 m in width. It is composed of eight flat faces for equipment mounting purposes, creating an octagon shape, as shown in Fig. 1 (a). The top floor is an 1-m^2 sandwich aluminium panel. The interface between the spacecraft and the launch vehicle is achieved via four aluminium brackets located around cut-outs at the base of the structure. The total mass including the interface brackets is around 64 kg.

The spacecraft structure supports a dummy telescope mounted on a baseplate through a tripod; its mass is around 140 kg. Besides, as depicted in Fig. 1 (b), a bracket connects to one of the eight walls the so-called wheel elastomer mounting system (WEMS) device which is loaded with an 8-kg dummy inertia wheel. The WEMS device acts as a mechanical filter which mitigates high-frequency disturbances coming from the inertia wheel through the presence of a soft elastomeric interface between its mobile part, *i.e.* the inertia wheel and a supporting metallic cross, and its fixed part, *i.e.* the bracket and by extension the spacecraft. Moreover, the WEMS incorporates eight mechanical stops designed to limit the axial and lateral motions of the inertia wheel during launch, which gives rise to strongly nonlinear dynamic phenomena.



(a)



(b)

Figure 1: (a) SmallSat spacecraft equipped with an inertia wheel supported by the WEMS device; (b) detailed description of the WEMS components.

Experimental evidence of nonlinear modal interactions using the wavelet transform

To reveal nonlinear modal interactions in the SmallSat dynamics, the wavelet transform amplitude [5] of a vertical (Z) acceleration signal measured on the WEMS device at $1g$ level is depicted in Fig. 2 over 5 – 35 Hz. The excitation frequency is clearly visible throughout the wavelet, but higher harmonic components of at least comparable amplitude can also be noticed. In particular, a significant level of response around 60 Hz is pointed out in Fig. 2 for sweep frequencies just below 30 Hz. This corresponds to a 2:1 interaction between two internally resonant modes of the structure, namely mode 3 involving an out-of-phase motion of the inertia wheel and the WEMS bracket, and mode 7 consisting in an axial motion of the telescope supporting panel. The existence of this interaction is confirmed in Fig. 3 (a), where the raw acceleration signal measured at the centre of the instrument panel is plotted at 0.1 g (in blue) and 1 g (in black). A high-amplitude response at 1 g is observed between 20 and 30 Hz, which can be confidently attributed to a nonlinear resonance as no linear mode of the panel is located in this interval. One also remarks the presence of two resonances predicted by linear modal analysis around 46 and 56 Hz. At the 0.1 g level for which the satellite behaves linearly, there is no sign of the 2:1 modal interaction in 20 – 30 Hz, proving that it is an inherently nonlinear phenomenon activated for sufficiently large energies.

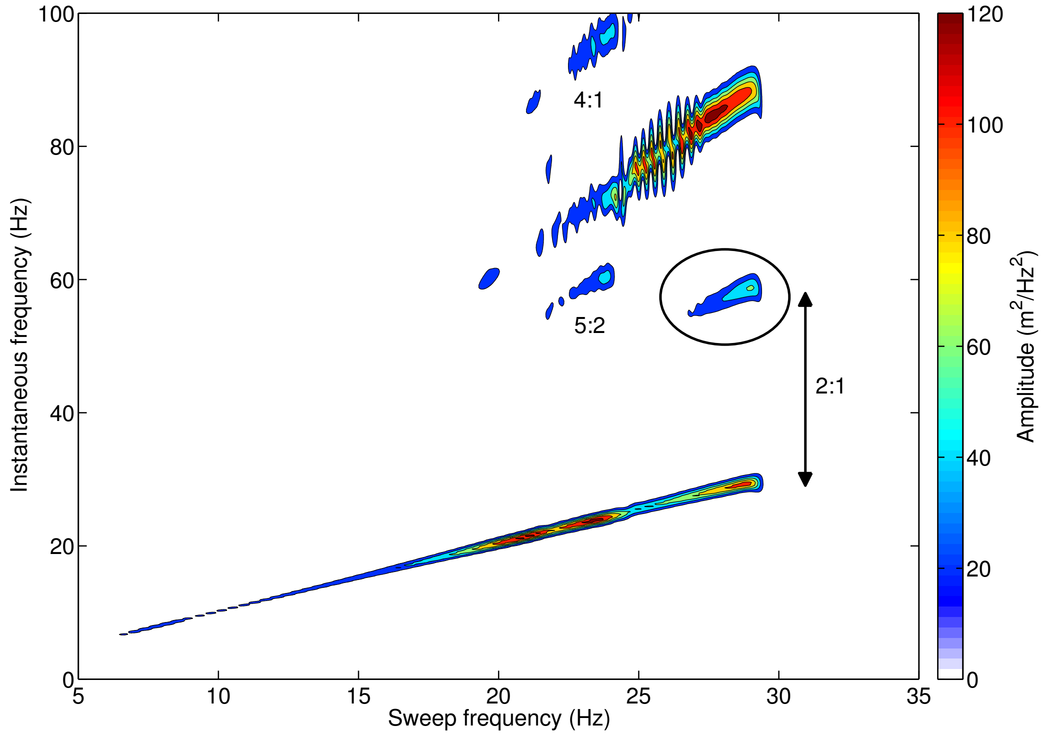
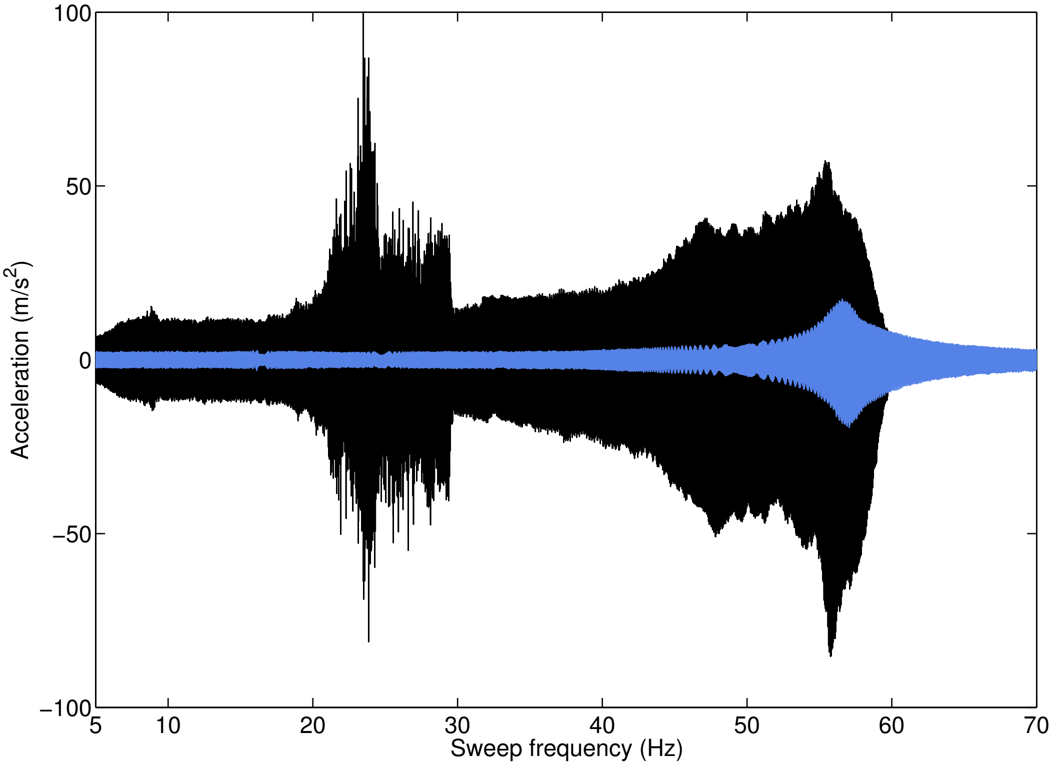


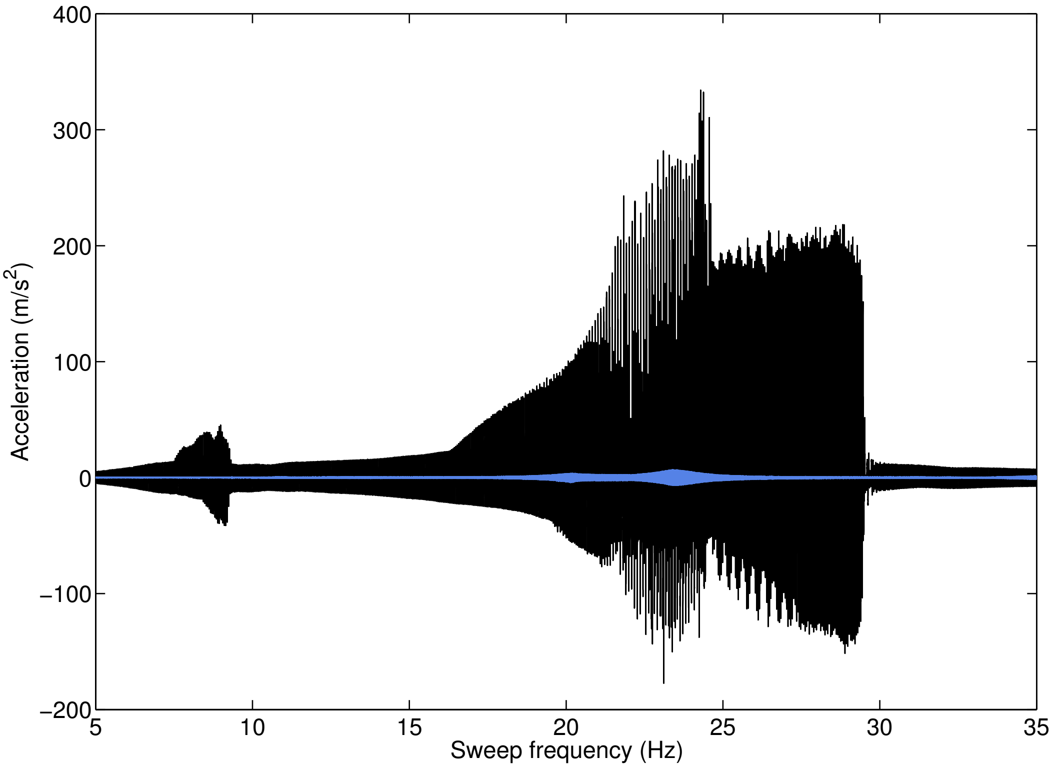
Figure 2: Experimental evidence of nonlinear modal interactions using the wavelet transform amplitude of a vertical (Z) acceleration signal measured on the WEMS device at 1 g level over 5 – 35 Hz. A 2:1 interaction between modes 3 and 7 is encircled.

Unlike what is frequently discussed in the literature [1], the ratio of the linear natural frequencies of modes 3 and 7 involved in the nonlinear interaction is not an integer, but is equal to 2.48. However, the frequency of nonlinear modes may vary according to the excitation level. This is clearly visible in Fig. 3 (b) where the raw acceleration signal corresponding to Fig. 2 is plotted at 0.1 g (in blue) and 1 g (in black). At 0.1 g, the linear resonance frequency of mode 3 can be located around 23 Hz, while it is shifted up to 29 Hz at 1 g. This means that a 2:1 ratio between the frequencies of modes 3 and 7 can still be realised due to the energy dependence of nonlinear dynamics. Indeed, the frequency of mode 3 increases rapidly as soon as nonlinearity is activated, whereas the frequency of mode 7 remains unchanged as it involves no WEMS motion. This is therefore the experimental evidence of a 2:1 interaction between modes with noncommensurate linear frequencies.

It should also be stressed that this 2:1 modal interaction may jeopardise the integrity of the structure as it is accompanied by an energy transfer from a local mode of the spacecraft with low effective mass, *i.e.* mode 3, to a global mode with high effective mass, *i.e.* mode 7. In addition, the time series at 1 g in Fig. 3 (a) shows that the nonlinear resonance of the instrument panel is associated with larger accelerations (around 100 m/s^2) than the linear resonance of the panel (around 80 m/s^2). Furthermore, the 2:1 interaction is not an isolated phenomenon as other internal resonances, such as



(a)



(b)

Figure 3: (a) Confirmation of the existence of the 2:1 modal interaction encircled in Fig. 2 through the raw acceleration signal measured at the centre of the instrument panel at 0.1 g (blue) and 1 g (black); (b) shift of the resonance frequency of mode 3 between 0.1 g (blue) and 1 g (black).

5:2 or 4:1 interactions, can also be observed in Fig. 2. Fig. 4 eventually shows that an additional 2:1 modal interaction can be evidenced by analysing a lateral (X) acceleration signal at the WEMS device using the wavelet transform. This modal interaction is interesting because, at this sensor, the only visible frequency component is 45 Hz despite the fact that the excitation frequency is 22.5 Hz. The present discussion implies that important, and potentially dangerous, dynamic phenomena can be missed when ignoring nonlinearity.

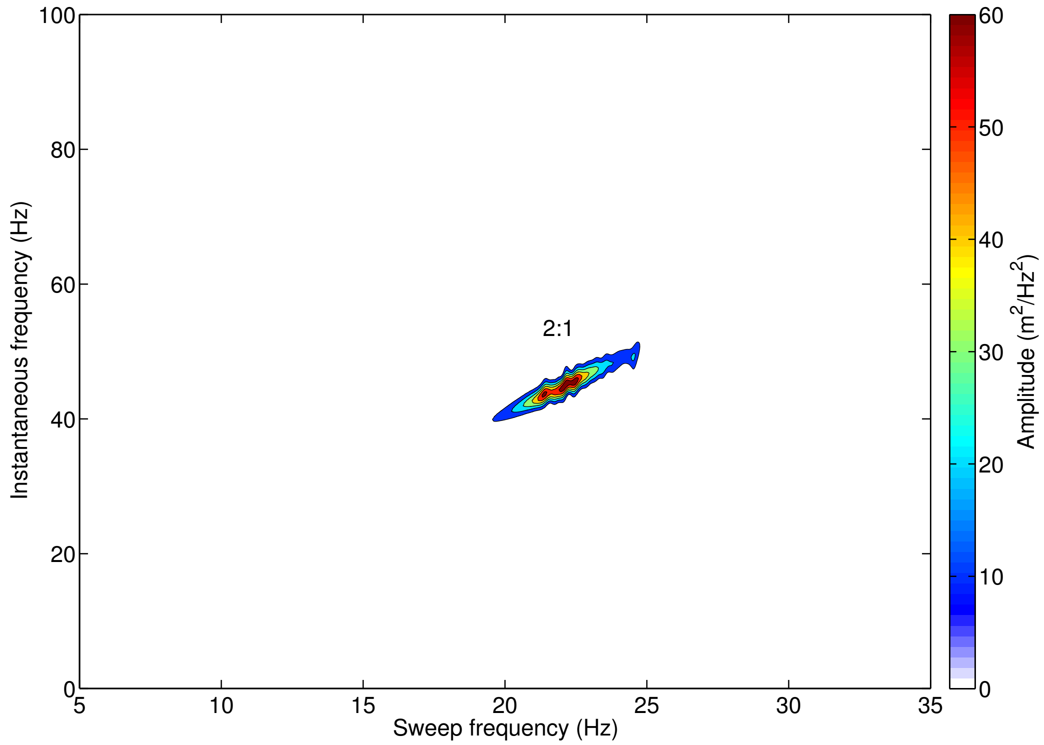


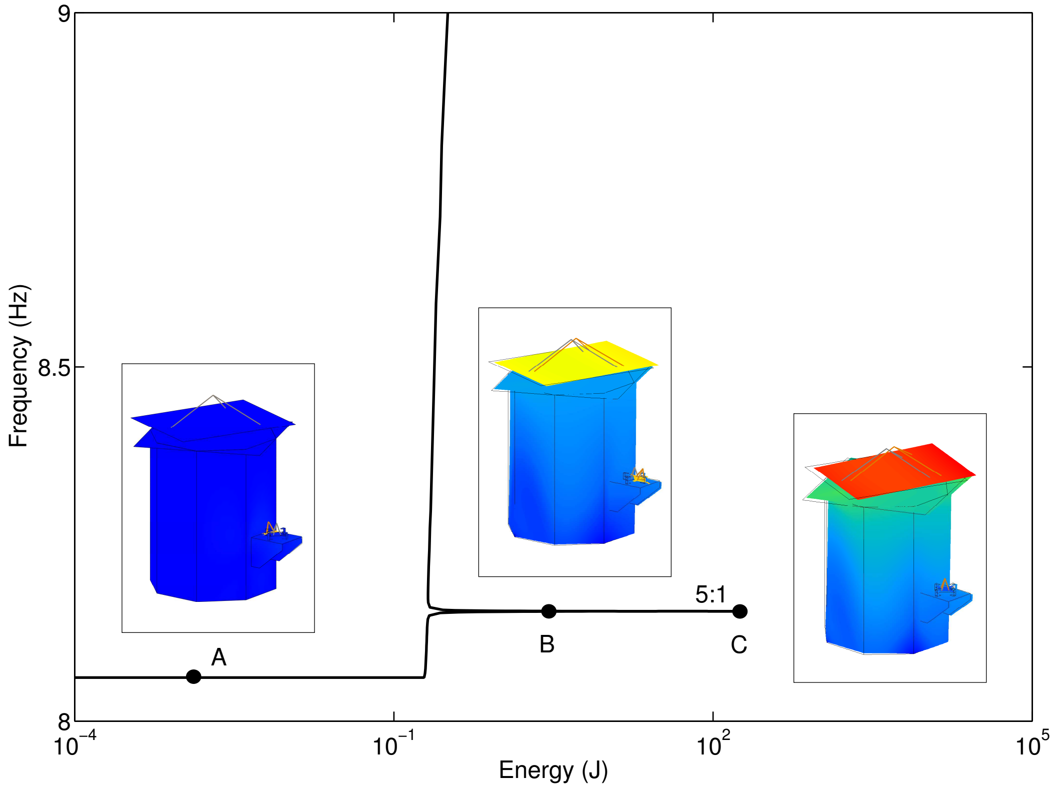
Figure 4: Evidence of another 2:1 nonlinear modal interaction using the wavelet transform amplitude of a lateral (X) acceleration signal measured on the WEMS device at 1 g level over 5 – 35 Hz.

Numerical confirmation of nonlinear modal interactions using numerical continuation

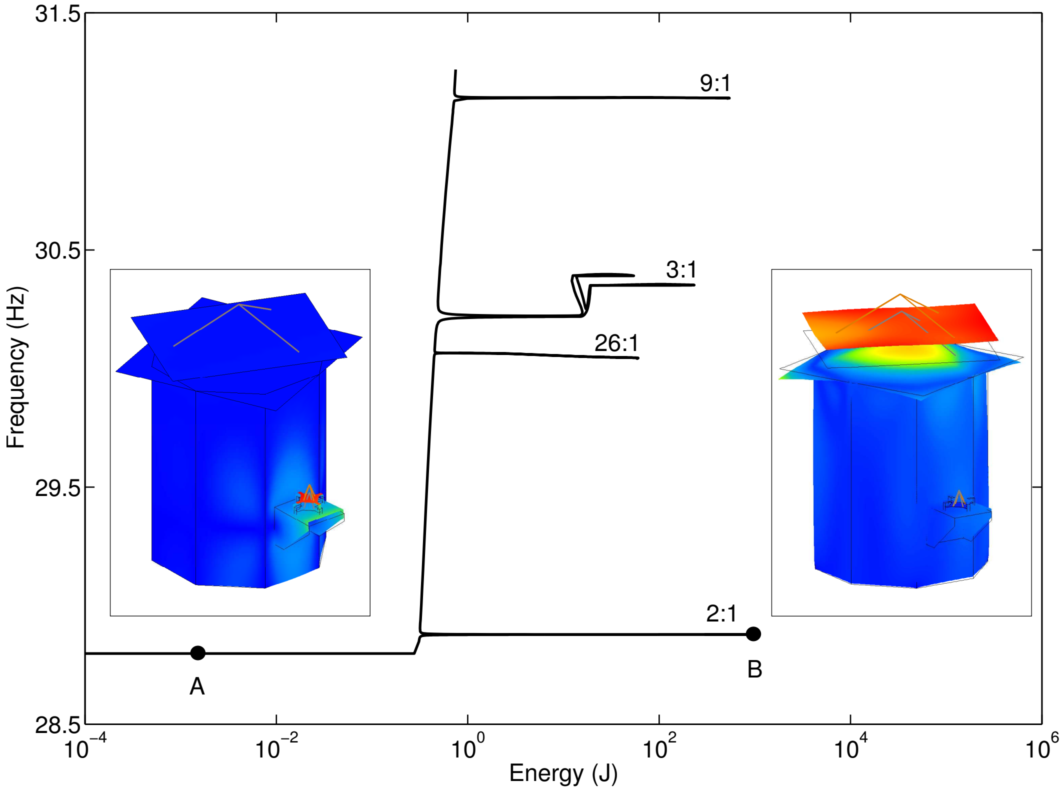
The existence of modal interactions in a nonlinear system can be reliably predicted based on the analysis of its nonlinear normal modes (NNMs). Following Rosenberg's extended definition [3], NNMs are rigorously defined as nonnecessarily synchronous, periodic motions of the unforced, conservative system. In this section, the algorithm proposed in Ref. [6] is exploited to calculate the NNMs of the SmallSat satellite. To this end, a finite element model of the structure was developed. A detailed description of this model can be found in Ref. [7]. The frequencies associated with NNMs may vary with the amplitude of excitation, and for this reason, NNMs are usually depicted in a so-called frequency-energy plot (FEP). A mode in a FEP is represented by a point at a frequency corresponding to the minimal period of the periodic motion, and at an energy equal to the total conserved energy accompanying the motion. A branch in a FEP details the complete frequency-energy dependence of the considered mode.

The FEP of the first NNM of the spacecraft is illustrated in Fig. 5 (a). The plot is formed by a main backbone to which a "tongue" is attached. For low energies, the frequency of the NNM remains constant since no mechanical stop is activated. The corresponding modal shape at point A is identical to the first normal mode of the underlying linear structure. Beyond an energy threshold of about 0.2 J, multiple mechanical stops are impacted. The NNM frequency then rapidly increases because of the important difference between the stiffnesses of the elastomer plots and the mechanical stops.

When progressing along the nonlinear backbone curve, harmonic components of the fundamental frequency are generated. These harmonics may have a frequency close to the oscillation frequency of another NMM of the system. In this situation, a dynamic coupling between the two specific modes is established together with an energy transfer, leading to the appearance of a tongue of internal resonance. This phenomenon precisely results in Fig. 5 (a) in a 5:1 interaction. As energy increases along the branch of resonance, the fifth harmonic progressively becomes more important than the fundamental frequency. The modal shape depicted at point B has no linear counterpart, and is a mixing between NNM 1 and NNM 5. At the extremity of the tongue at point C, only the fifth harmonics remains, and the transition to NNM 5 is completed. Interactions between NNMs with similar topologies were previously reported in technical literature, *e.g.*,



(a)



(b)

Figure 5: (a) FEP of NNM 1 with different modal shapes illustrating the mechanism of an internal resonance; (b) FEP of NNM 3 showing a 2:1 internal resonance confirming the experimental observation in Fig. 2.

in Refs. [3, 8]. They were also observed in the case of a two-DOF vibro-impact system [9], and in a full-scale aircraft [10].

Fig. 5 (b) presents the FEP of the third NNM of the SmallSat. This is an interesting mode because it was proved experimentally to exhibit a 2:1 modal interaction in the previous section. Similarly to NNM 1 in Fig. 5 (a), the frequency of NNM 3 is constant at low energy because of the absence of impacts. Accordingly, the modal shape drawn at point A in Fig. 5 (b) is identical to the corresponding linear normal mode of the system. Beyond a certain energy threshold, mechanical stops are activated, and the NNM frequency severely increases due to the nonsmooth nature of the WEMS nonlinearities. Tongues of internal resonances are also created, during which one harmonic component of mode 3 excites another mode of the structure. More specifically, a 2:1 interaction can be distinguished. The modal shape depicted at its right end (point B) shows that a transition from a local mode involving the WEMS device to a more global mode of the instrument panel, located between 57 and 58 Hz, takes place. The excitation of this higher-frequency mode is possible thanks to the second harmonics generated by the nonlinear behaviour of mode 3. This is therefore the clear numerical confirmation of the experimental observation in Fig. 2. Note that other modal interactions, namely 3:1, 9:1 and 26:1, are predicted by the continuation algorithm but are not further investigated in the present paper. We refer the interested reader to Ref. [7] for a more in-depth analysis. The numerical reproduction of the 2:1 internal resonance briefly described in Fig. 4 is also achieved therein.

Conclusion

The objective of this paper was to demonstrate that the complex dynamics that can be obtained during numerical simulations of nonlinear systems with low dimensionality can also be observed in experimental conditions commonly endured by engineering structures in industry. In particular, the existence of 2:1 nonlinear interactions between modes with noncommensurate linear frequencies was revealed in the dynamics of a full-scale spacecraft structure by utilising the wavelet transform. A specific 2:1 interaction was also successfully correlated with the predictions obtained by applying a continuation algorithm to a finite element model of the spacecraft.

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